中国科学技术大学
University of Science and Technology of China

《人工智能数学原理与算法》

第2章：机器学习基础

2.2 线性代数基础

冯福利
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目录

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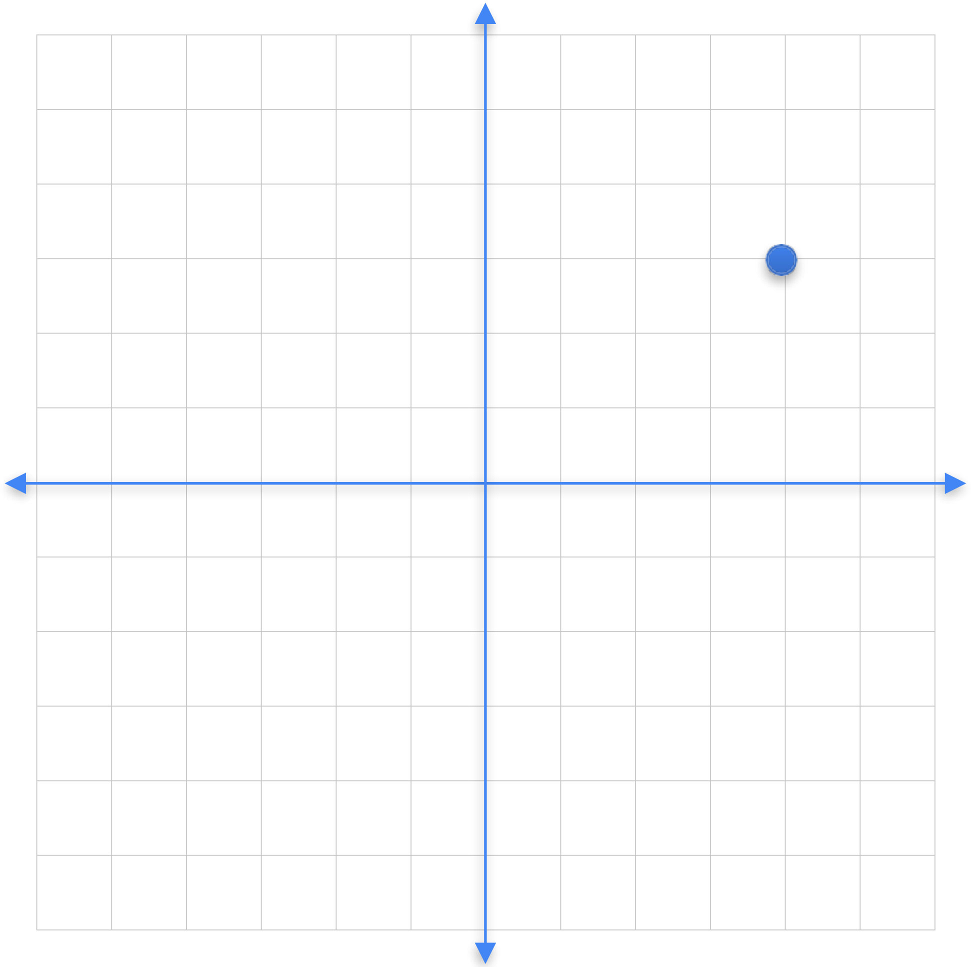
向量



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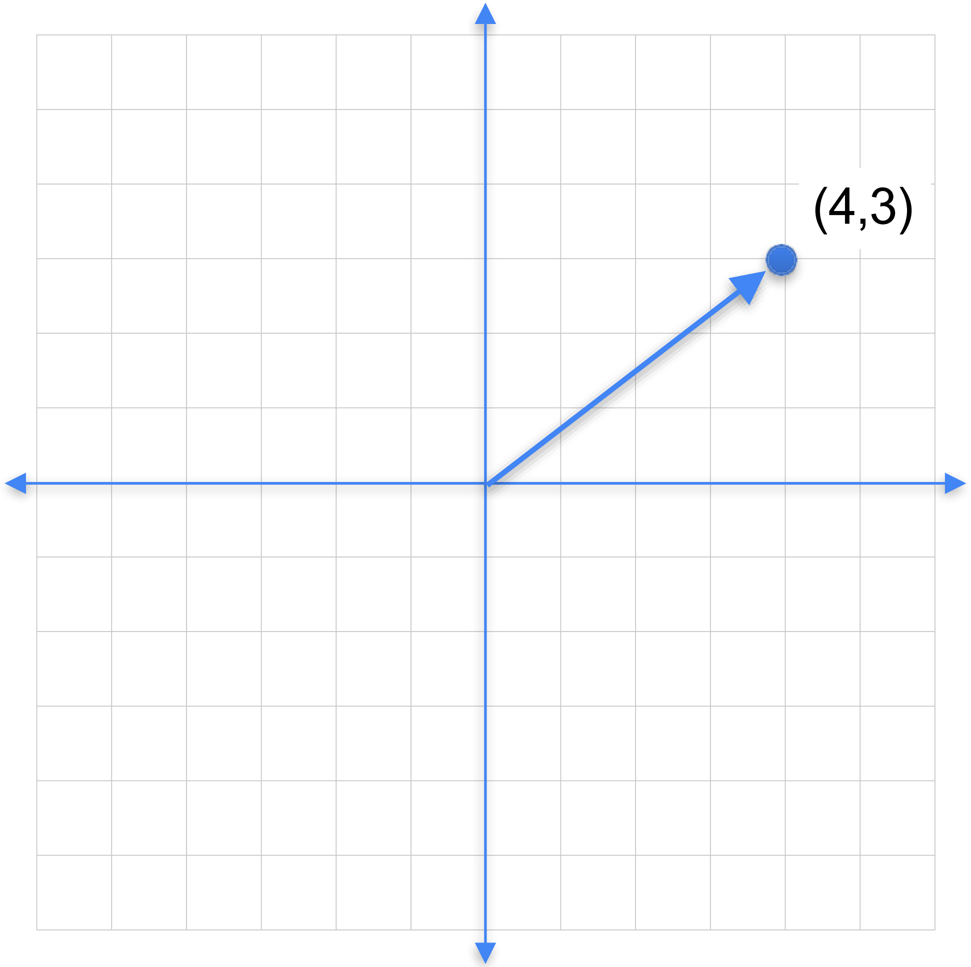

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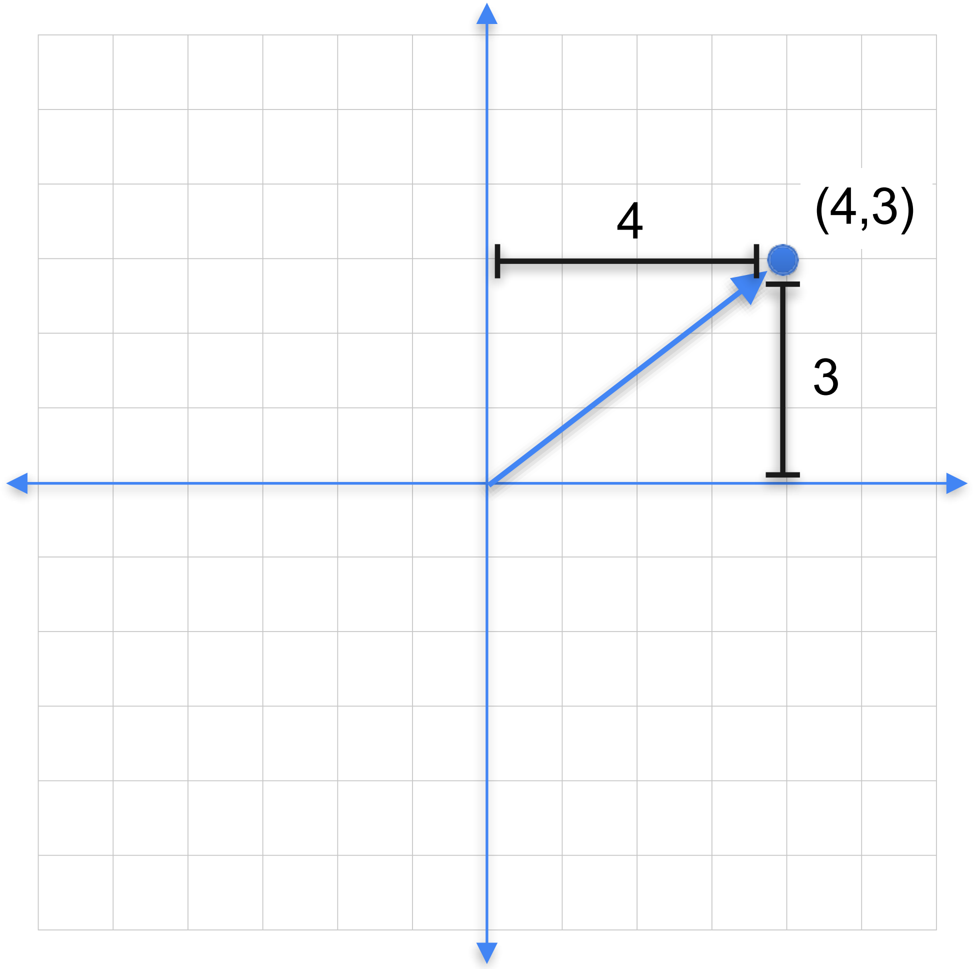

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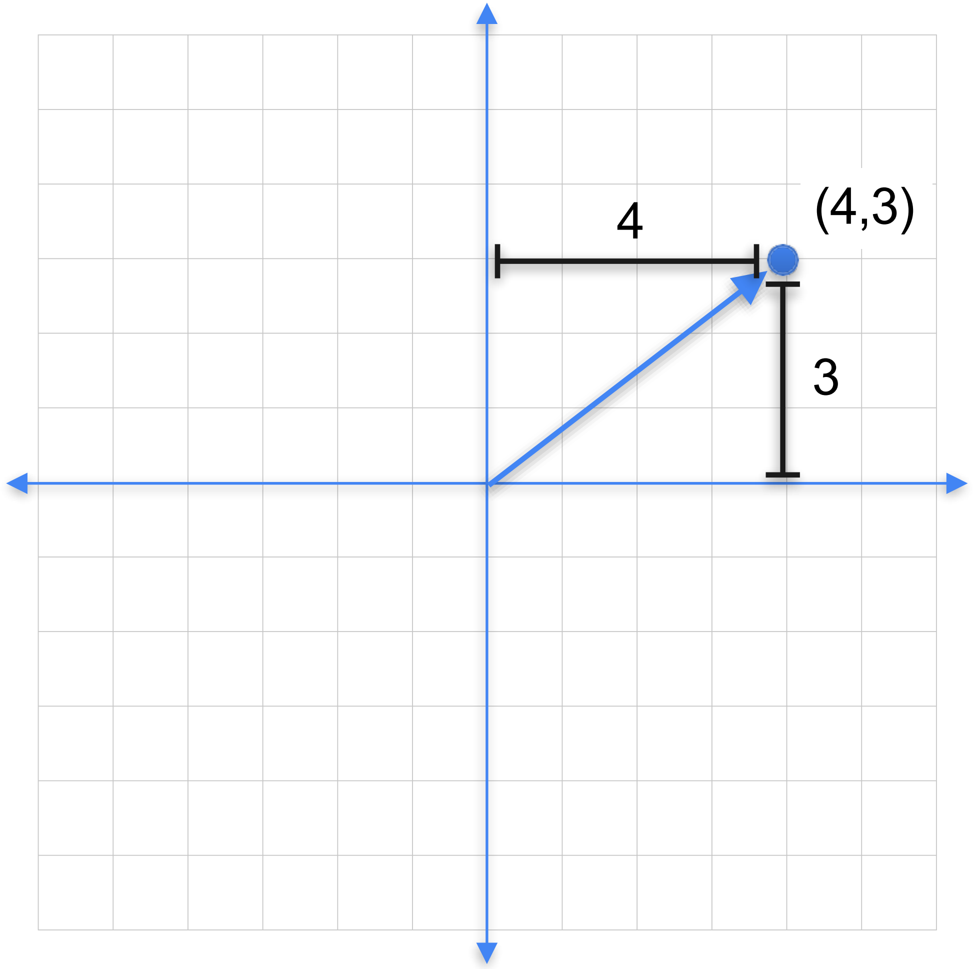

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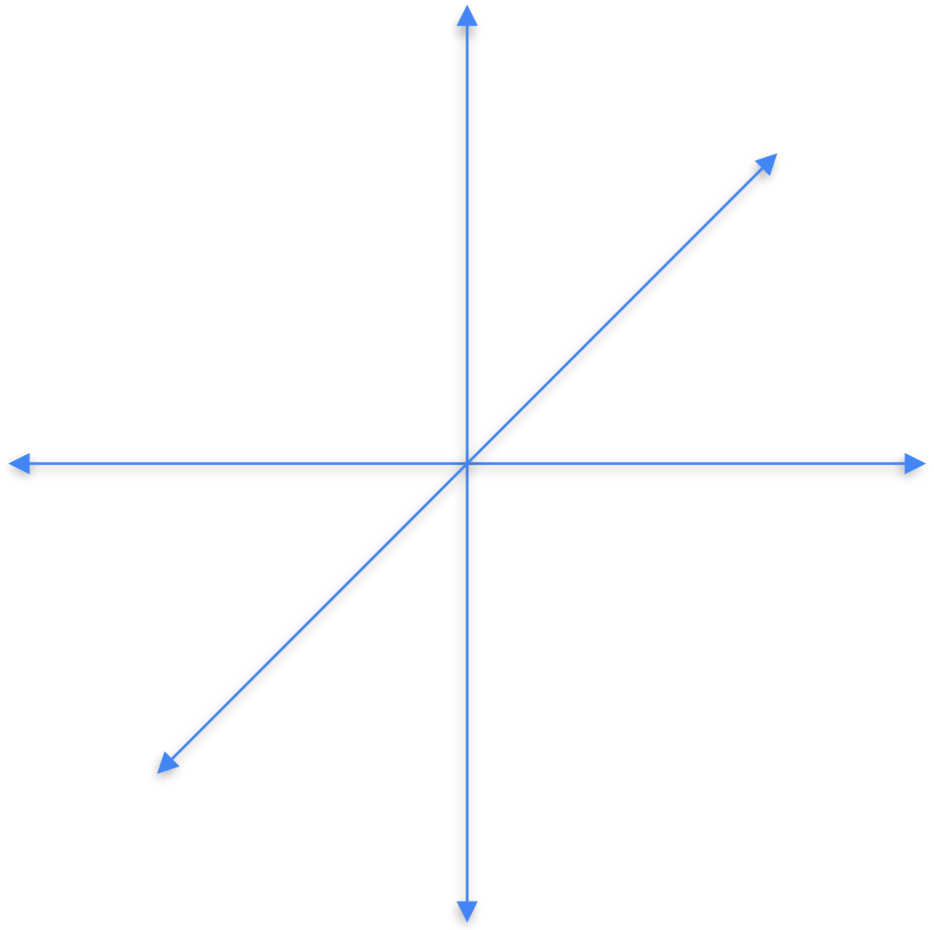

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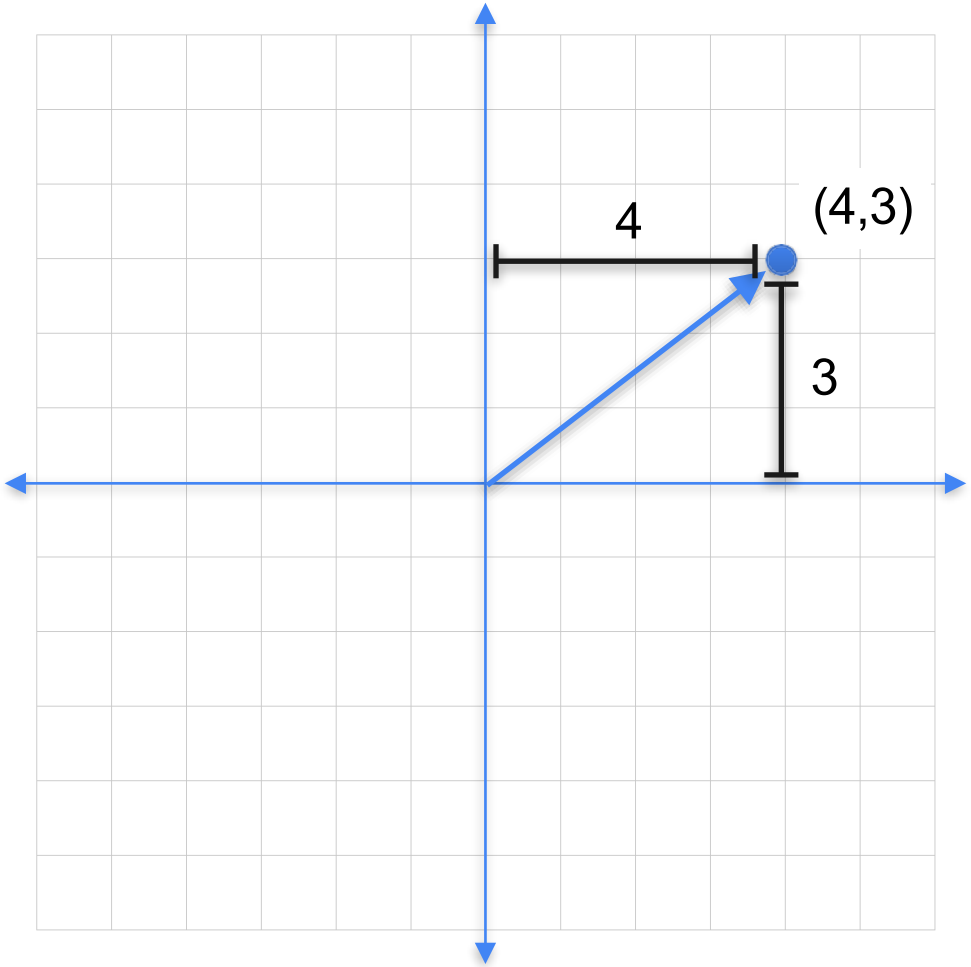

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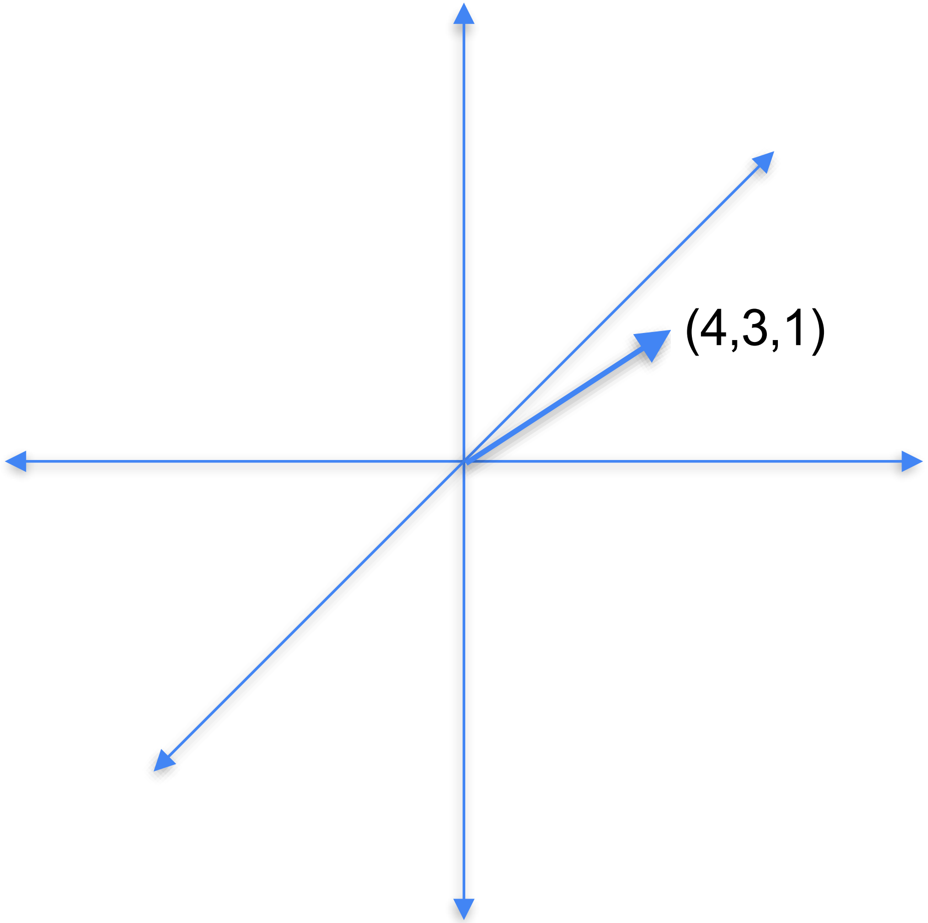

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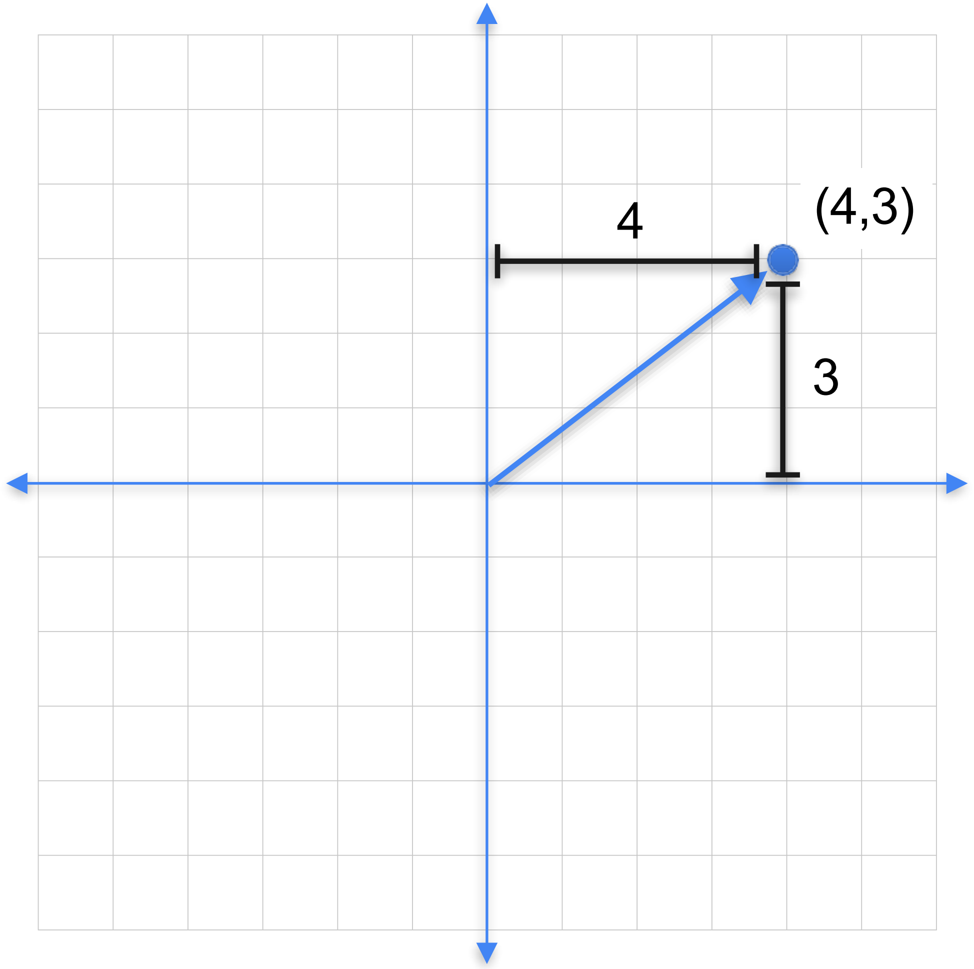

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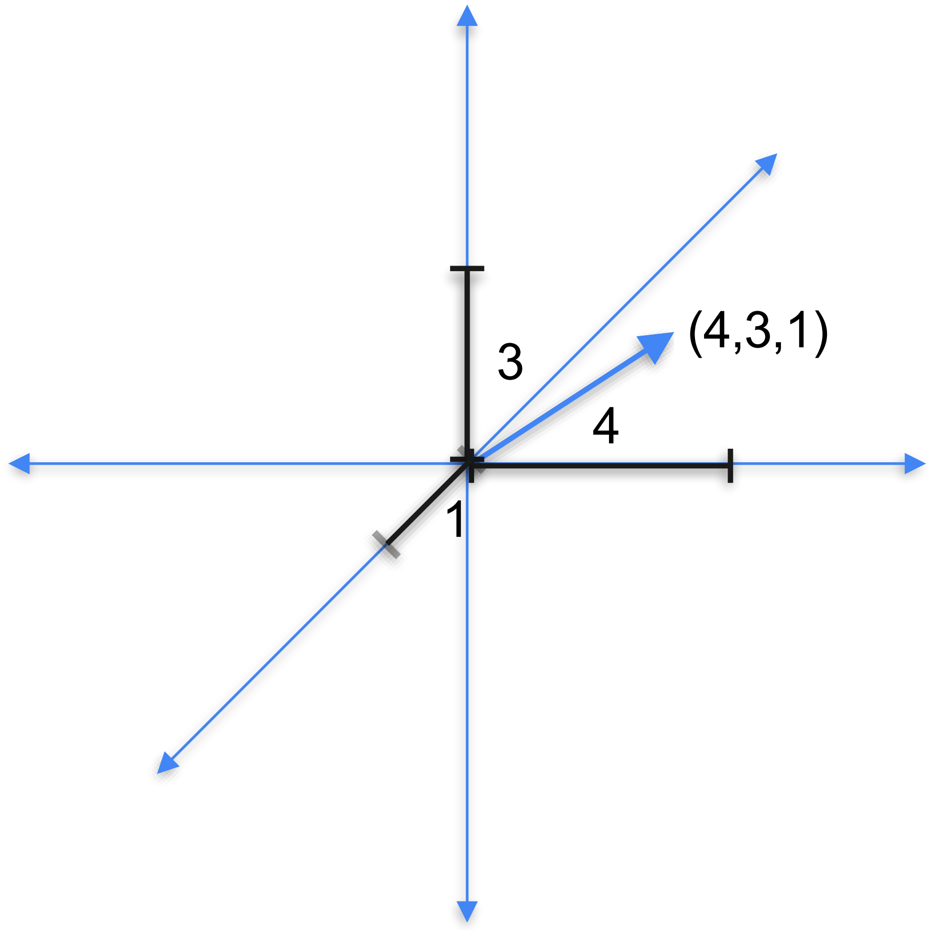

三维

1.向量及其属性

如何从下面的点走到上面的点？

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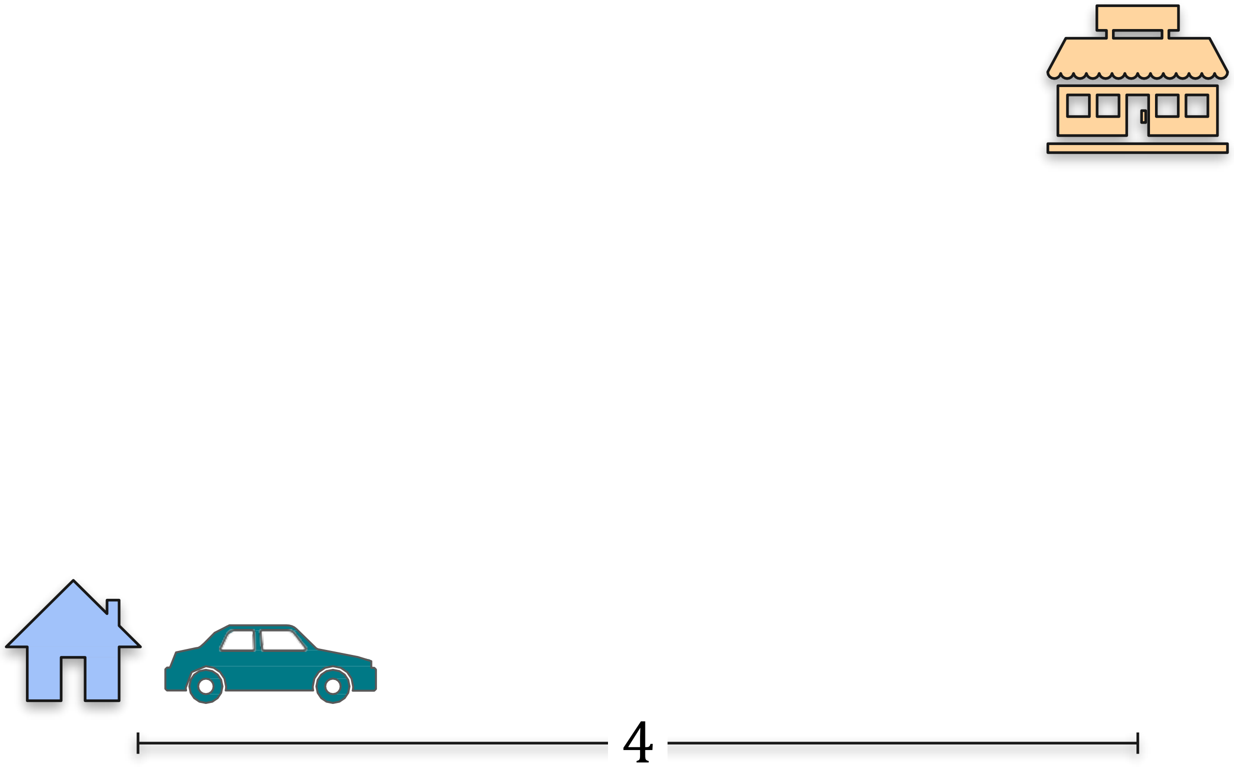

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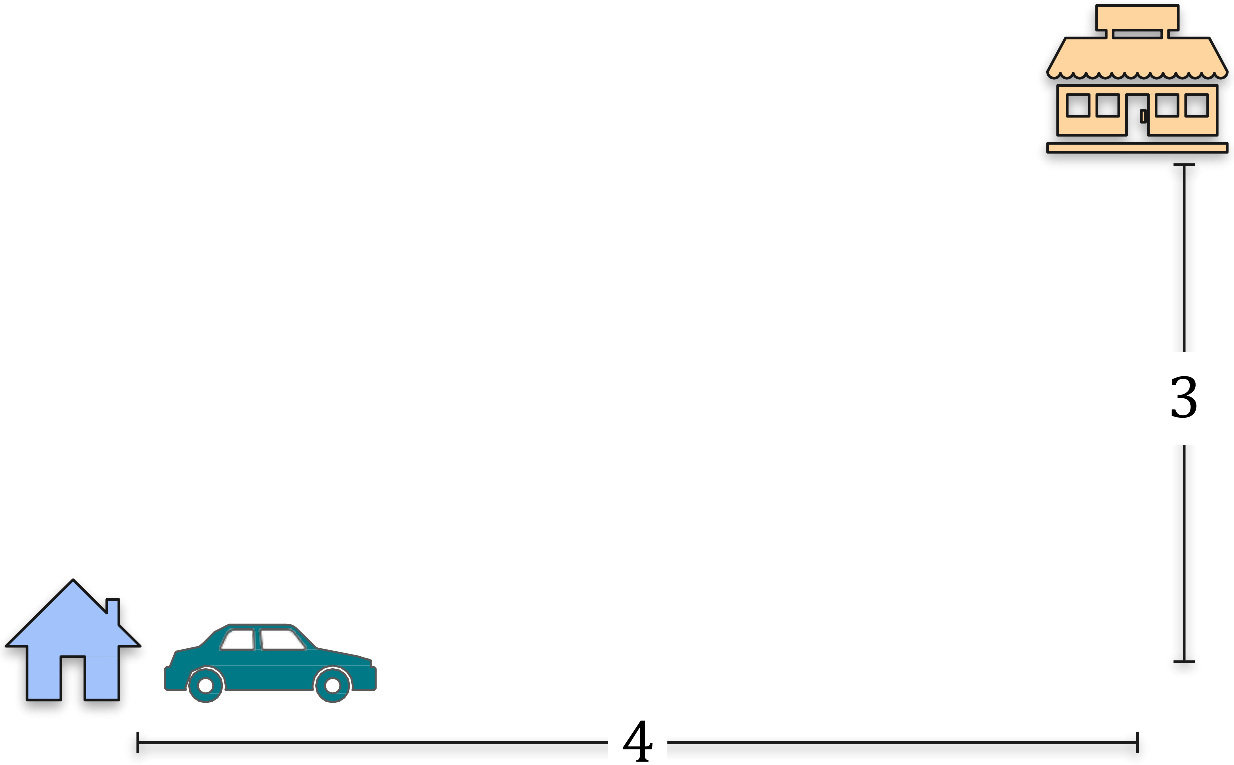

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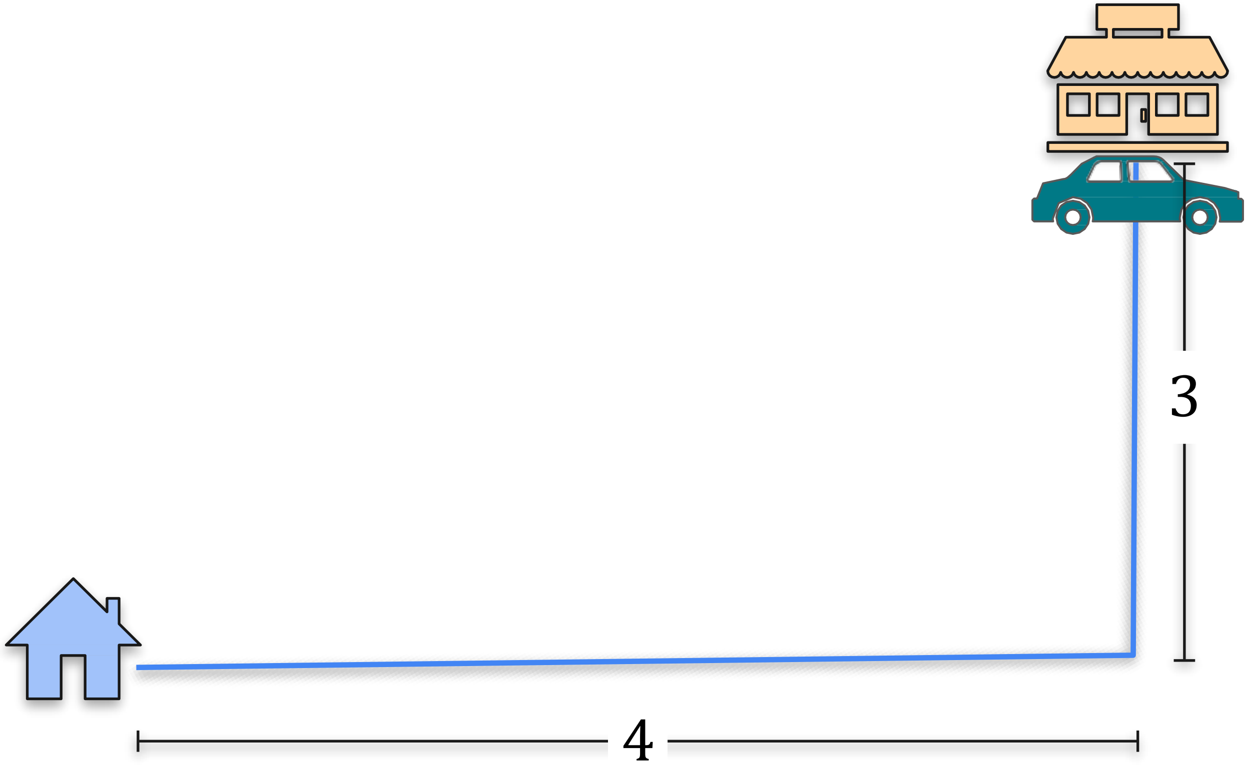

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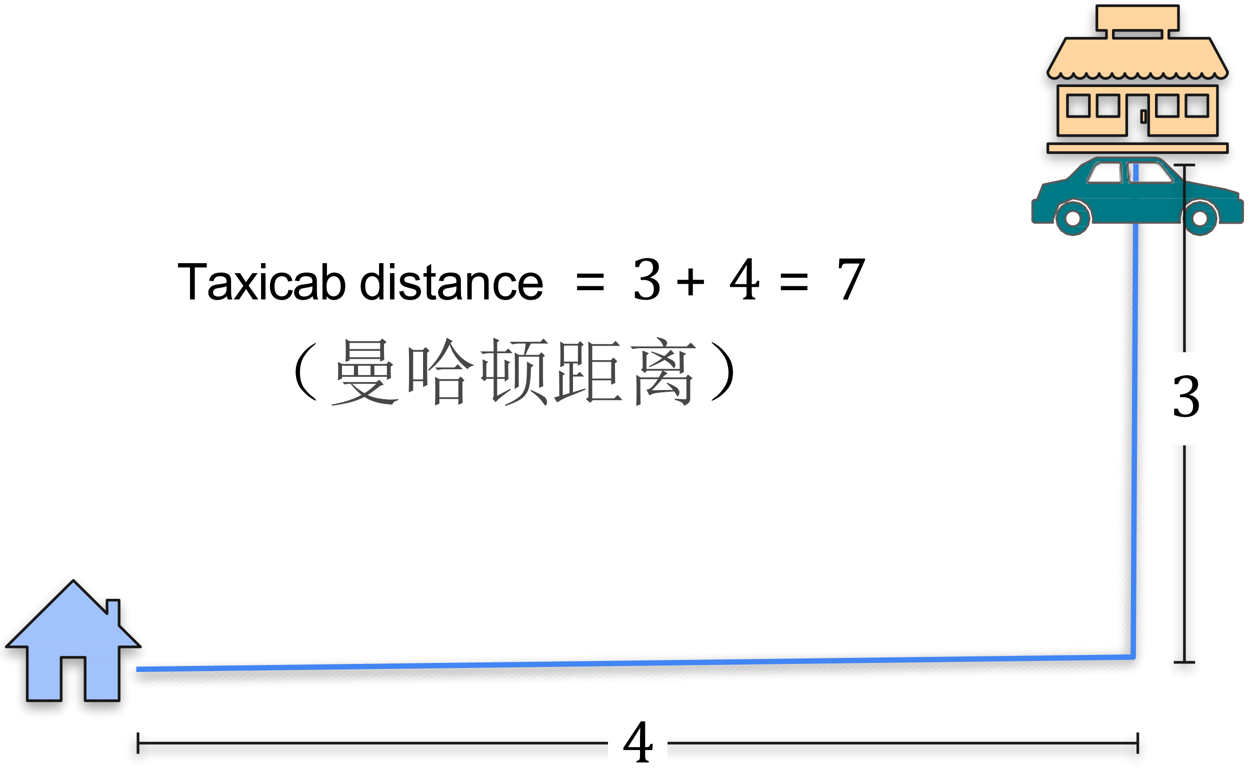
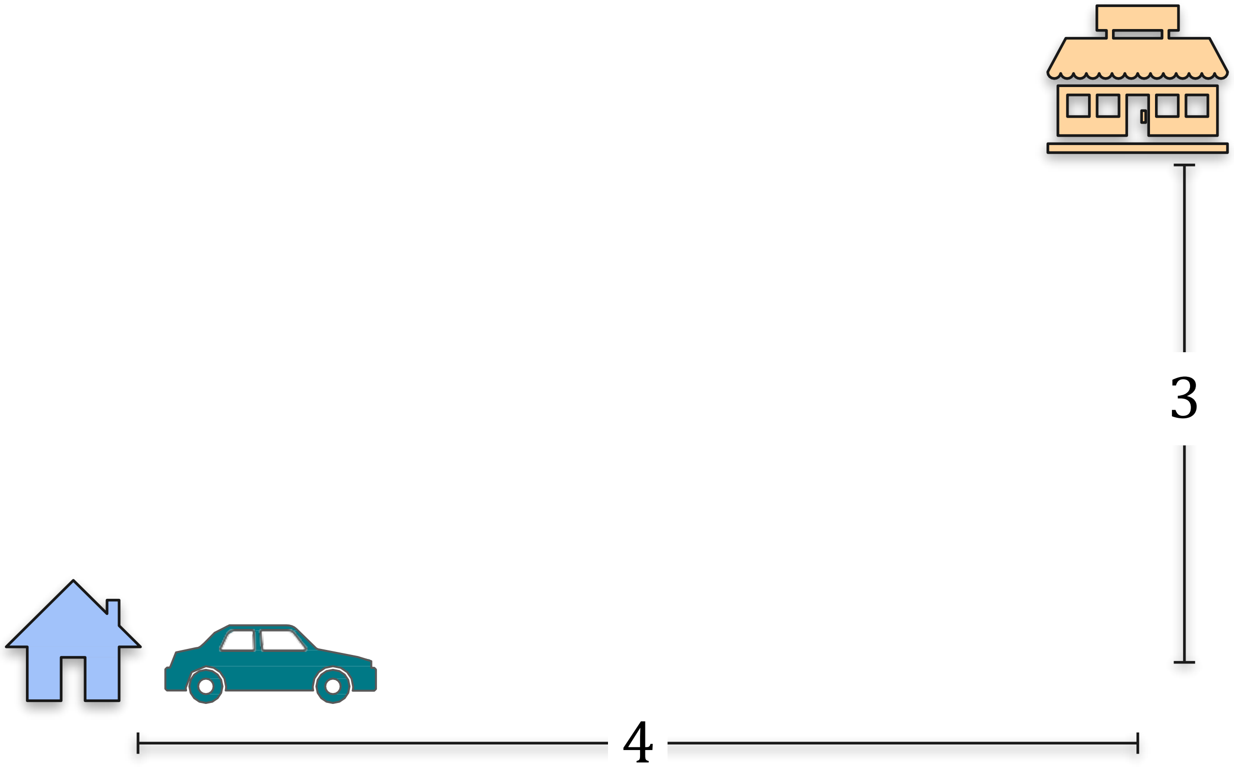


Taxicab distance = 3 + 4 = 7

（曼哈顿距离）

如何从下面的点走到上面的点？



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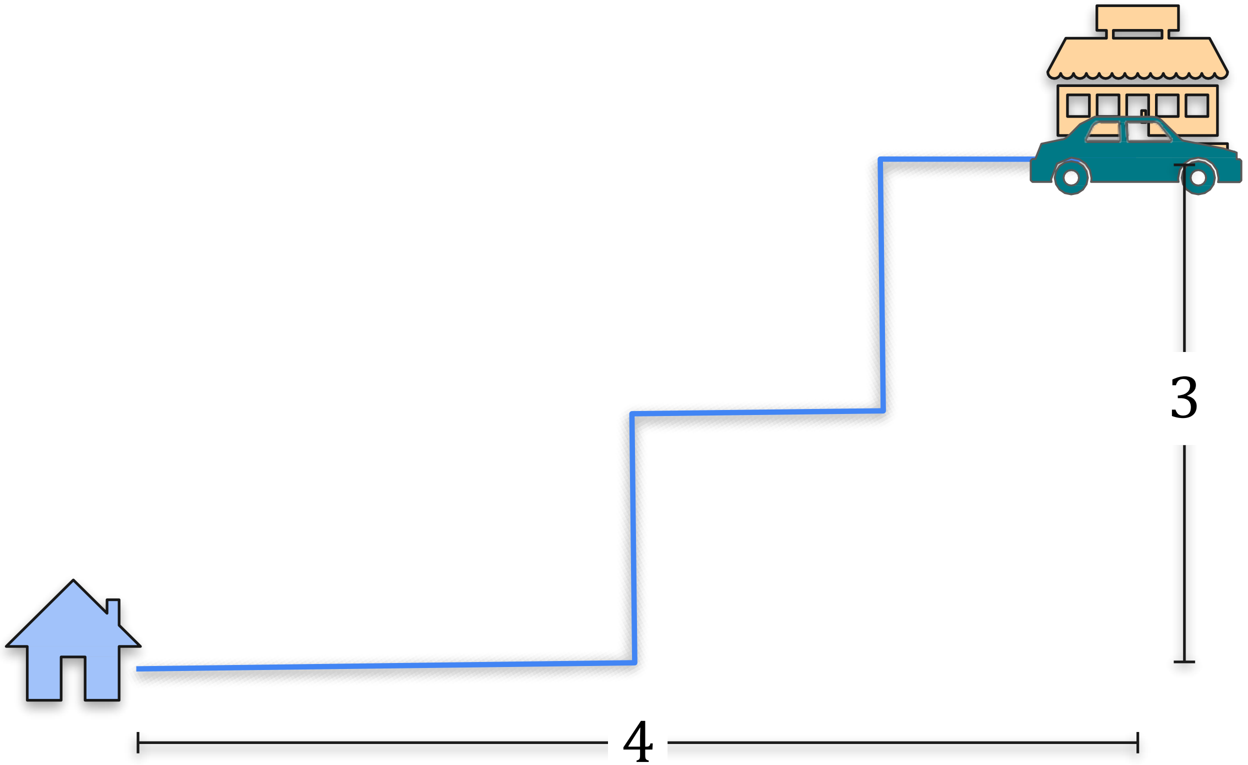

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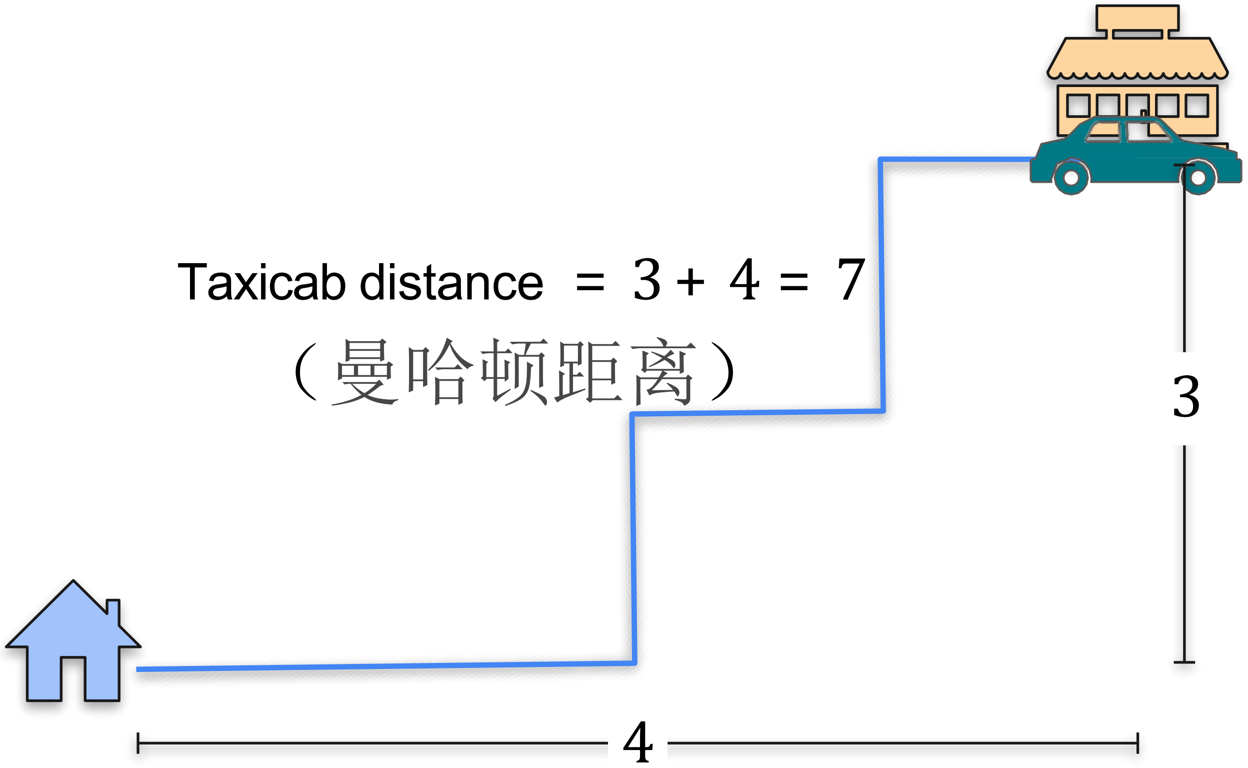

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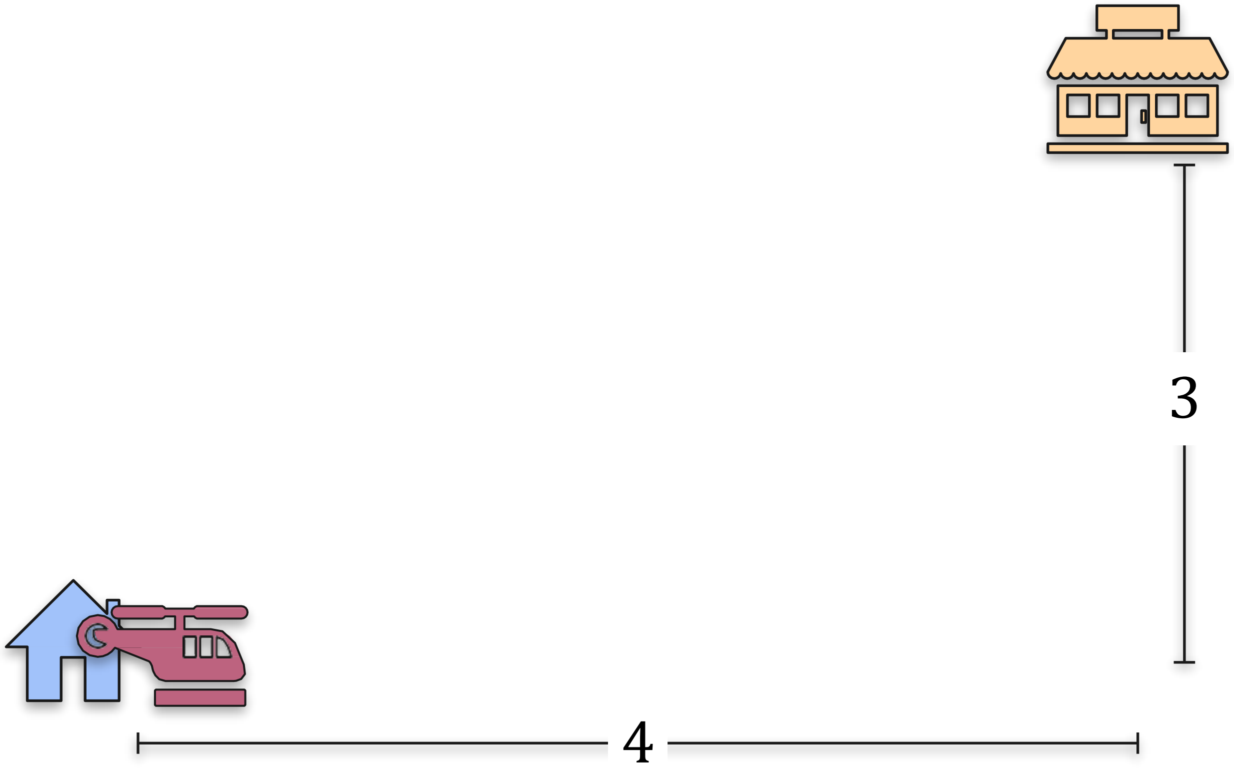

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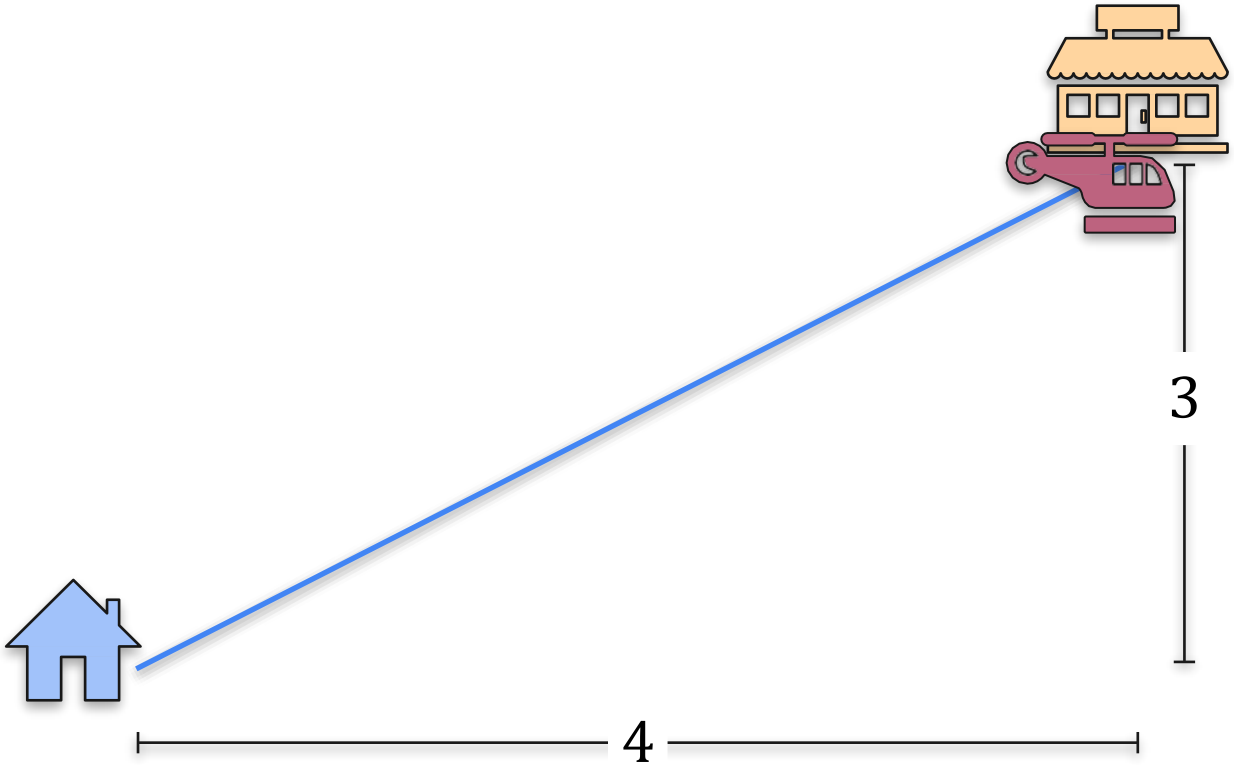

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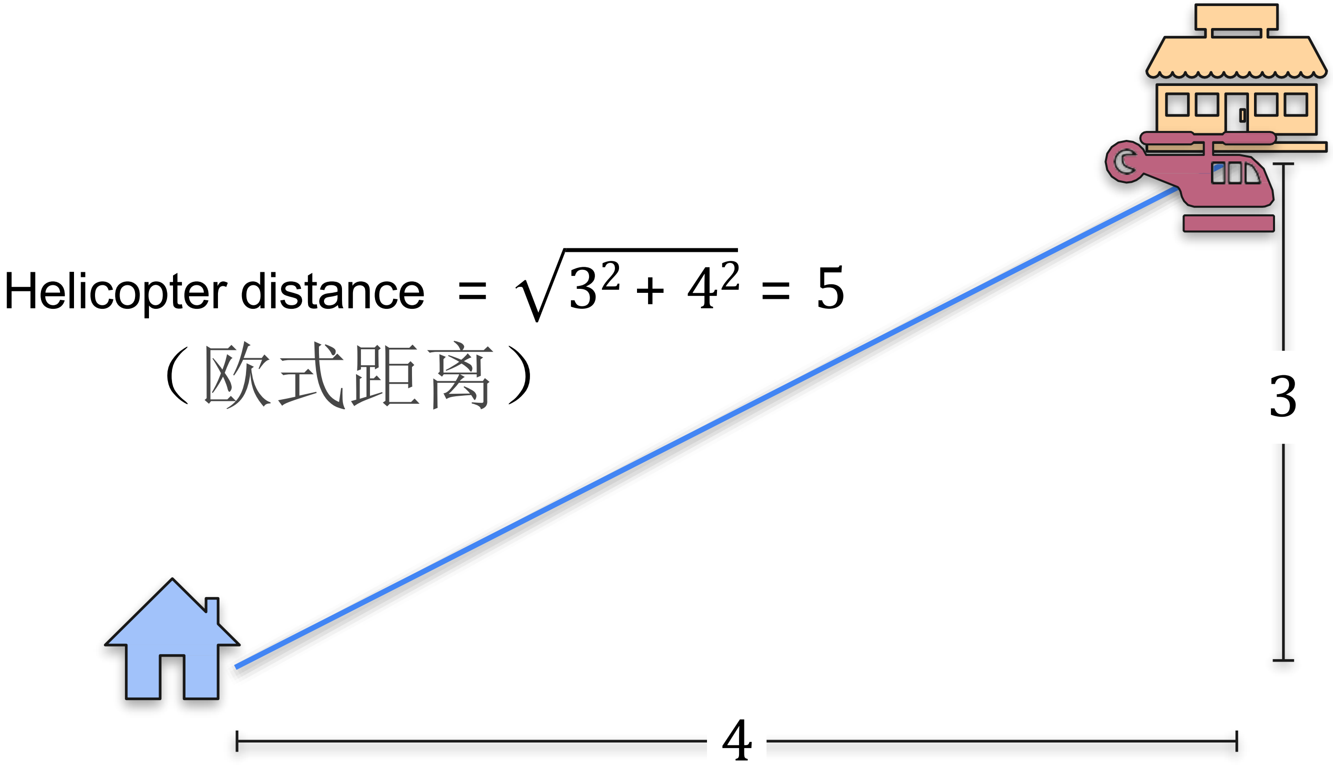
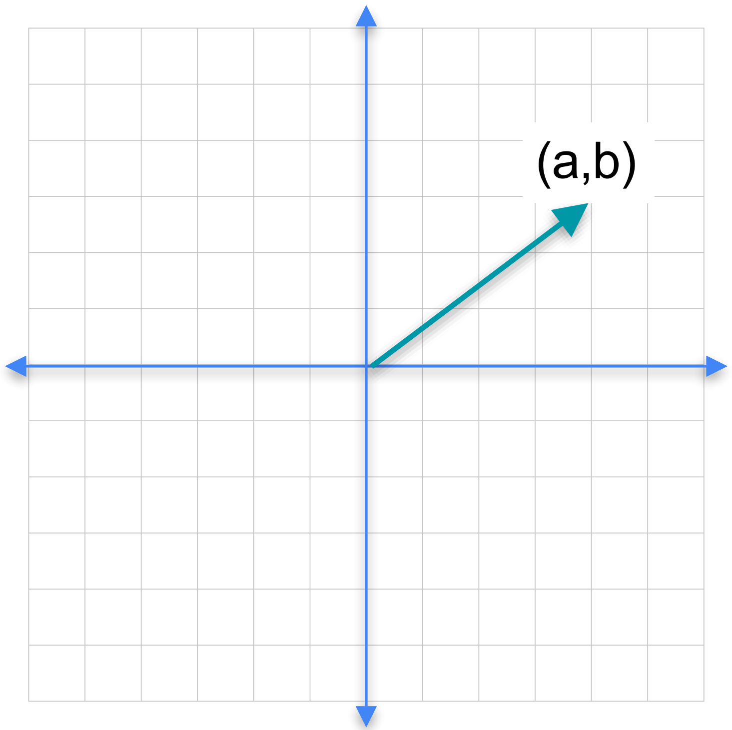


范数（Norm）



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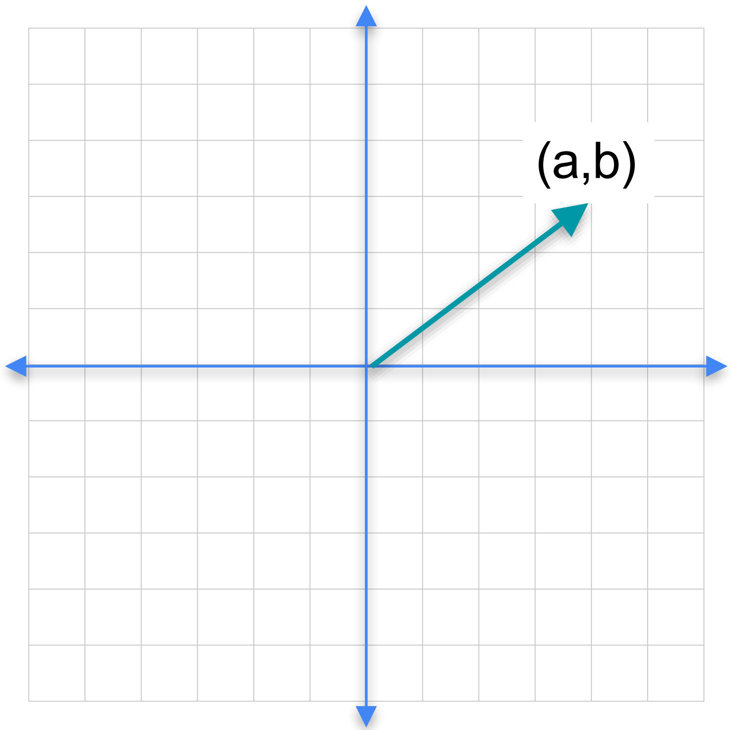
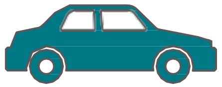
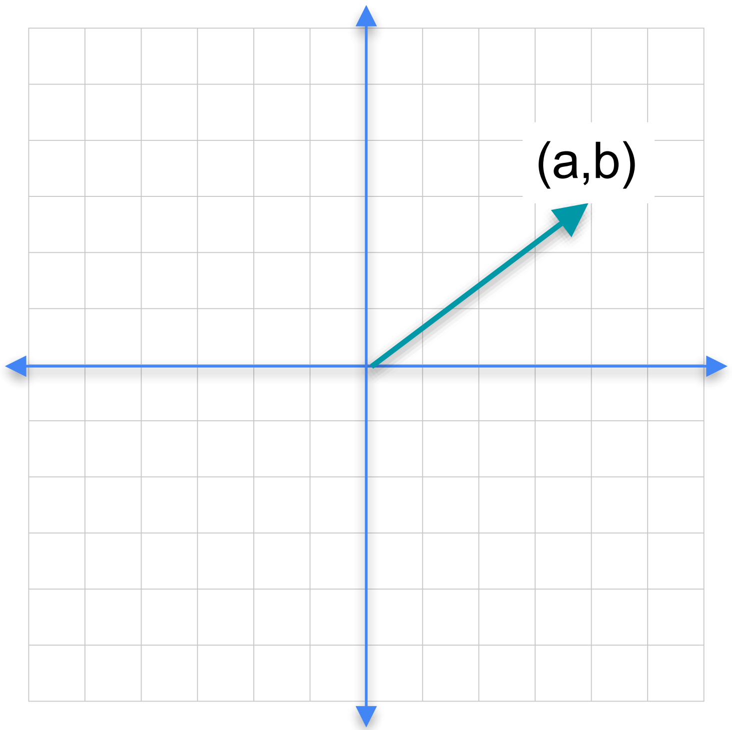
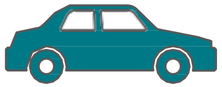


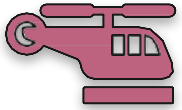


L1-范数 $= |(a,b)|_1 = |a| + |b|$

范数（Norm）



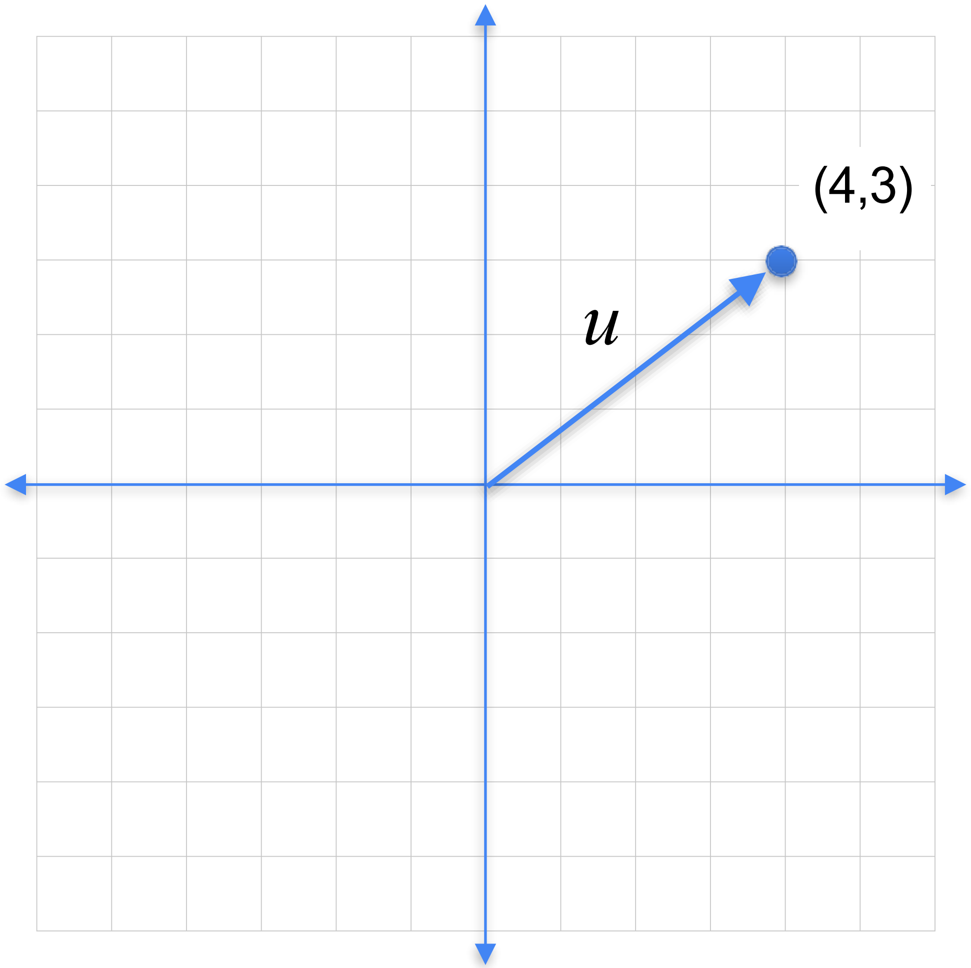




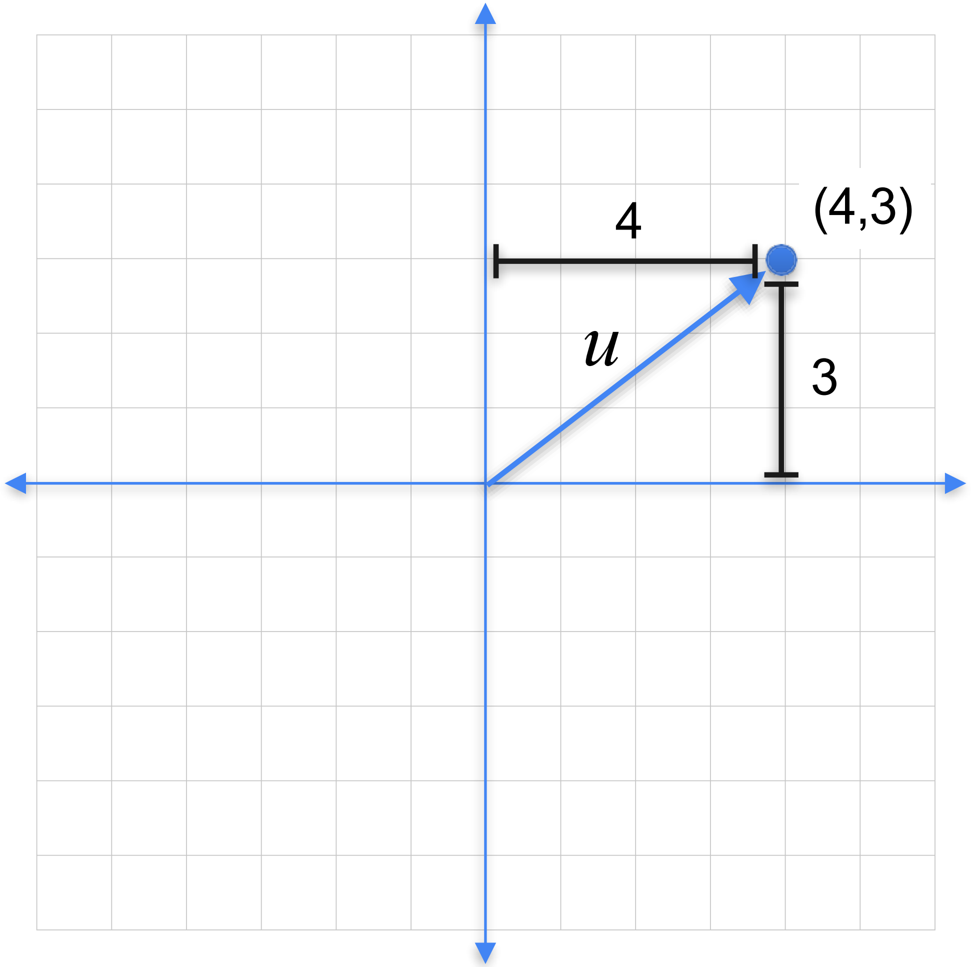
L1-范数 $= |(a,b)|_1 = |a| + |b|$

L2-范数 $= |(a,b)|_2 = \sqrt{a^2 + b^2}$

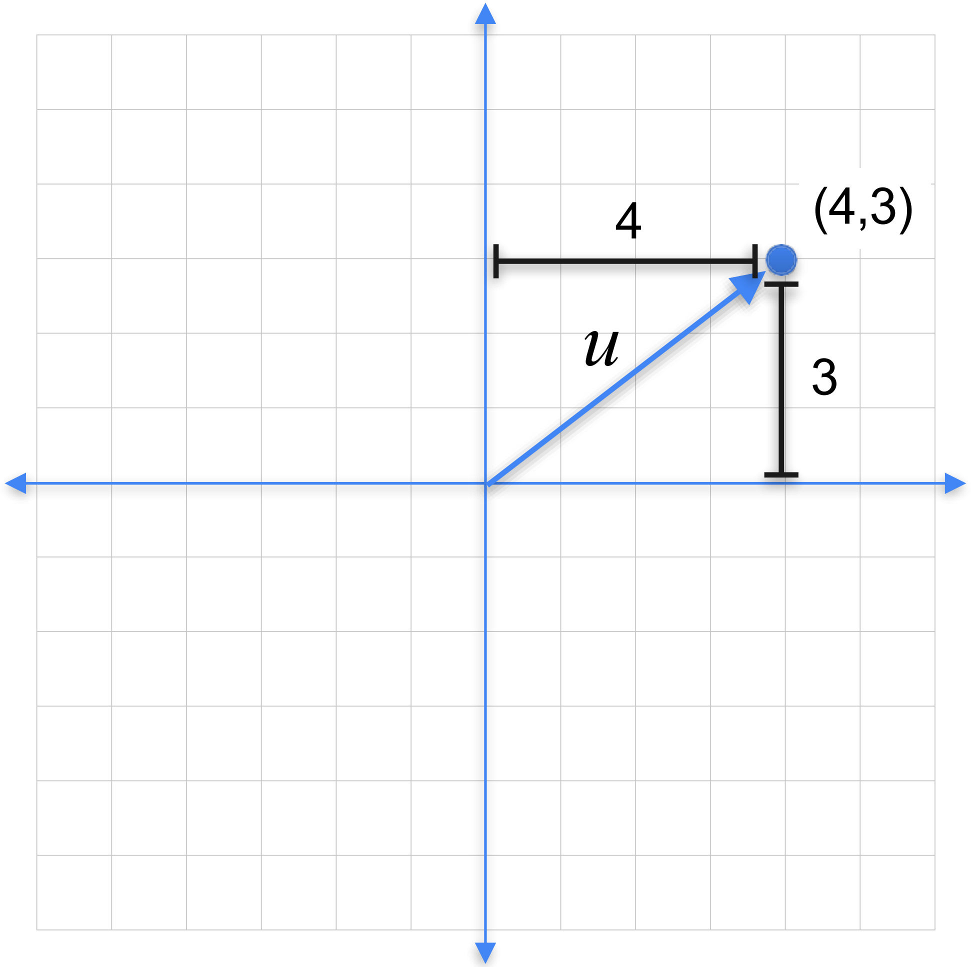
范数例子



范数例子



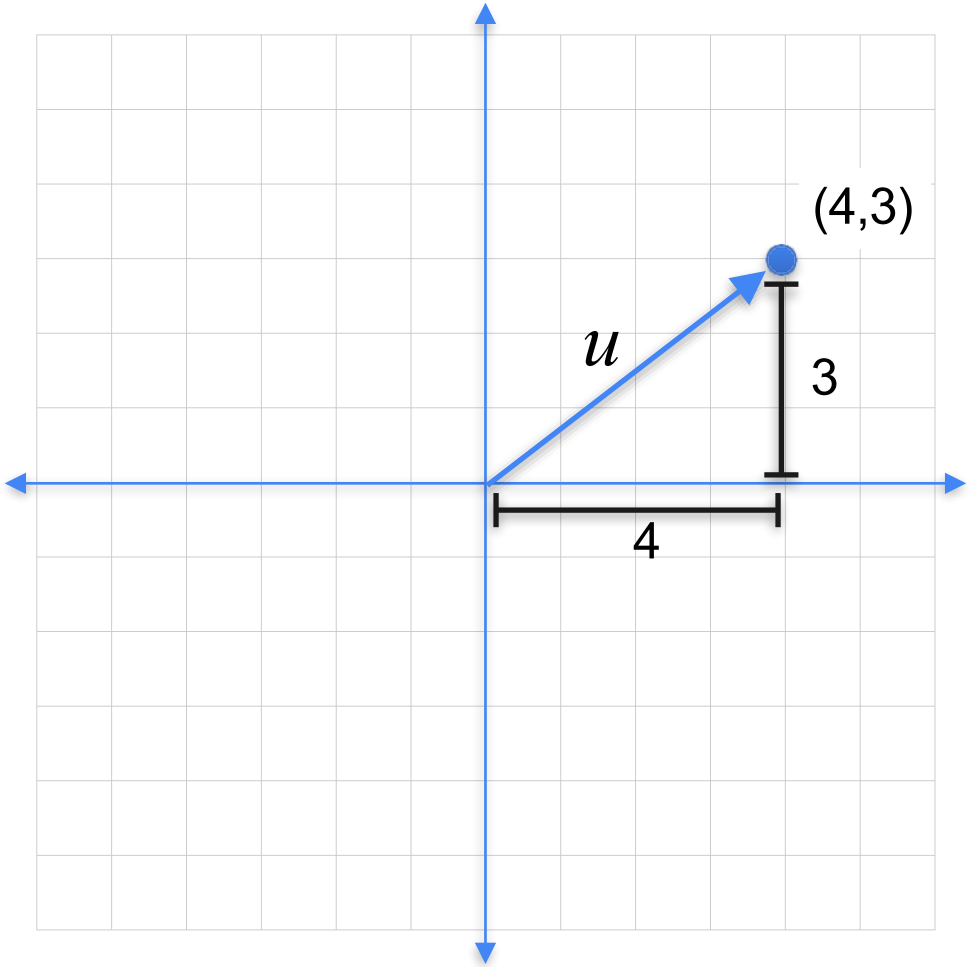
范数例子



L1-范数 = $|4| + |3| = 7$

L2-范数 = $\sqrt{4^2 + 3^2} = \sqrt{25} = 5$

向量的方向



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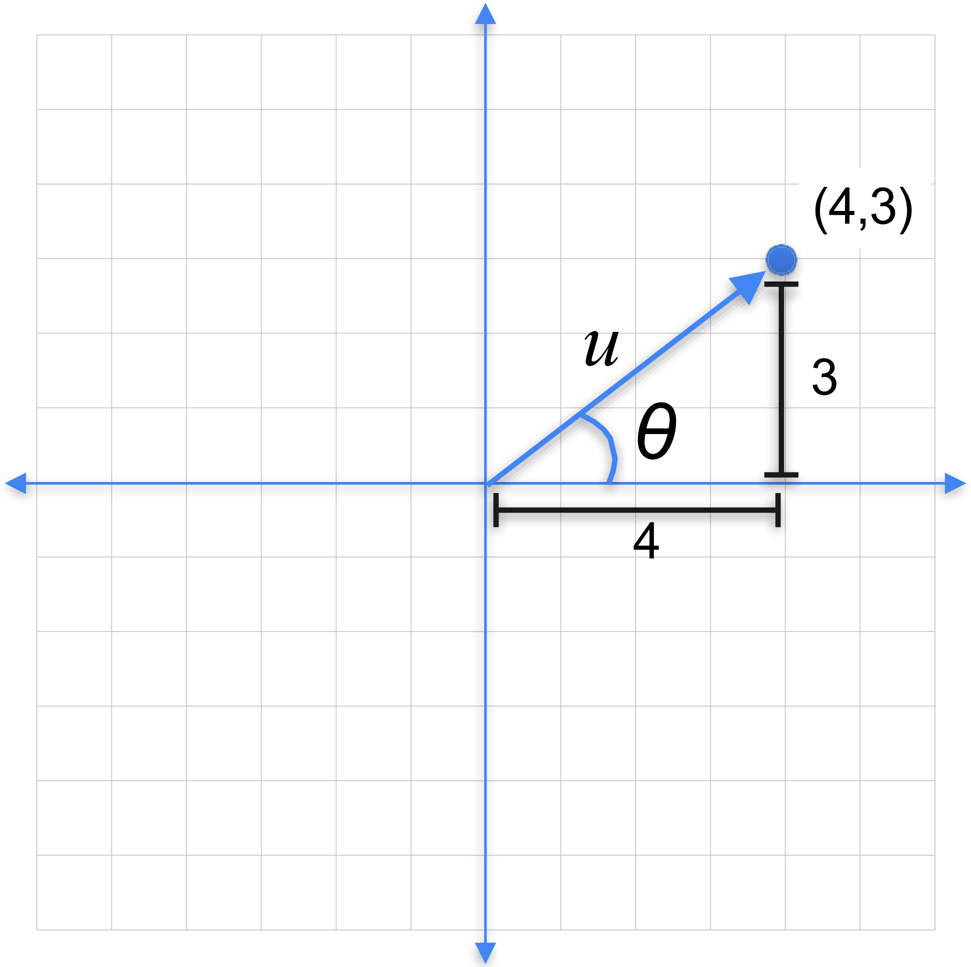

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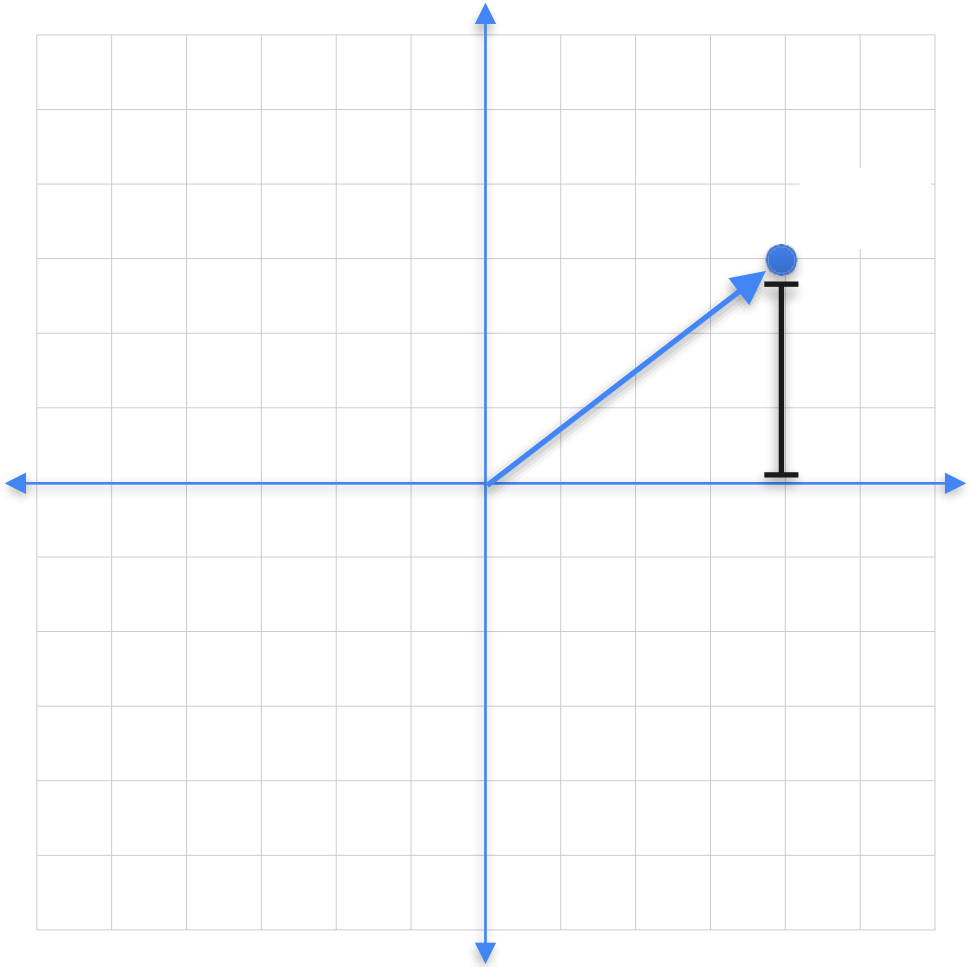
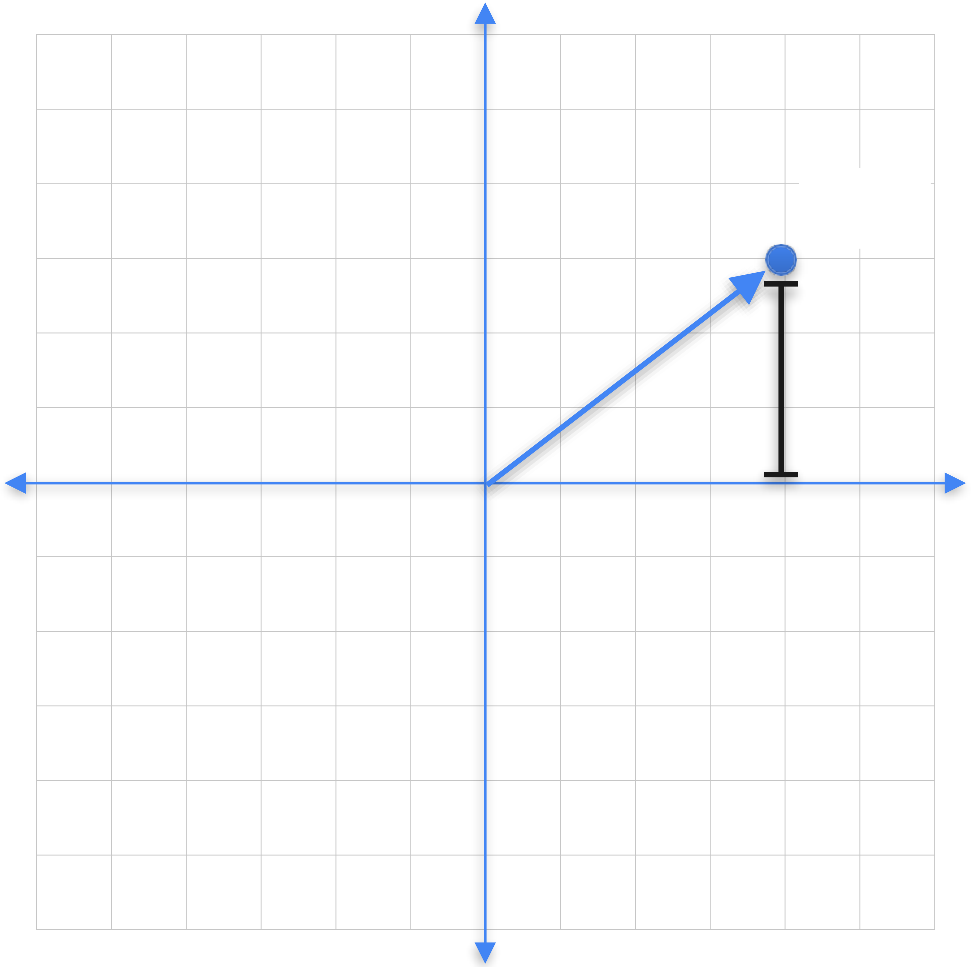


$\tan(\theta) = 3/4$

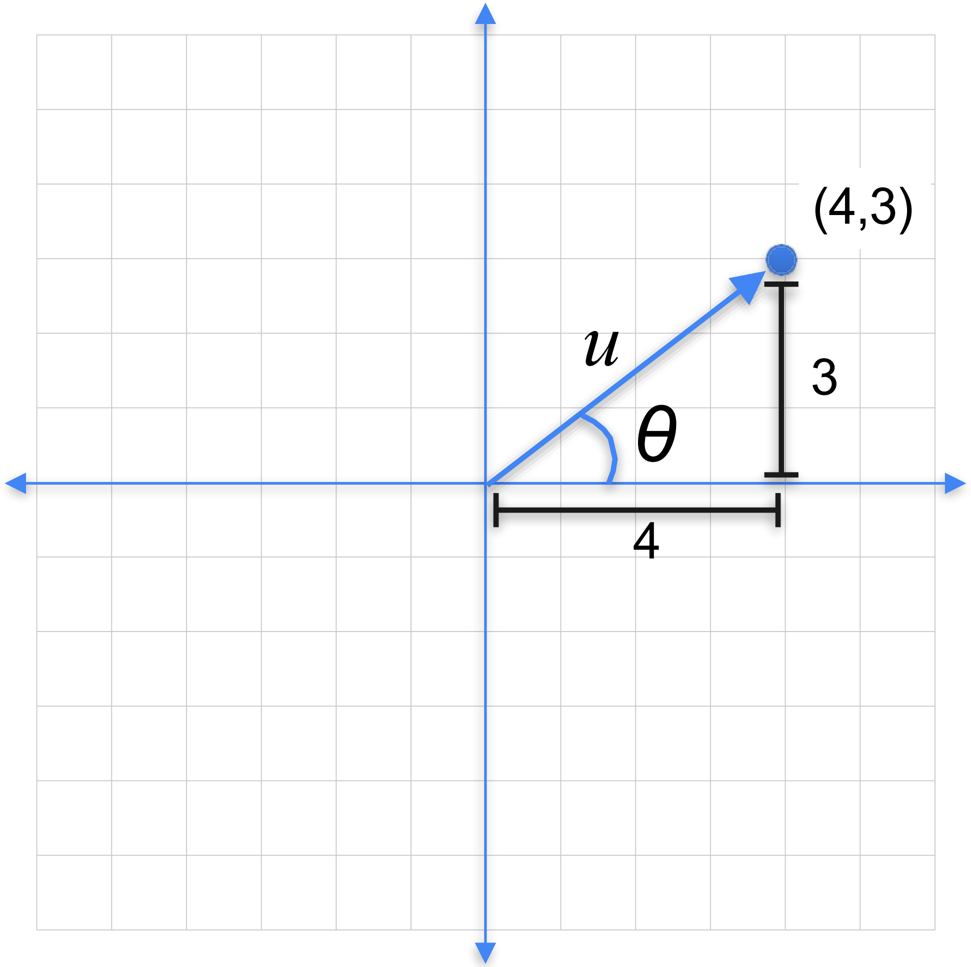
向量的方向



$\tan(\theta) = 3/4$

$\theta = \arctan(3/4) = 0.64$

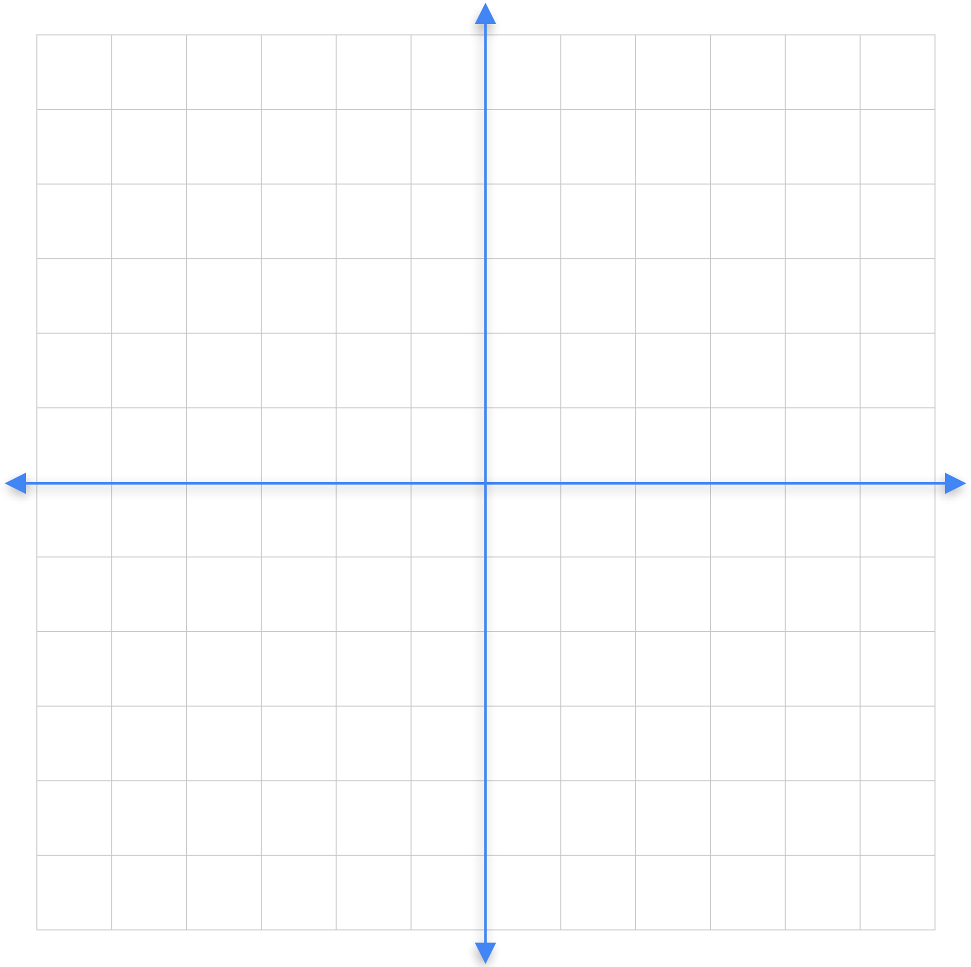
向量的方向



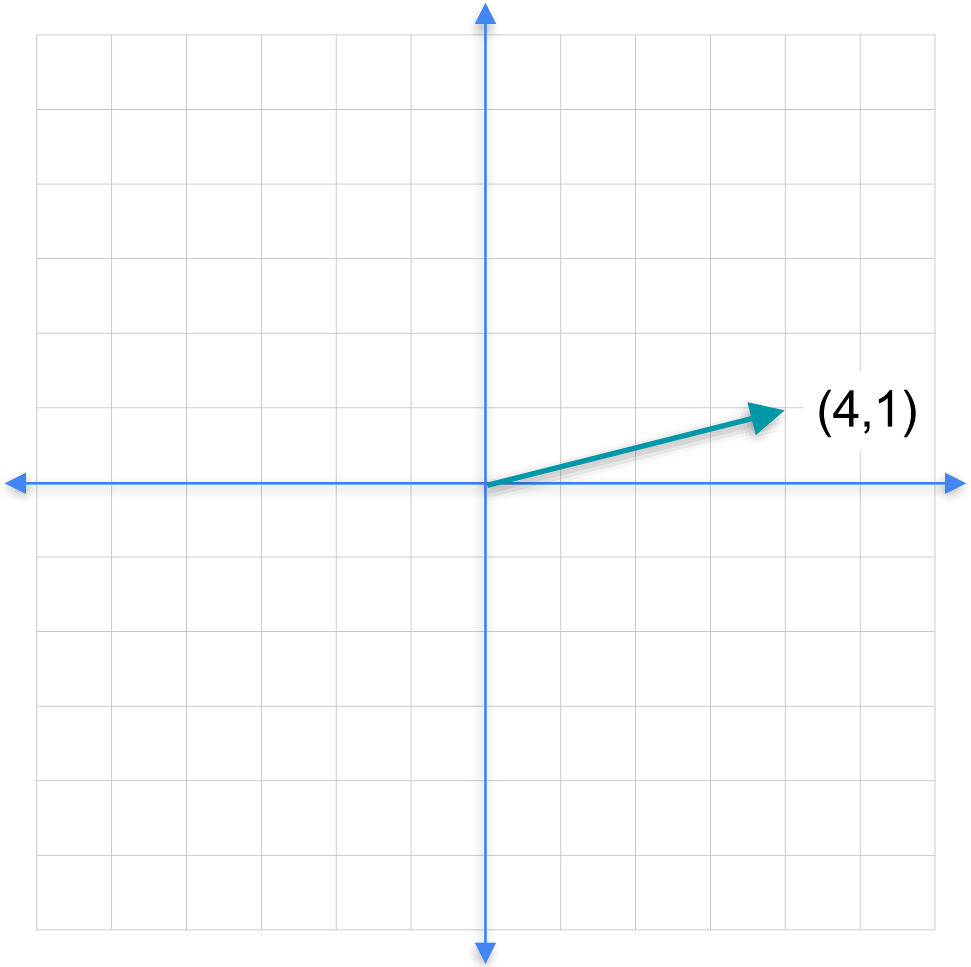
$\tan(\theta) = 3/4$

$\theta = \arctan(3/4) = 0.64 = 36.87°$

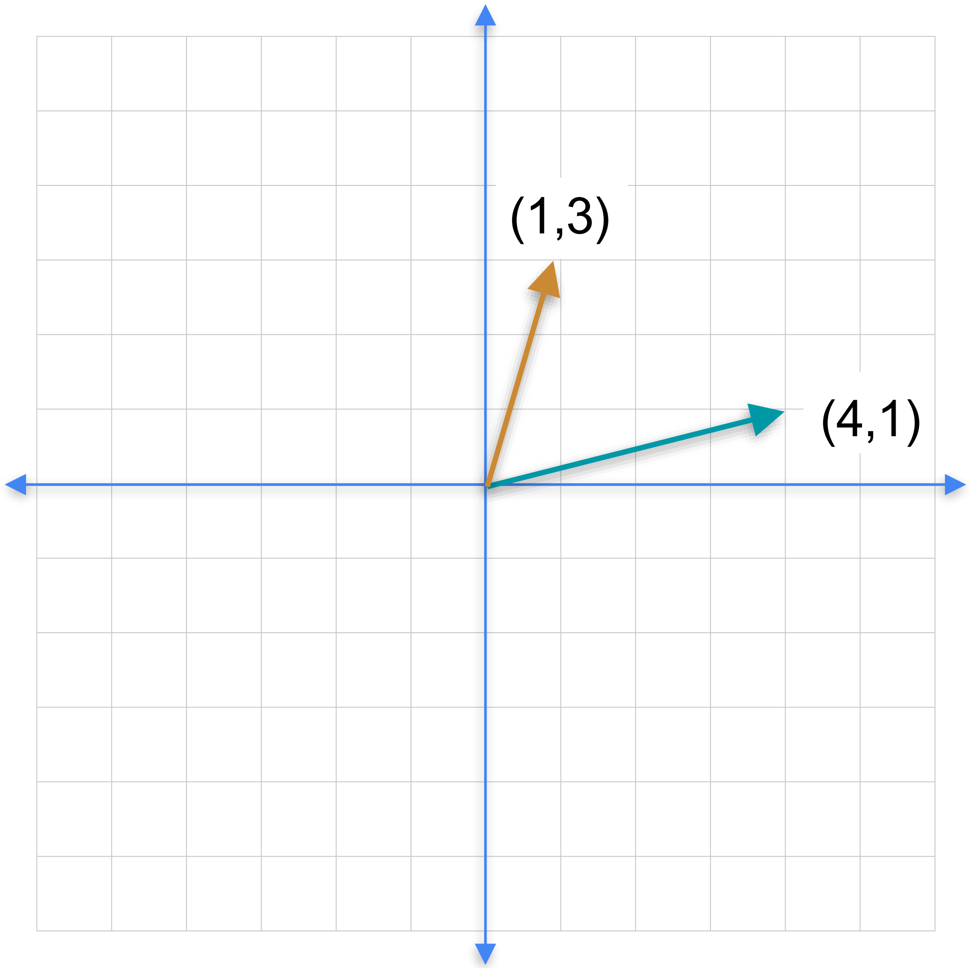
向量相加



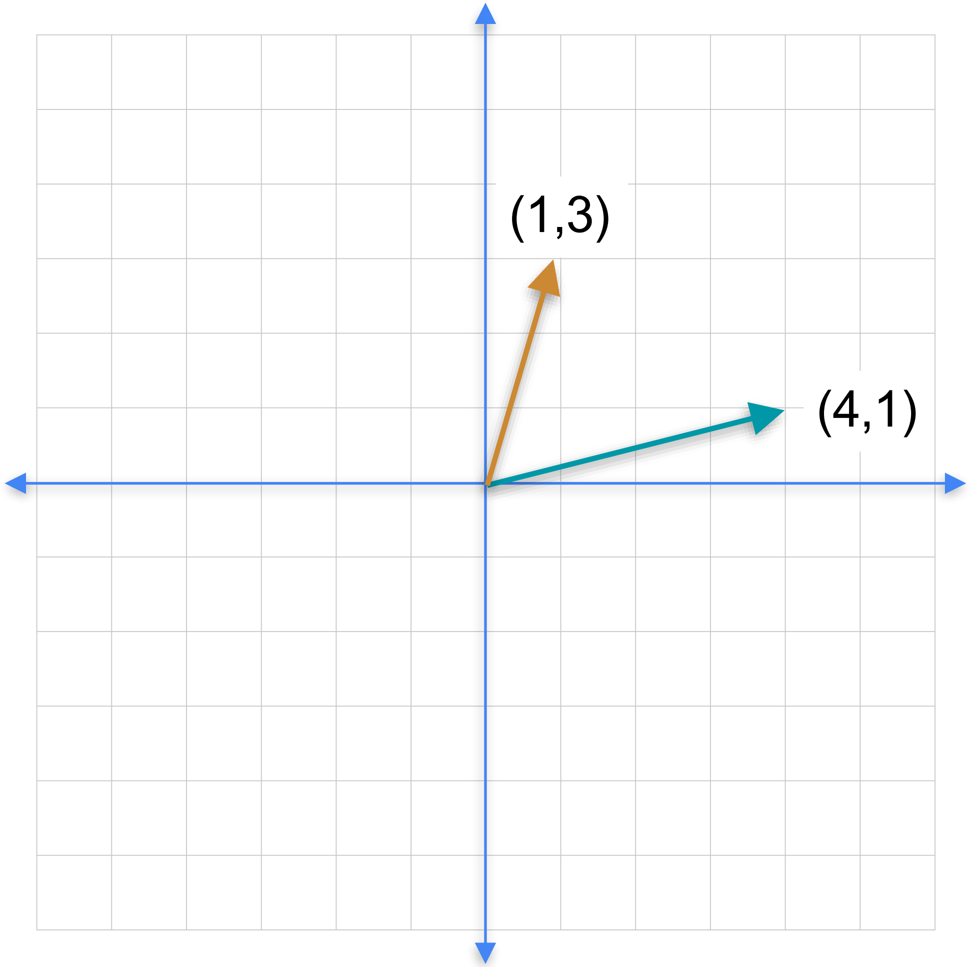
向量相加



向量相加

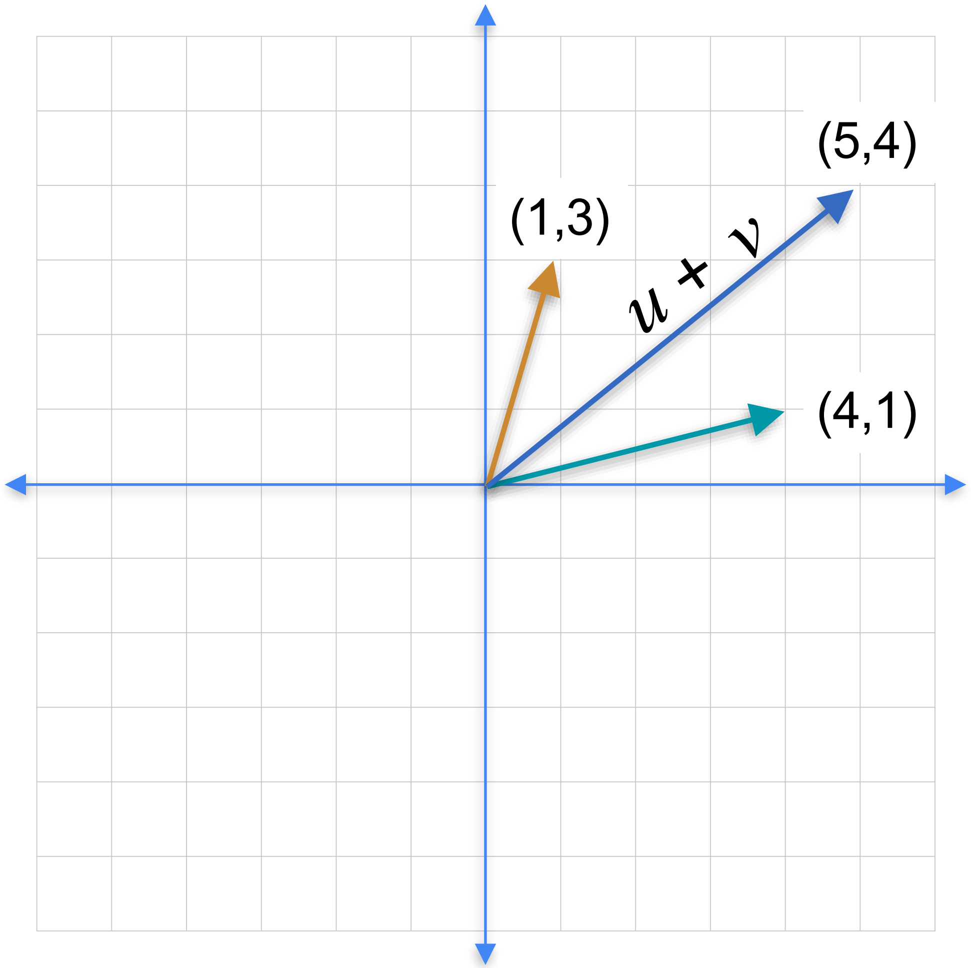


向量相加



$$u + v = (4 + 1, 1 + 3) = (5,4)$$

向量相加



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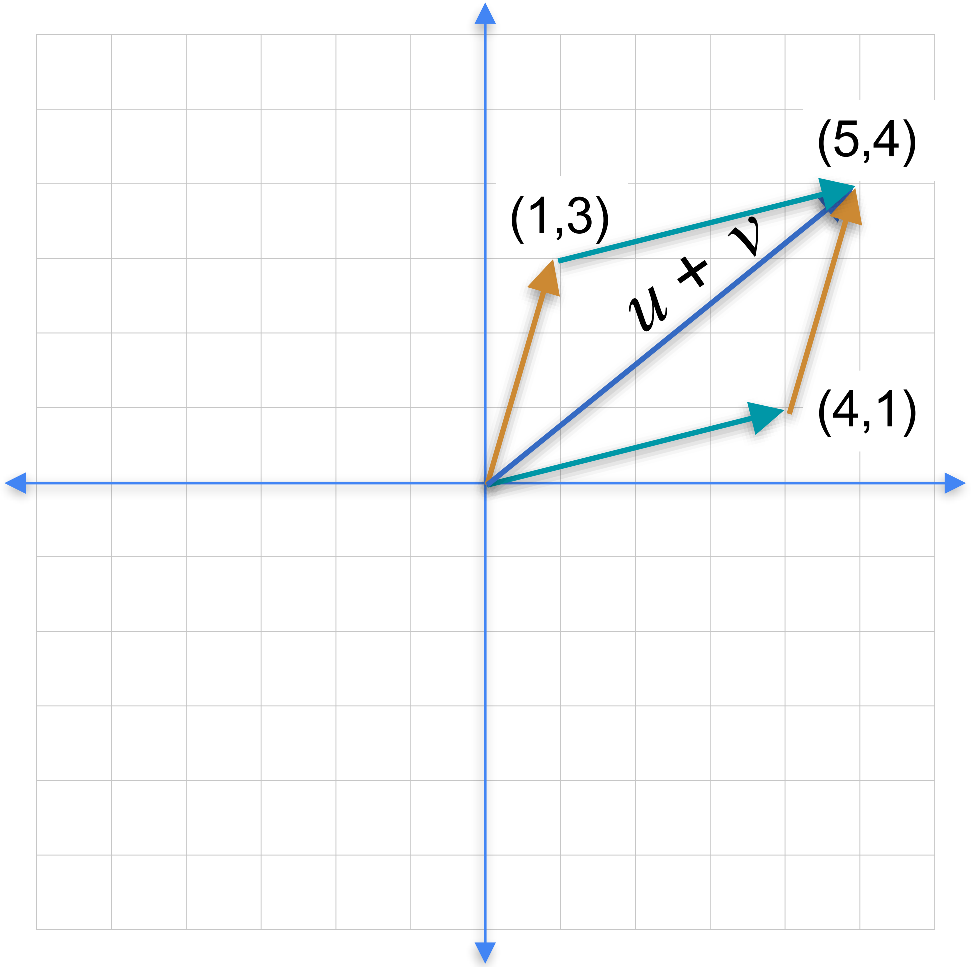
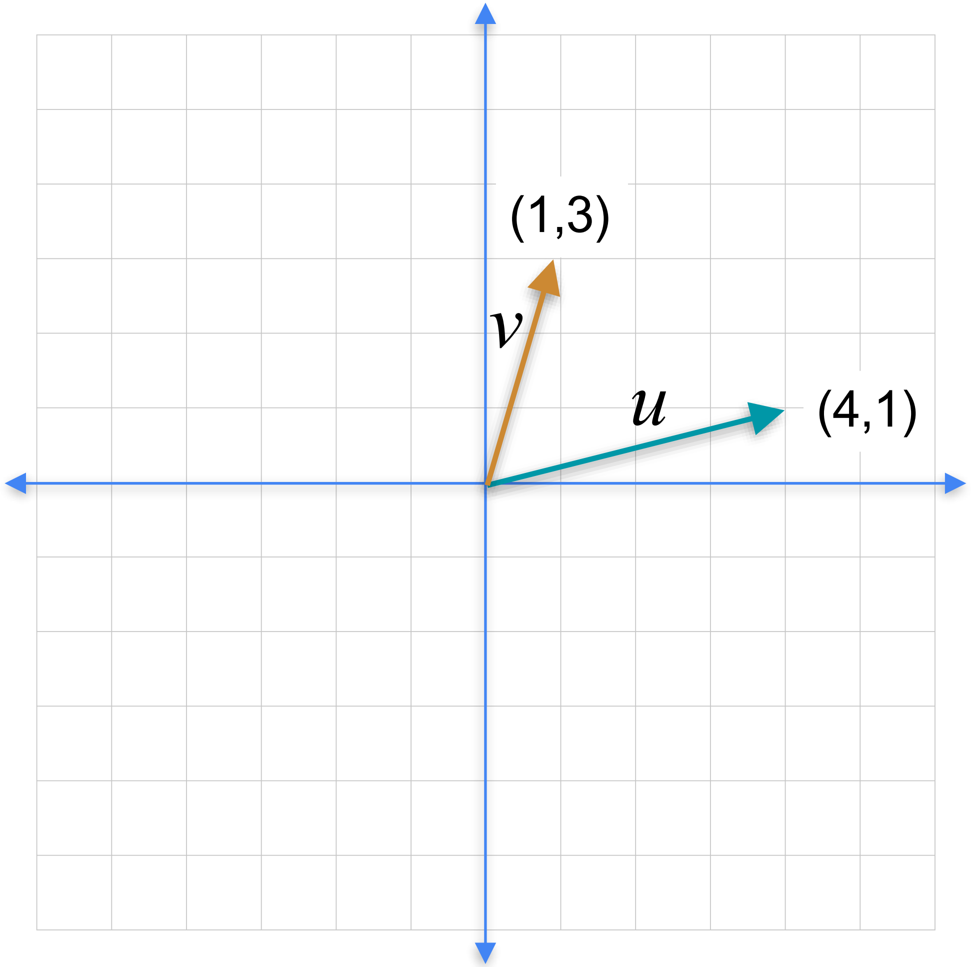
向量相加

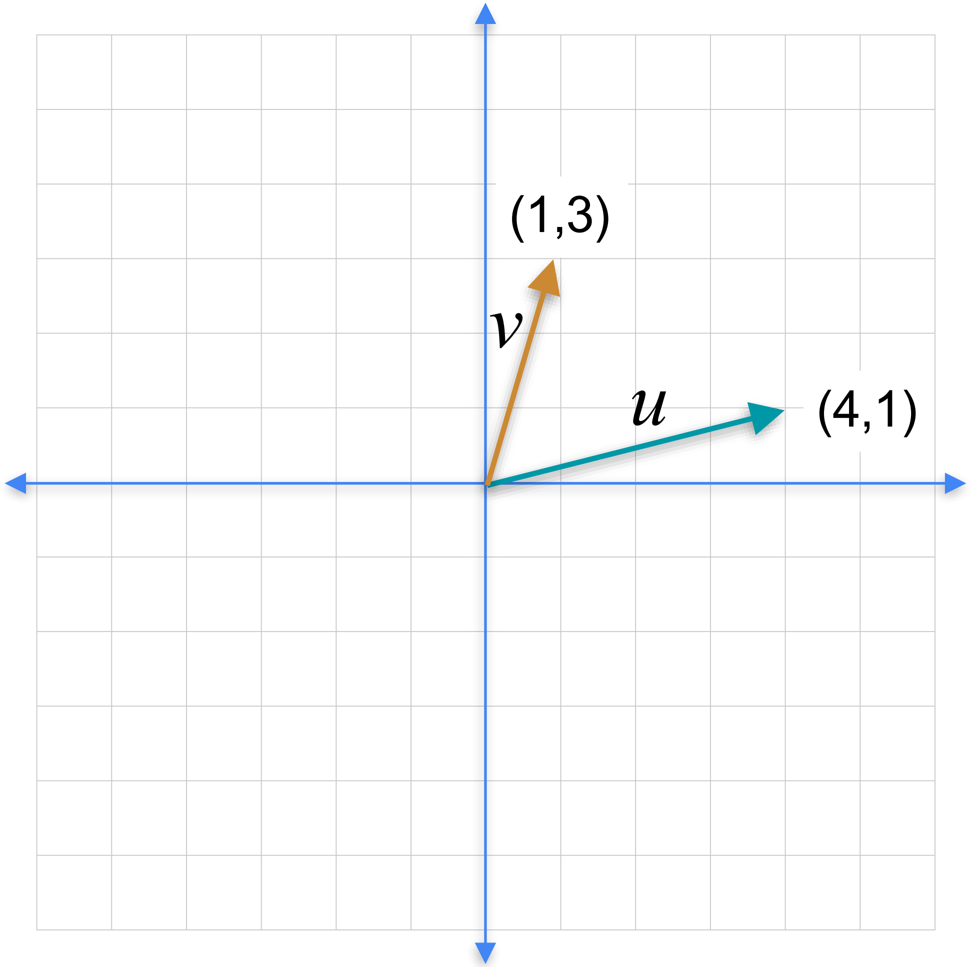


$$u + v = (4 + 1, 1 + 3) = (5,4)$$

向量相减

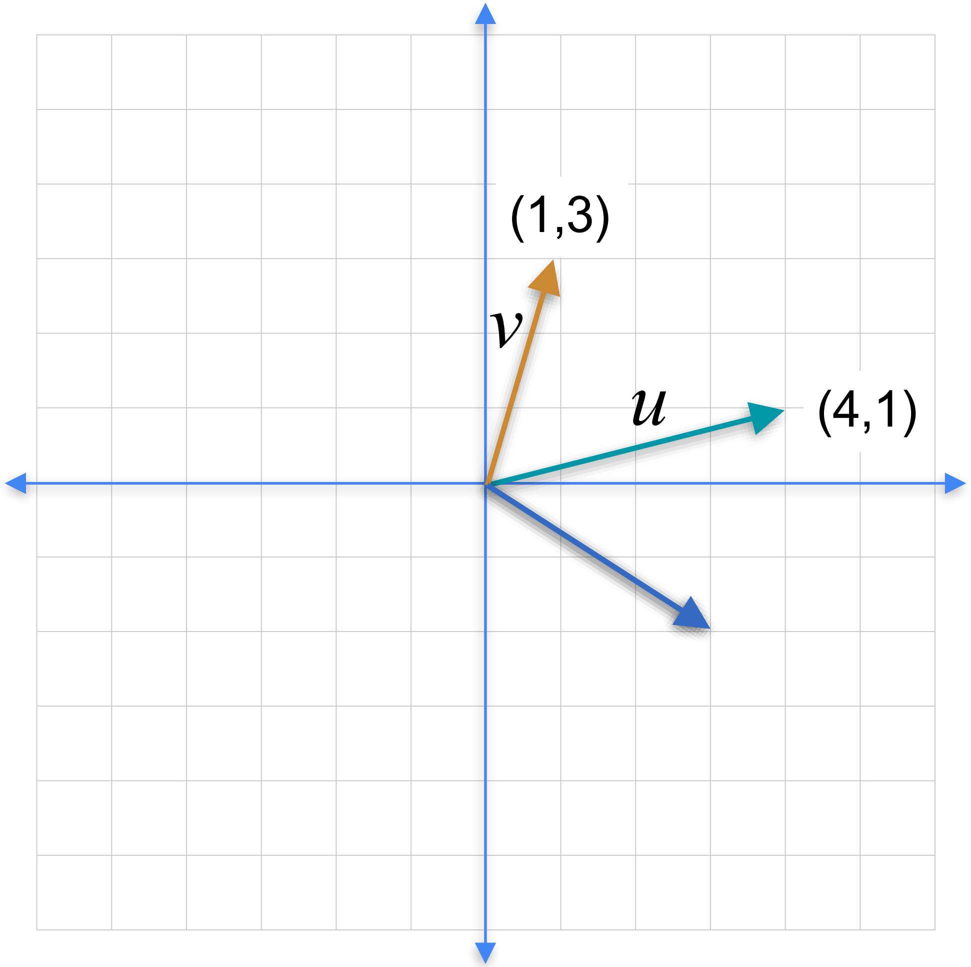


向量相减



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

向量相减



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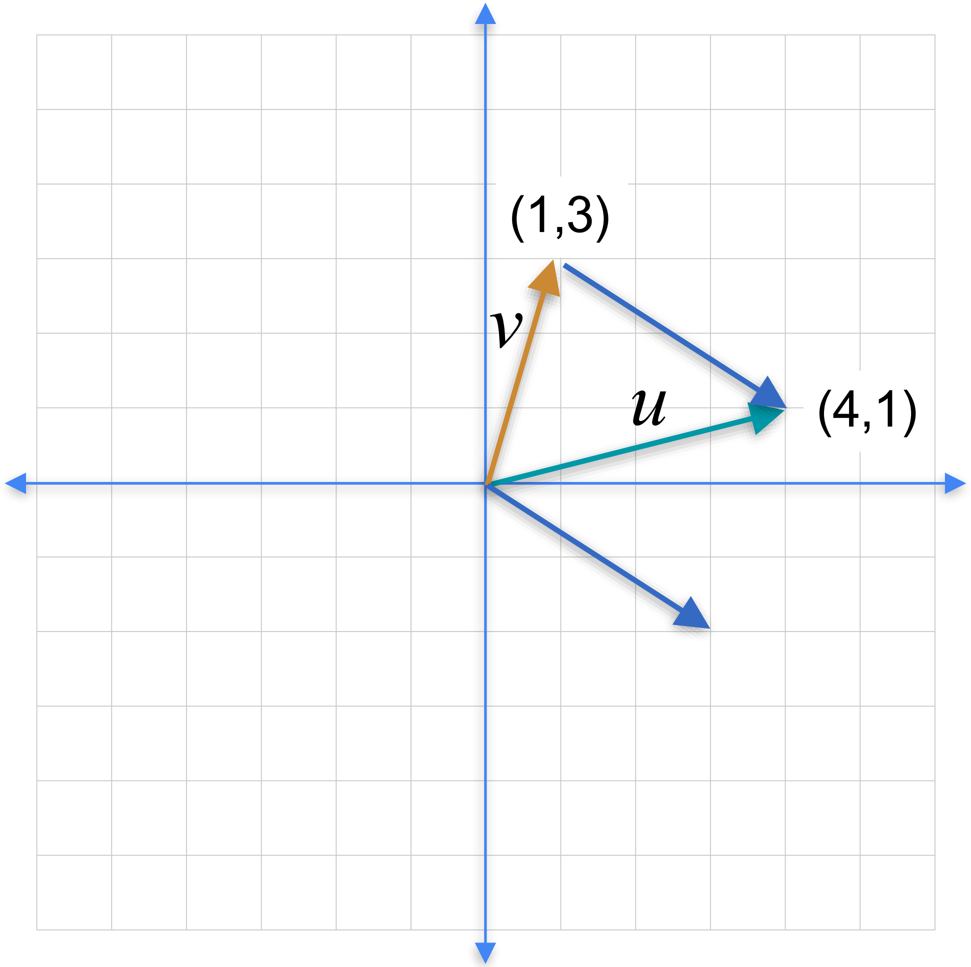

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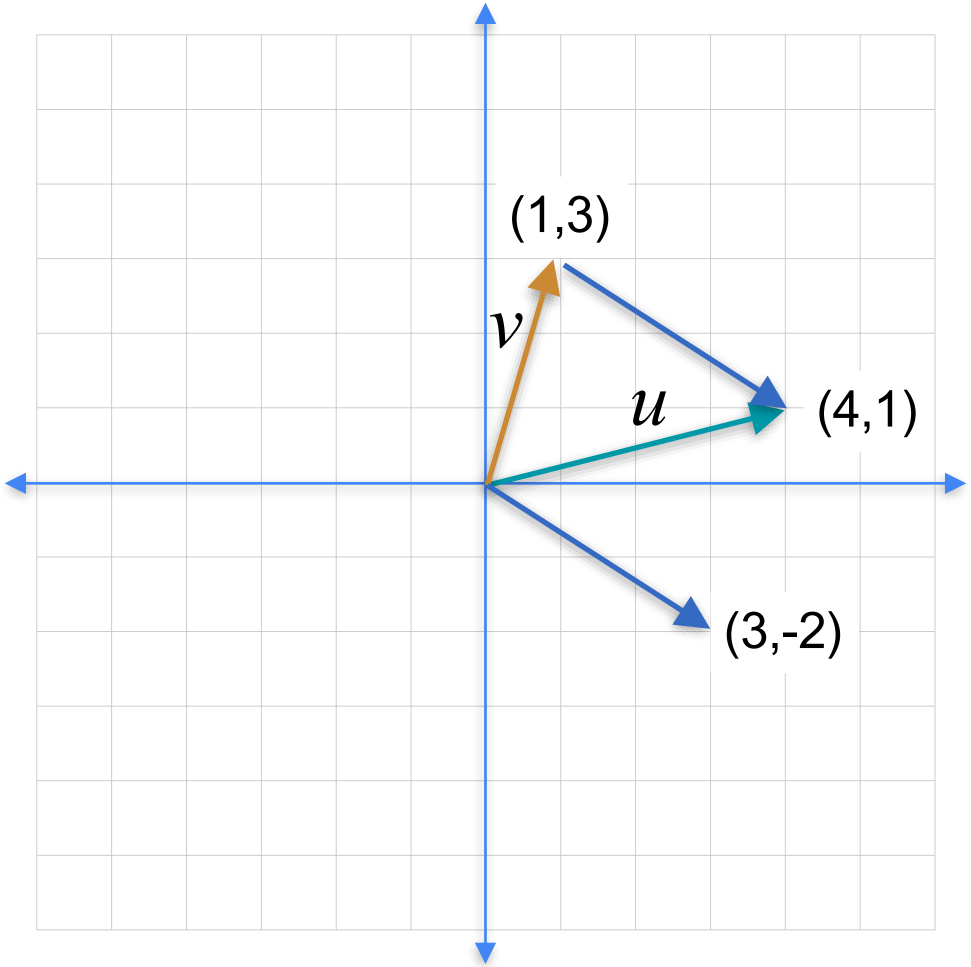
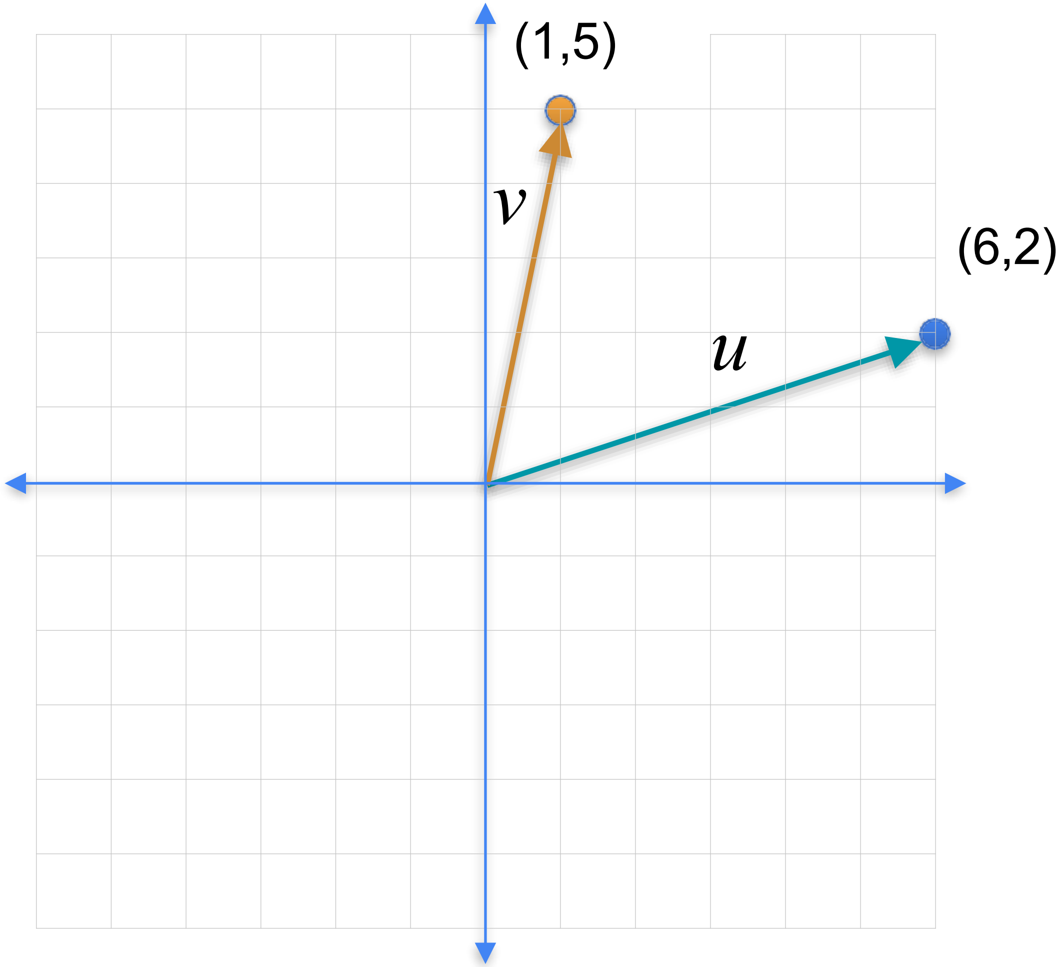


$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

向量距离



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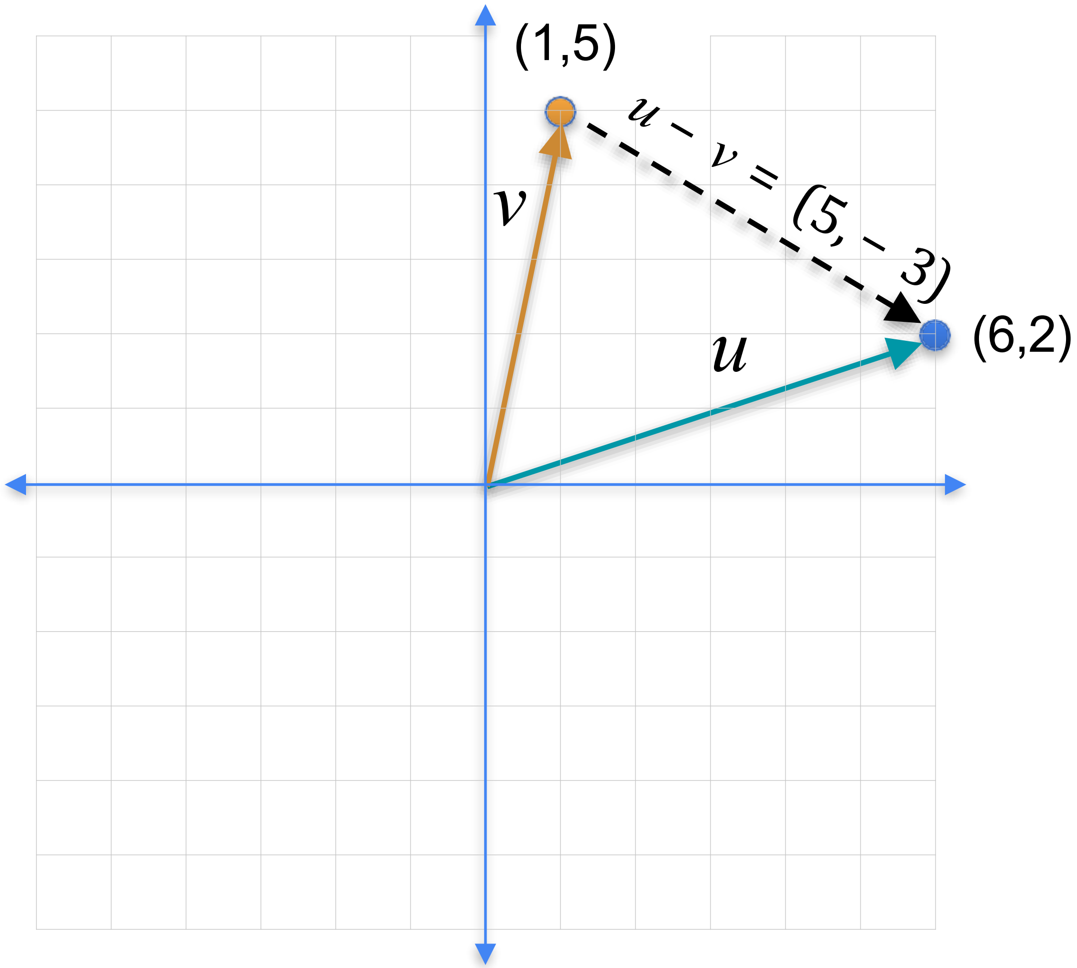

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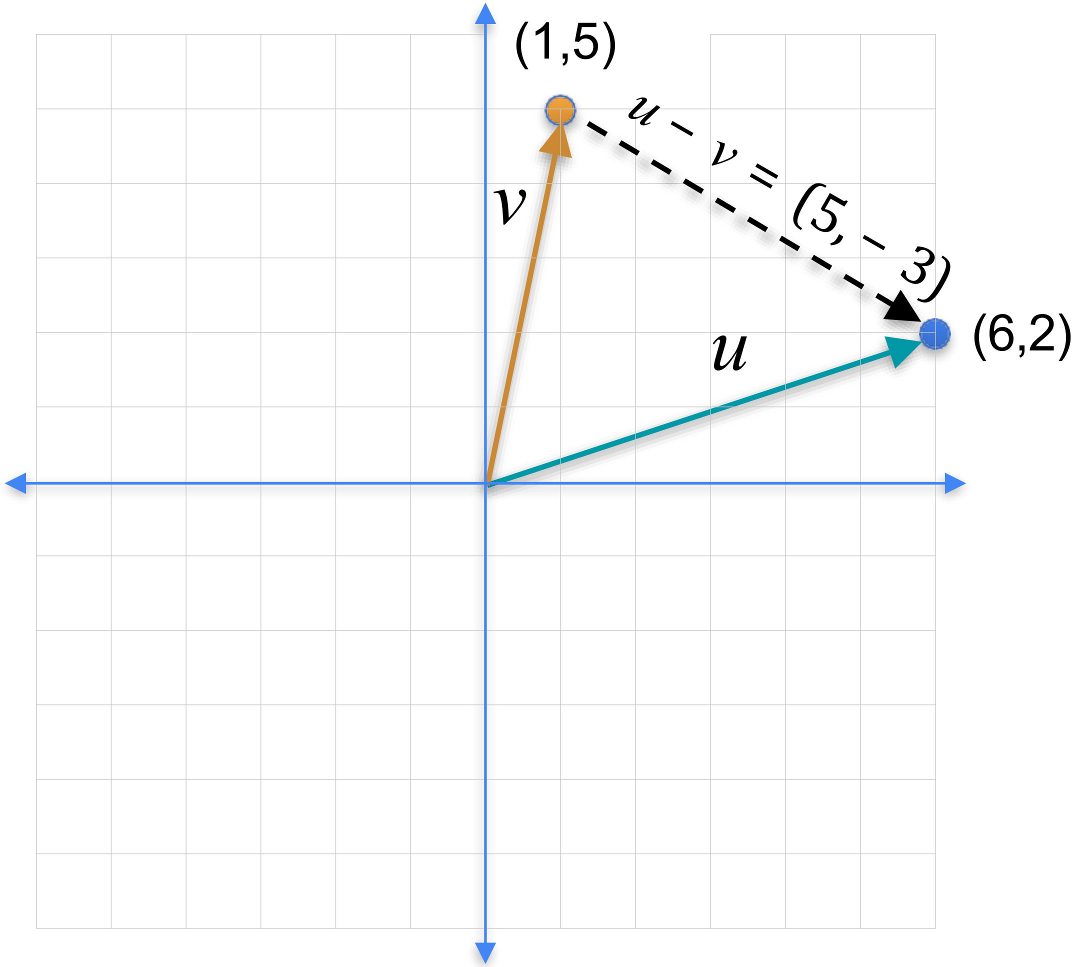
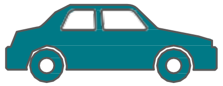
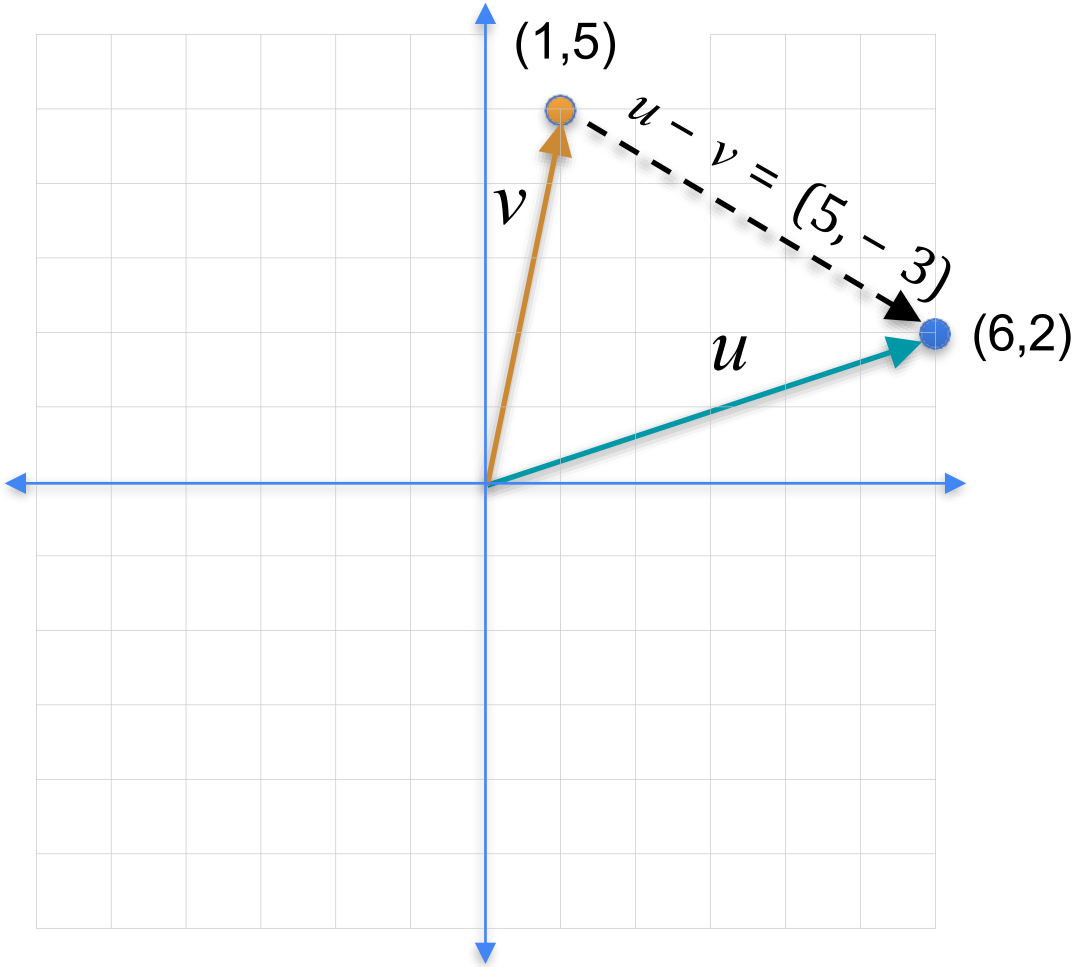
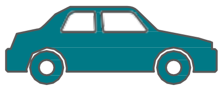




$$|u - v|_1 = |5| + |-3| = 8$$

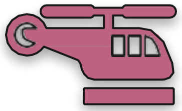
向量距离





L1-distance

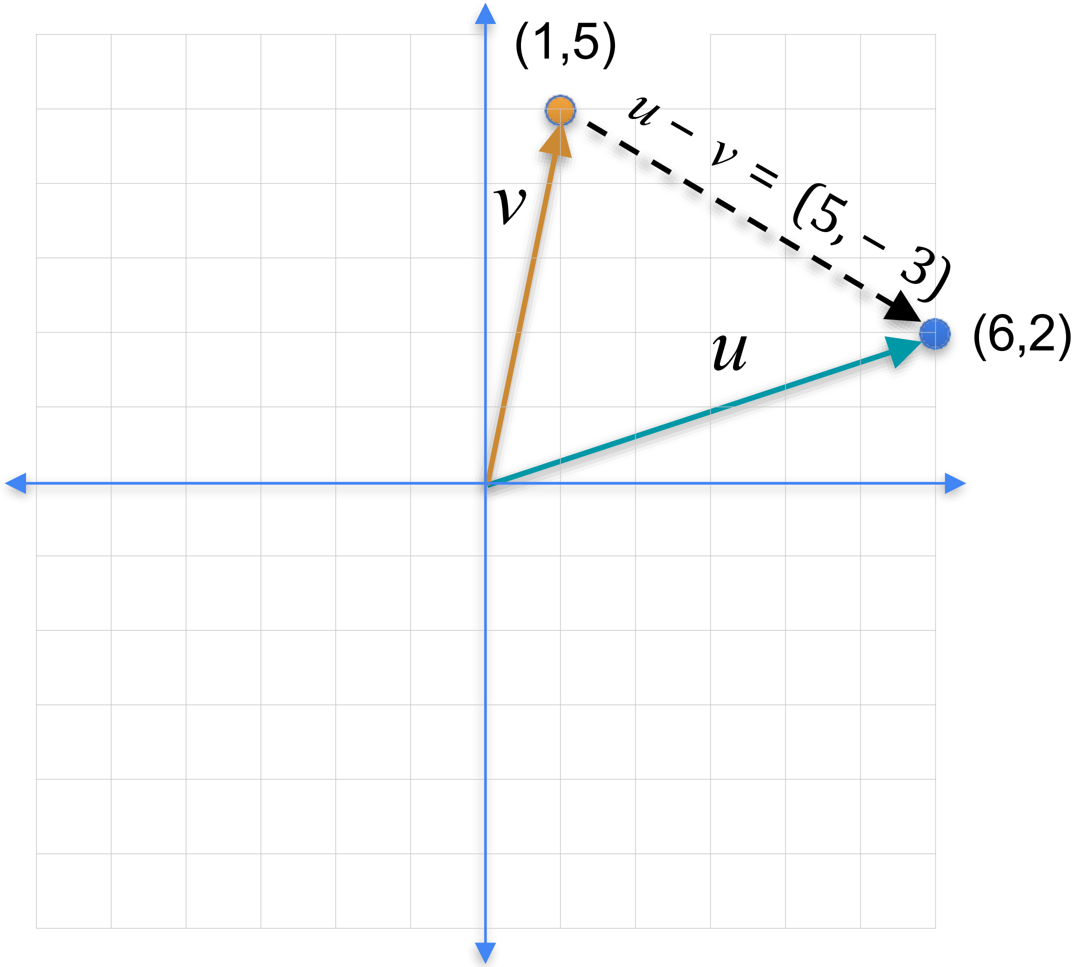
$$|u - v|_1 = |5| + |-3| = 8$$

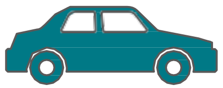


L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

向量距离





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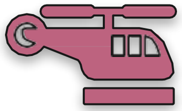

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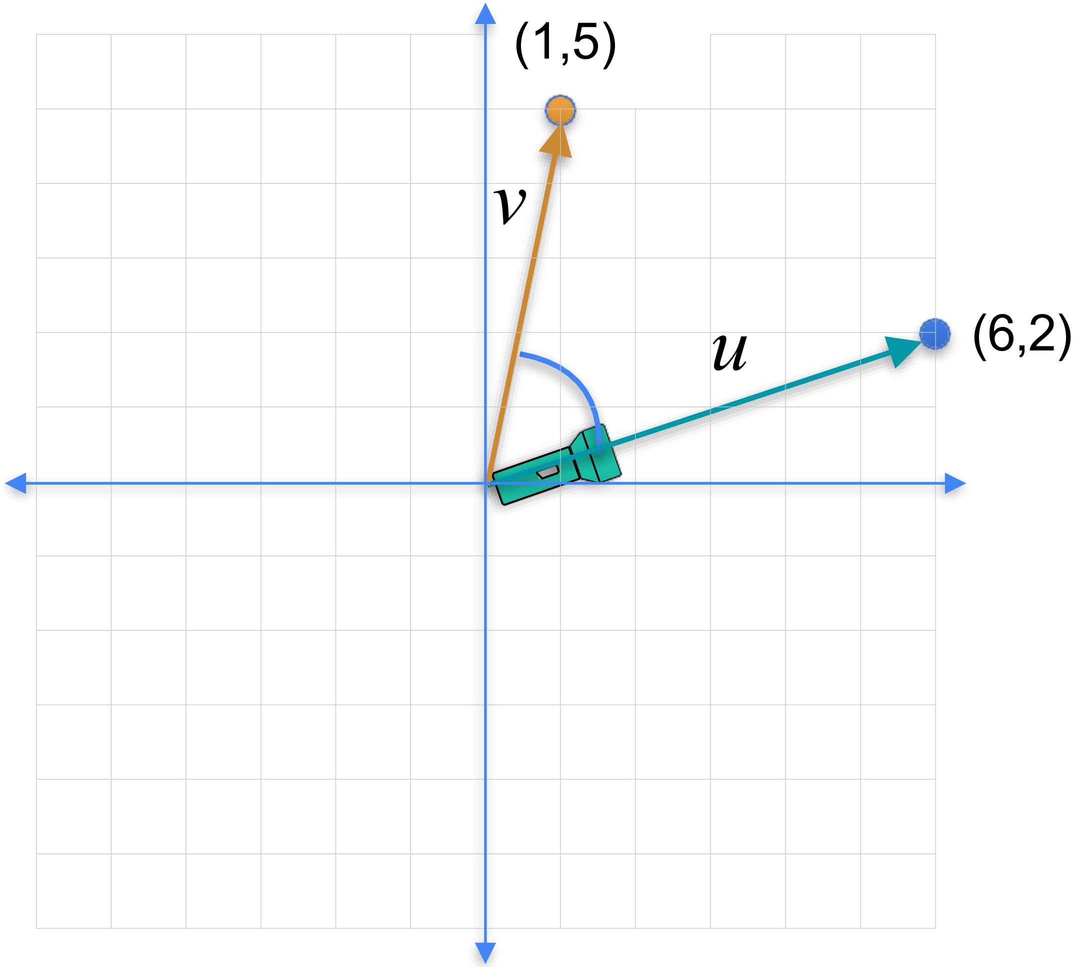

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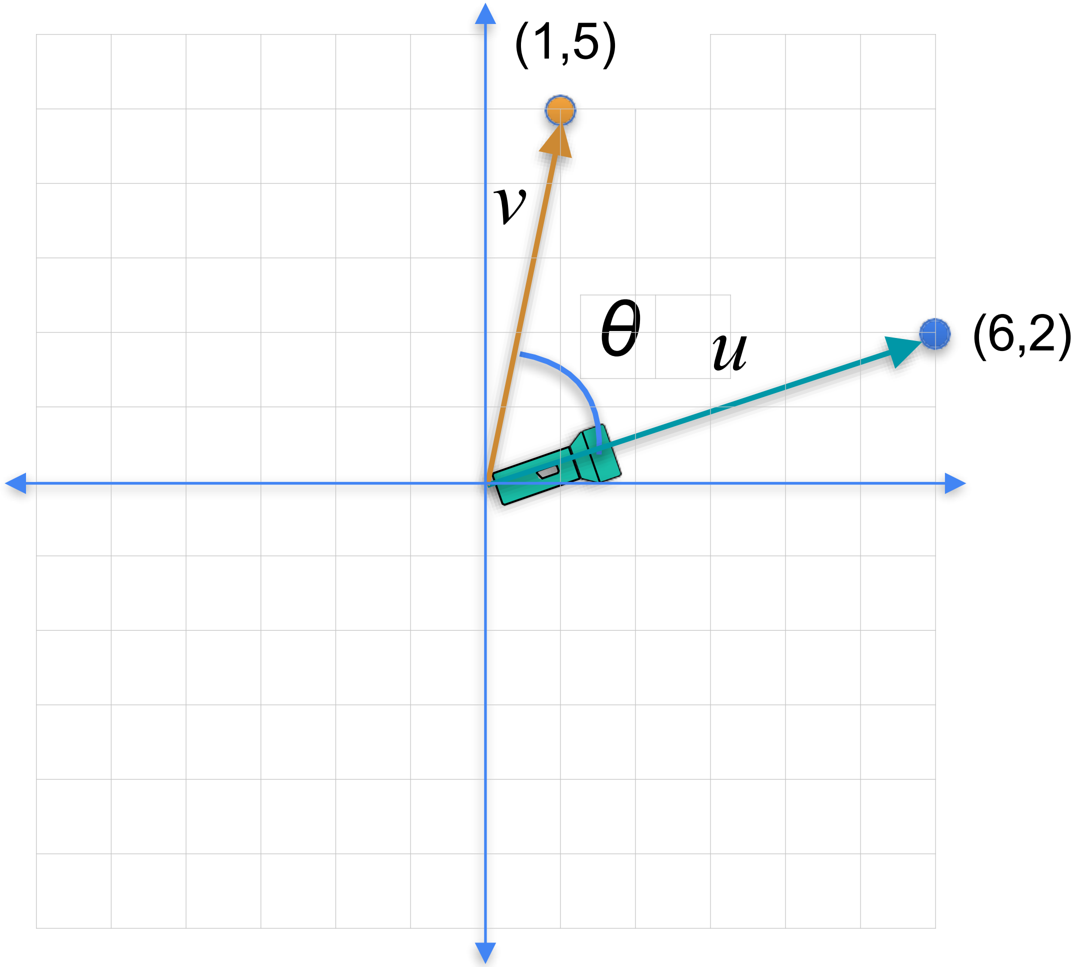

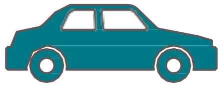

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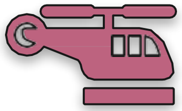

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

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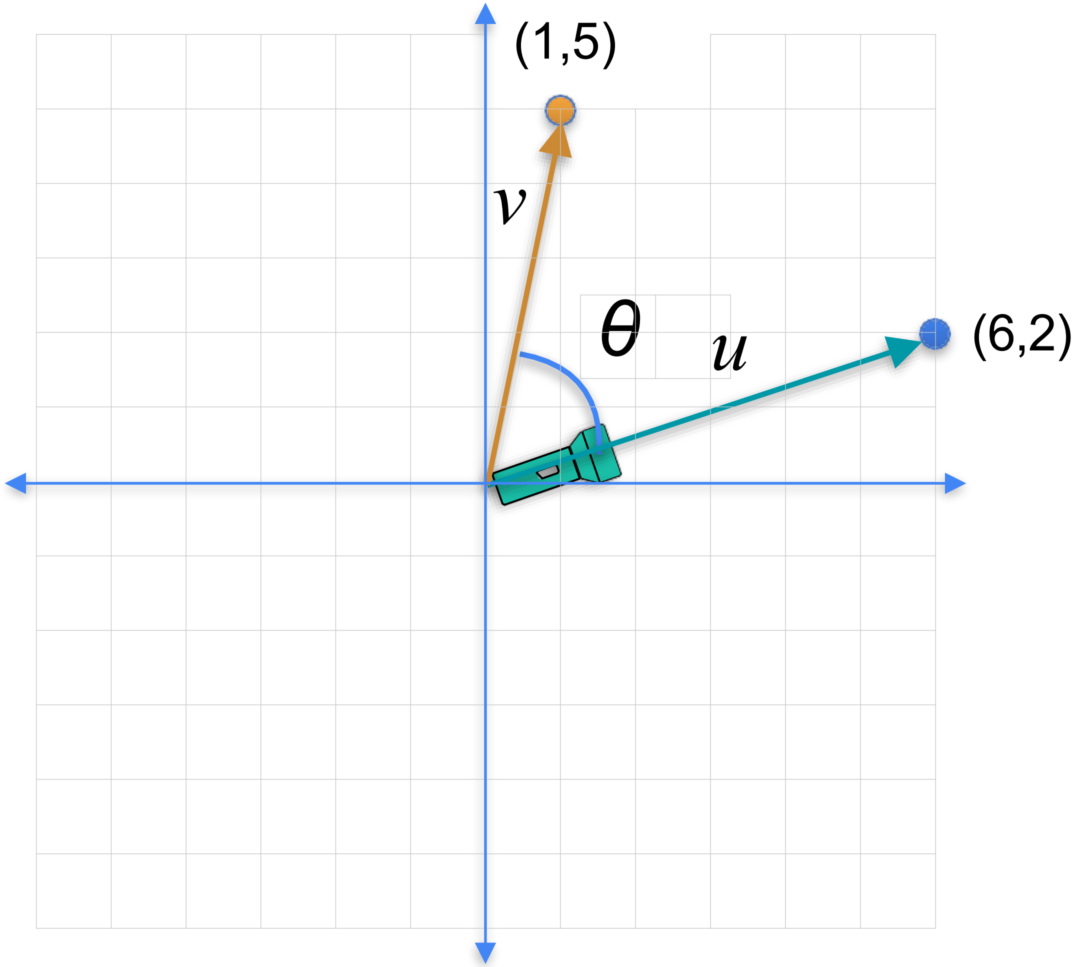

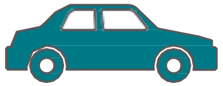

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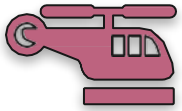

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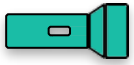
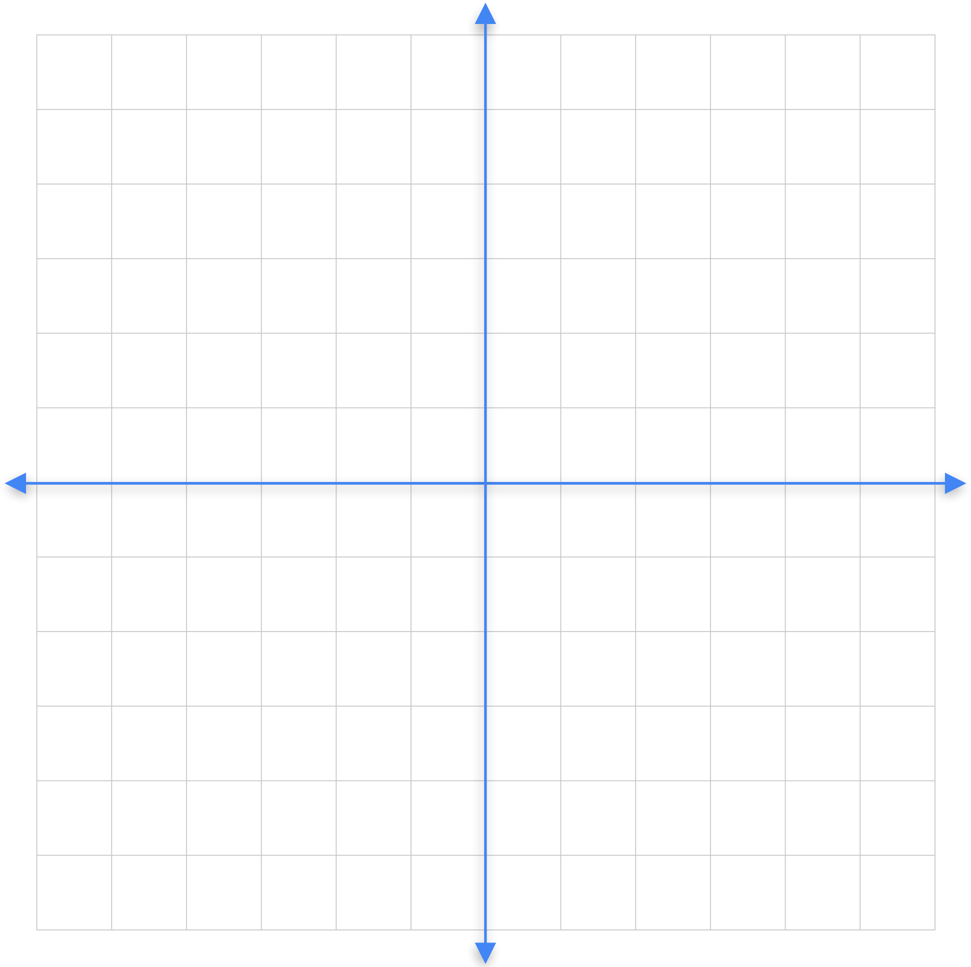


Cosine distance

$$\cos(\theta)$$

向量的数乘



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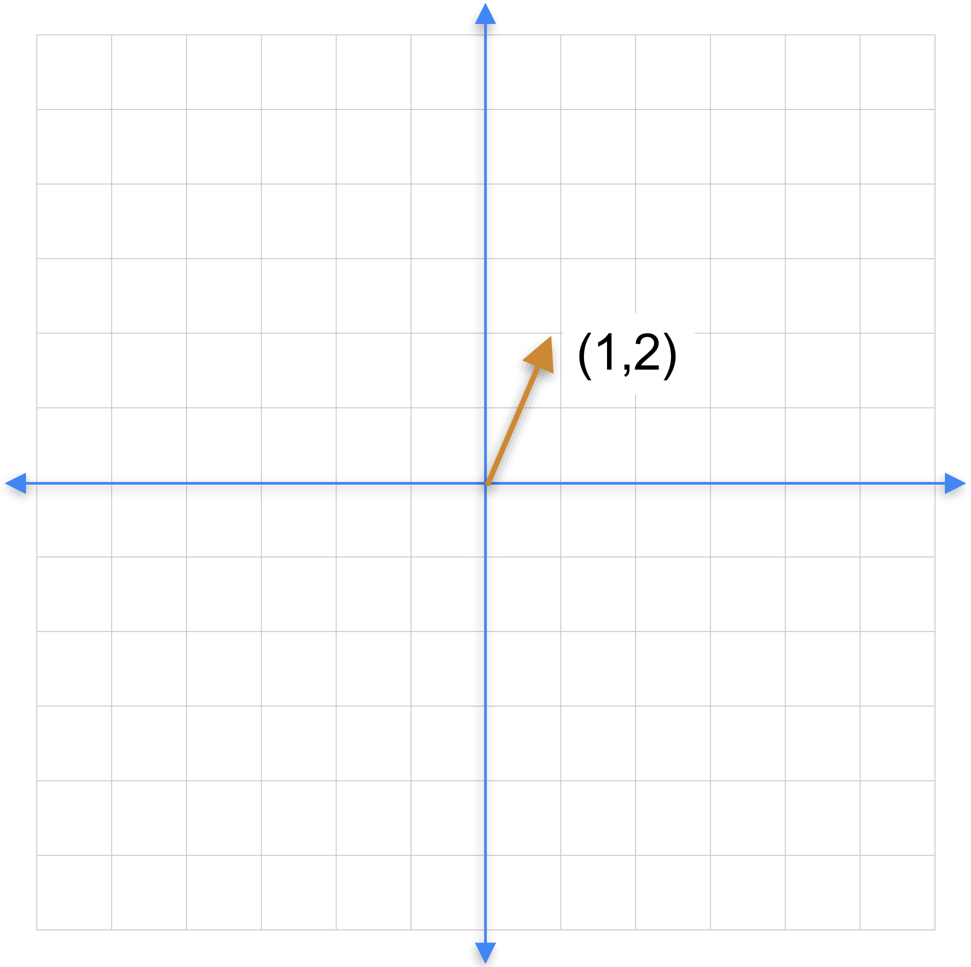

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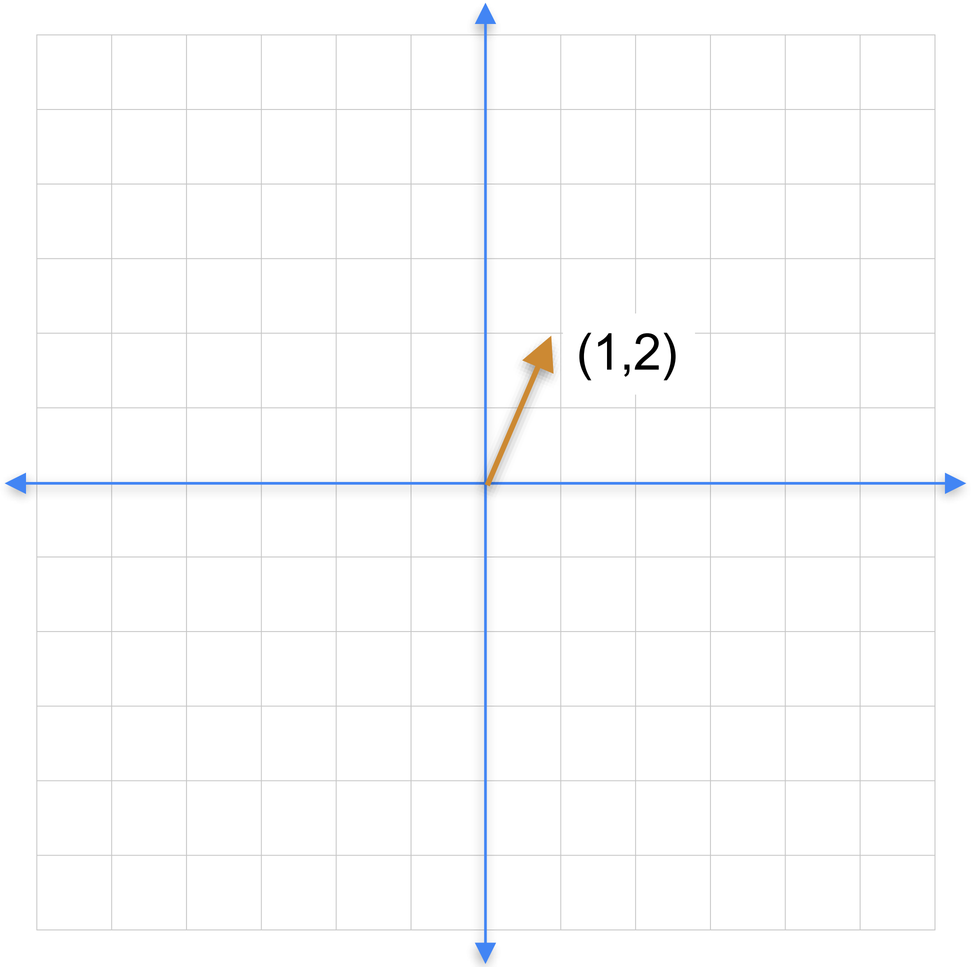
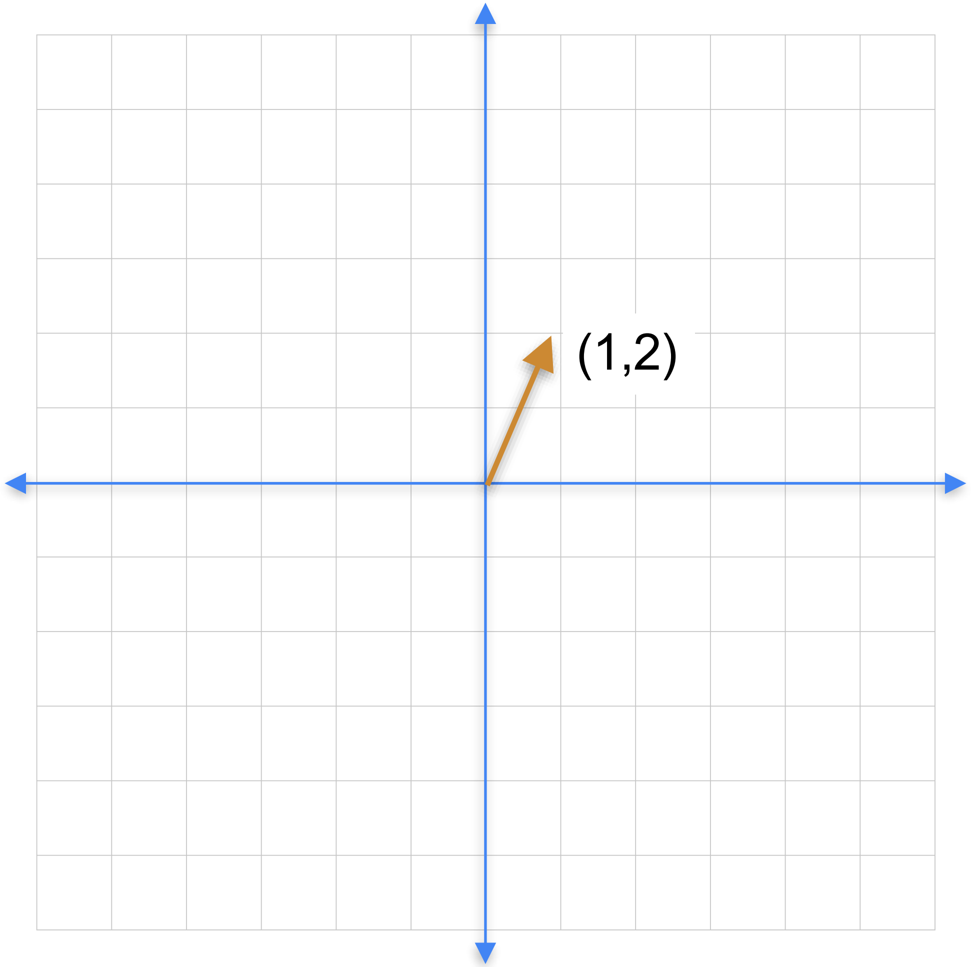


$u = (1,2)$

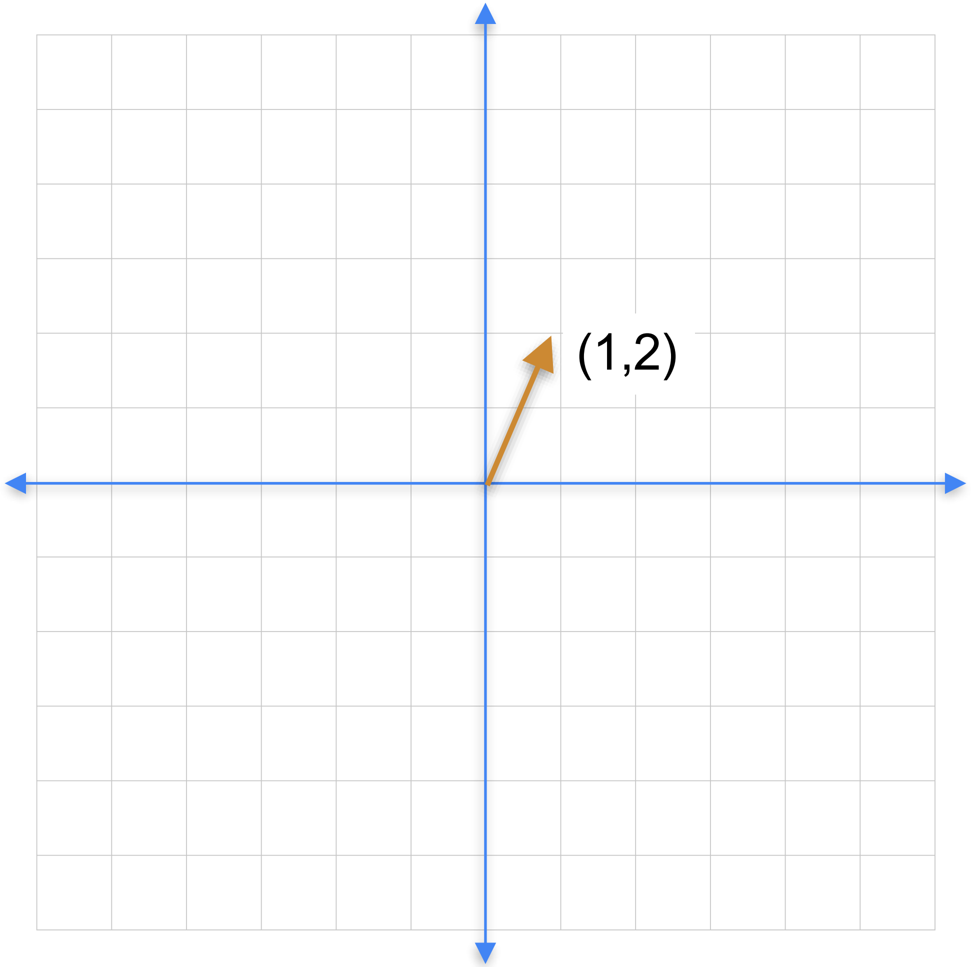
向量的数乘



$u = (1,2)$

$\lambda = 3$

向量的数乘

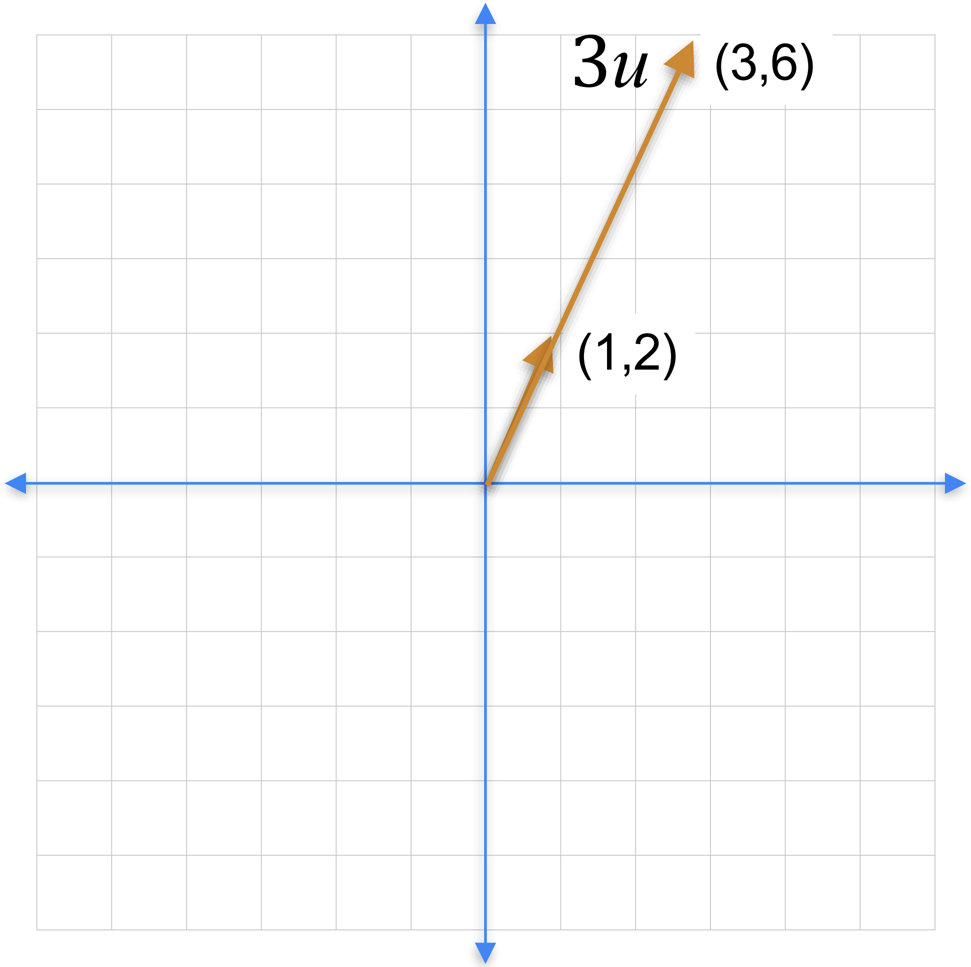


$u = (1,2)$

$\lambda = 3$

$\lambda u = (3,6)$

向量的数乘

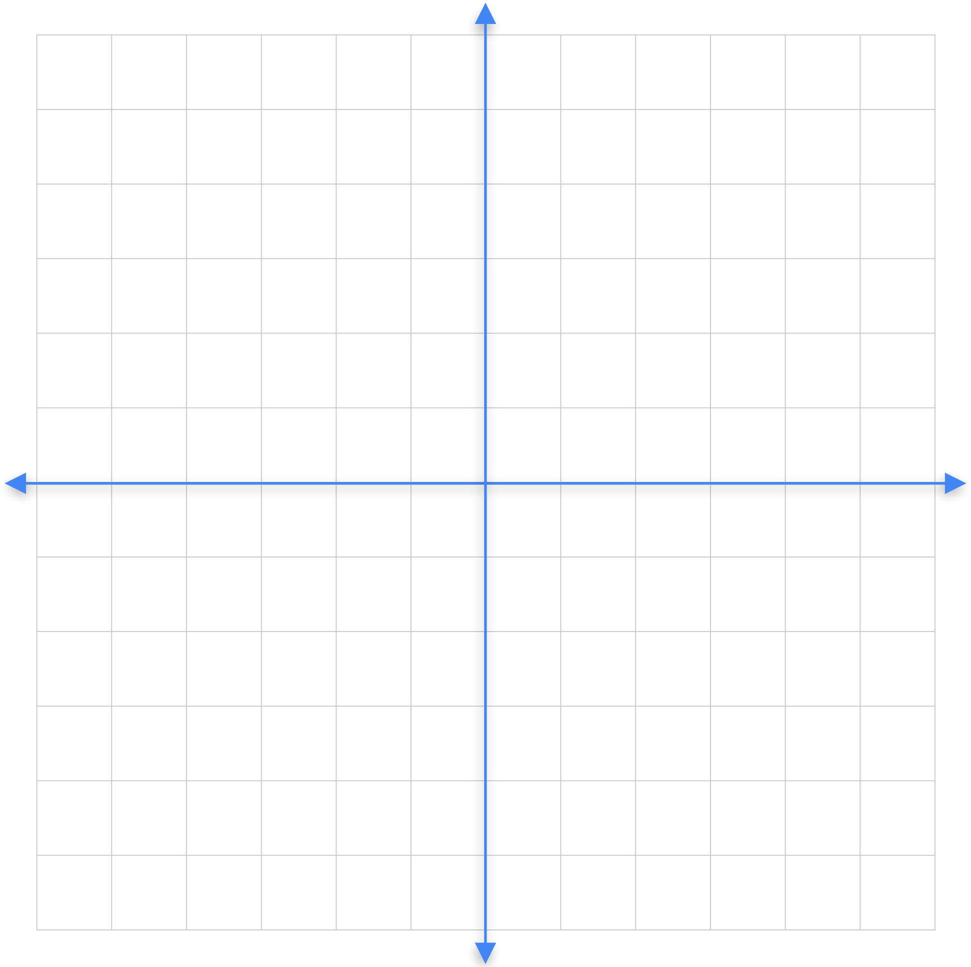


$u = (1,2)$

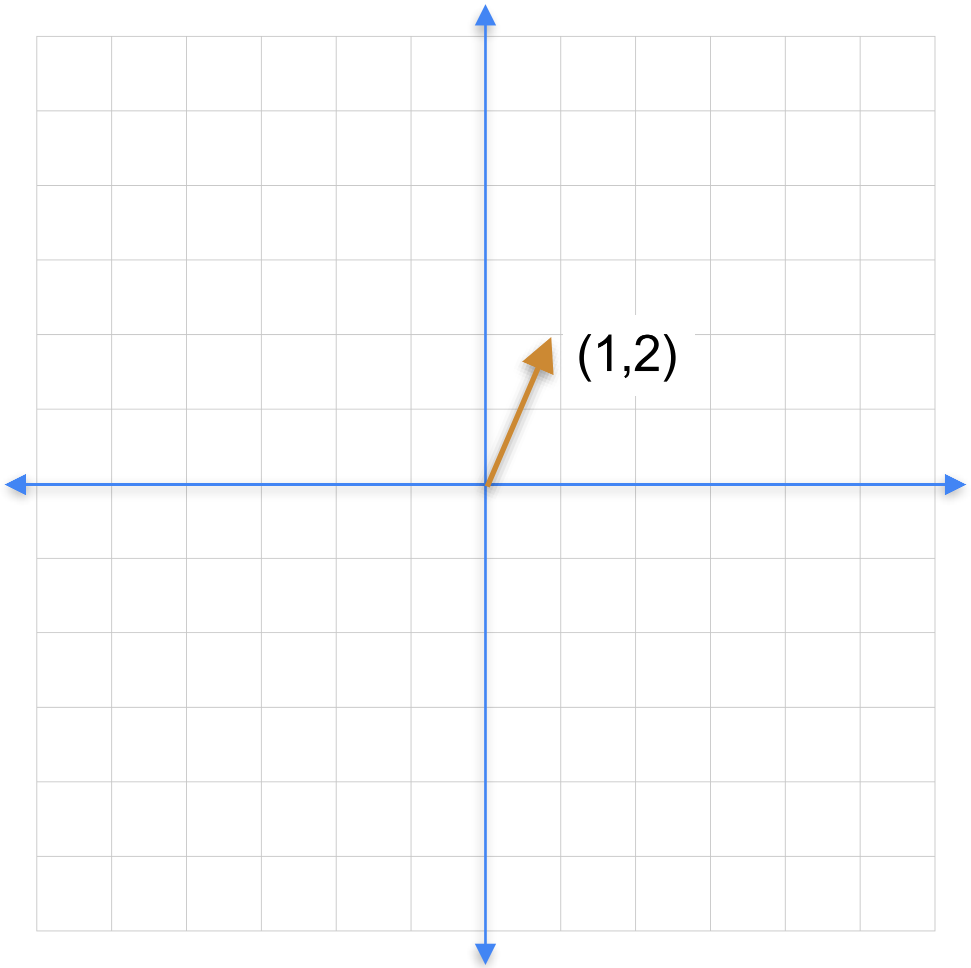
$\lambda = 3$

$\lambda u = (3,6)$

如果系数为负？

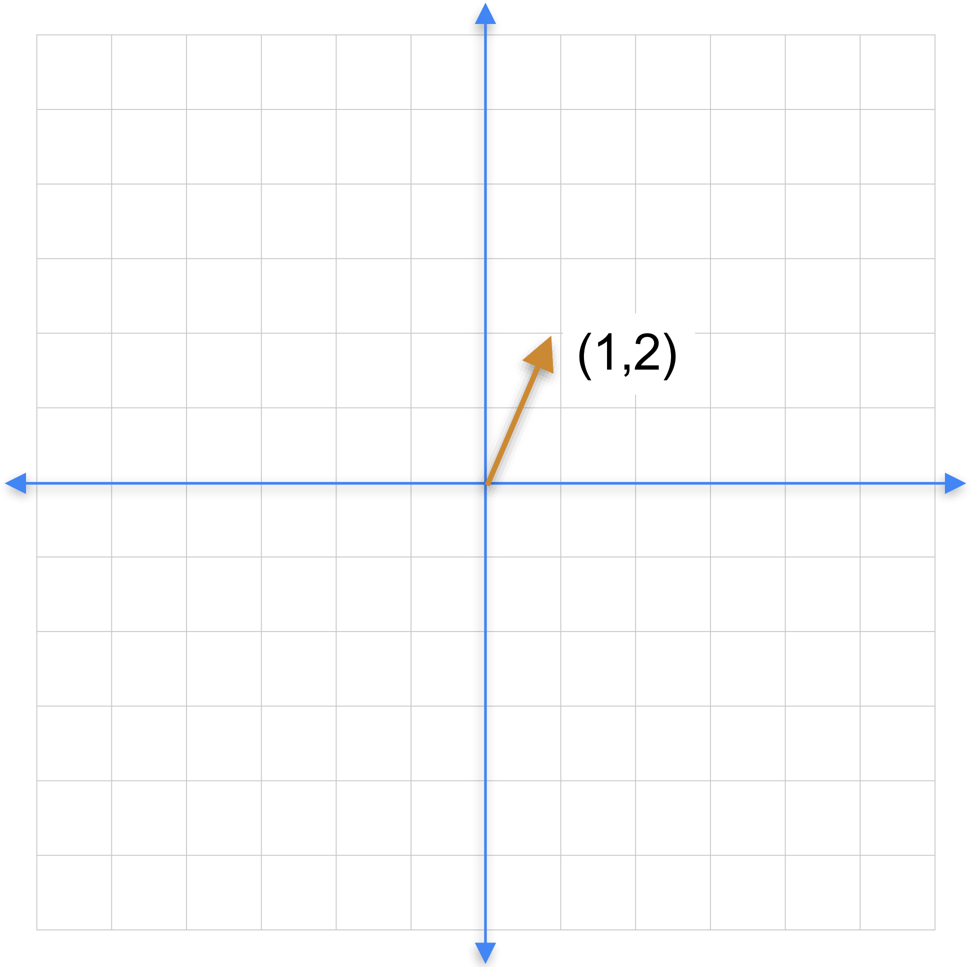


如果系数为负？



$u = (1,2)$

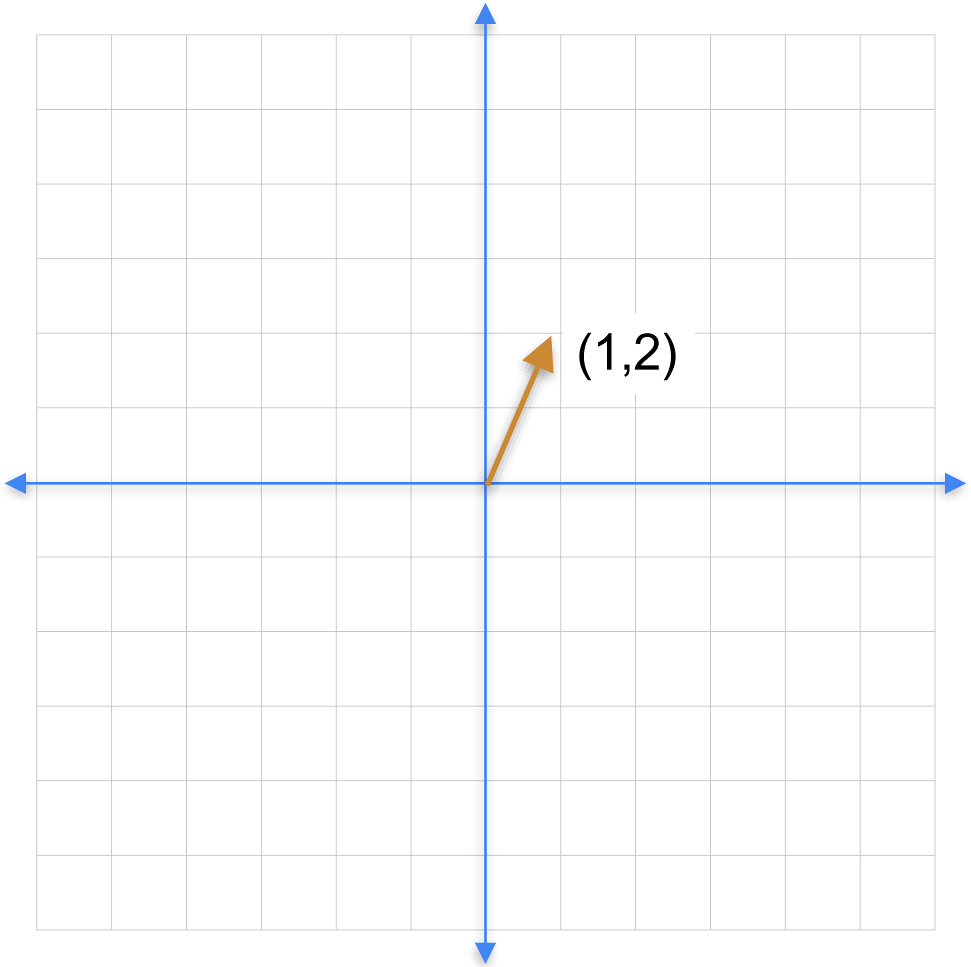
如果系数为负？



$u = (1,2)$

$\lambda = -2$

如果系数为负？

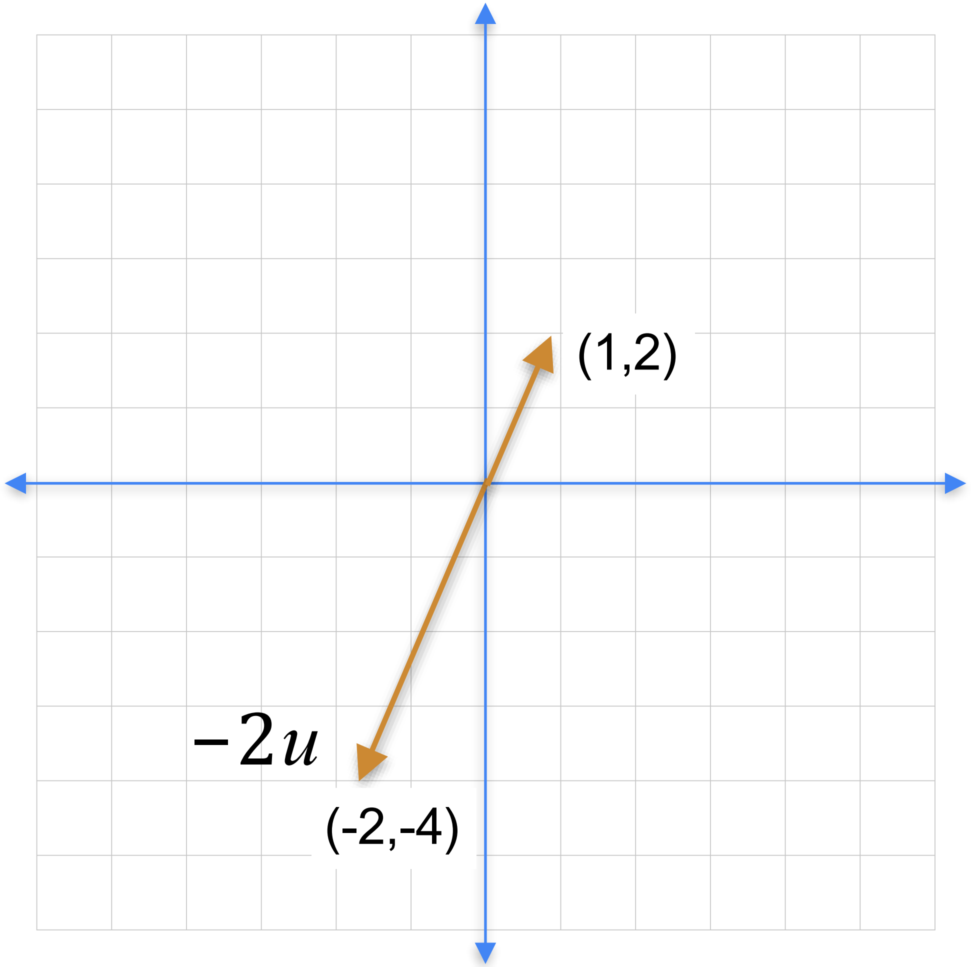


$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

如果系数为负？



$u = (1,2)$

$\lambda = -2$

$\lambda u = (-2, -4)$

线性操作如何简单表达？

线性操作如何简单表达？

数量
2个苹果
4个香蕉
1个樱桃

线性操作如何简单表达？

数量	价格
2个苹果	苹果：3元一个
4个香蕉	香蕉：5元一个
1个樱桃	樱桃：2元一个

线性操作如何简单表达？

数量	价格	总价格
2个苹果	苹果：3元一个	
4个香蕉	香蕉：5元一个	
1个樱桃	樱桃：2元一个	

线性操作如何简单表达？

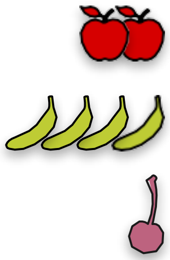
数量

2个苹果
4个香蕉
1个樱桃

价格

苹果：3元一个
香蕉：5元一个
樱桃：2元一个

总价格



2
4
1

线性操作如何简单表达？

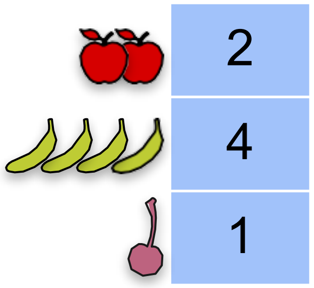
数量

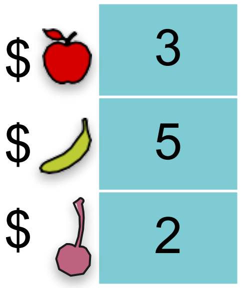
2个苹果
4个香蕉
1个樱桃

价格

苹果：3元一个
香蕉：5元一个
樱桃：2元一个

总价格





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价格
苹果：3元一个
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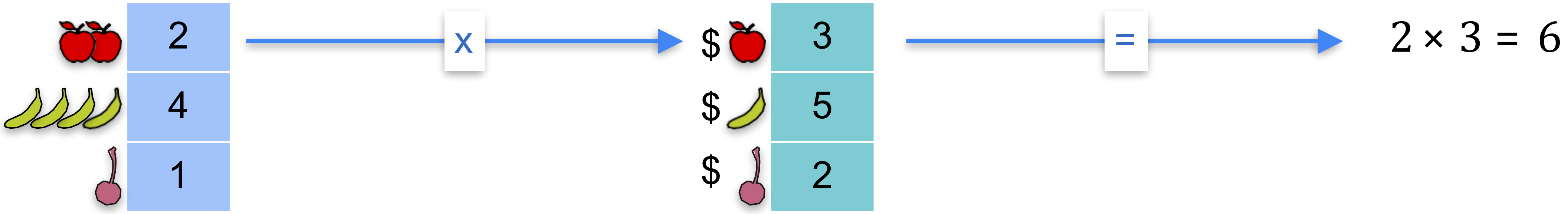
总价格



线性操作如何简单表达？

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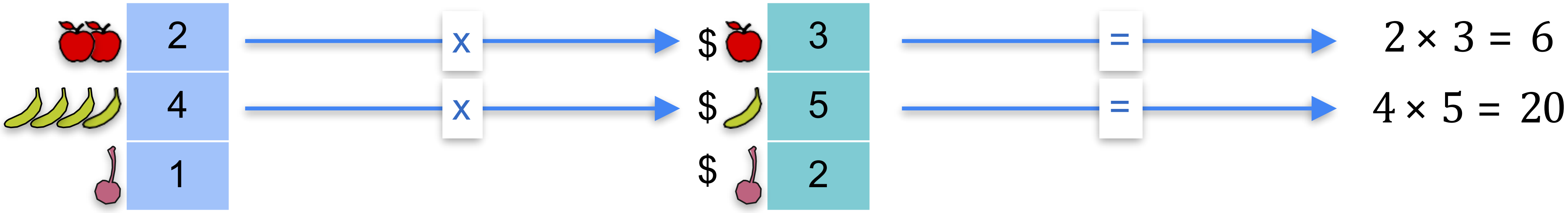
总价格



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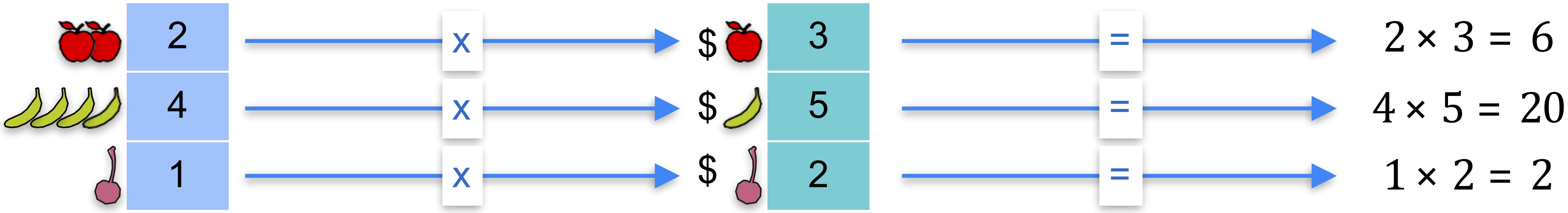
总价格



线性操作如何简单表达？

数量
2个苹果
4个香蕉
1个樱桃

价格
苹果：3元一个
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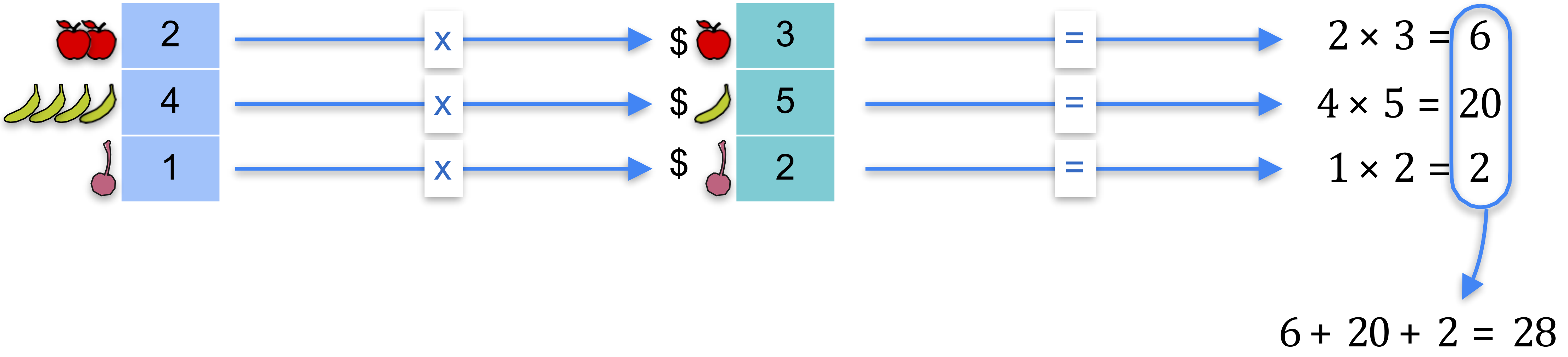
总价格



线性操作如何简单表达?

数量

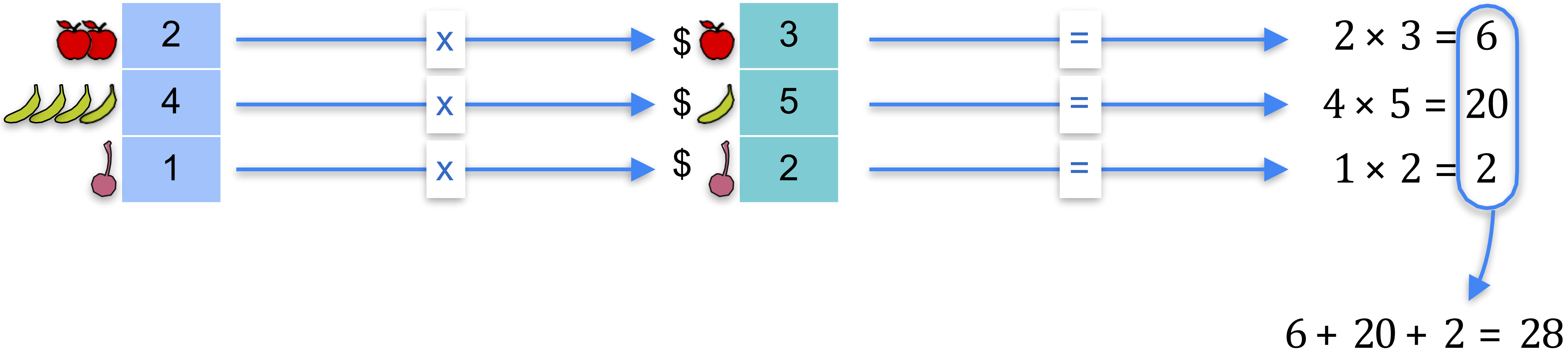
2个苹果
4个香蕉
1个樱桃

价格

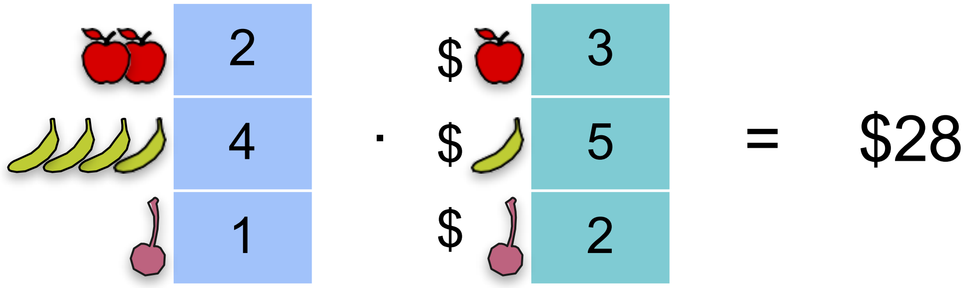
苹果：3元一个
香蕉：5元一个
樱桃：2元一个

总价格

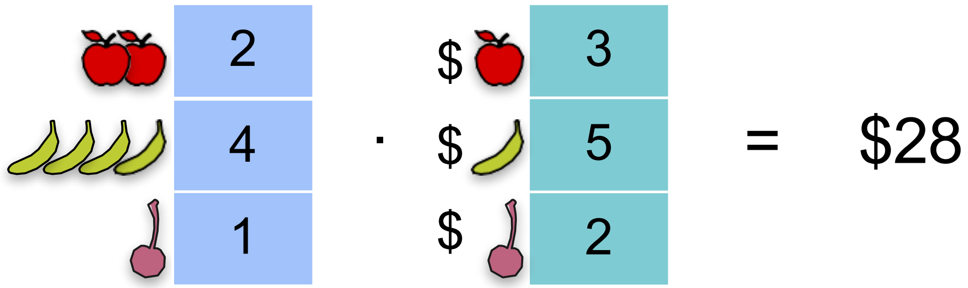
6 + 20 + 2 = 28



点积（dot product）

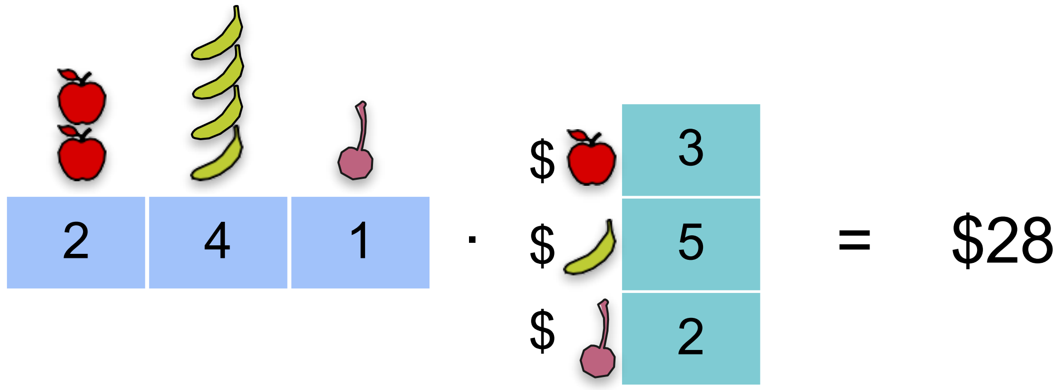


点积（dot product）



$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

点积（dot product）



$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

点积（dot product）

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = 28$$

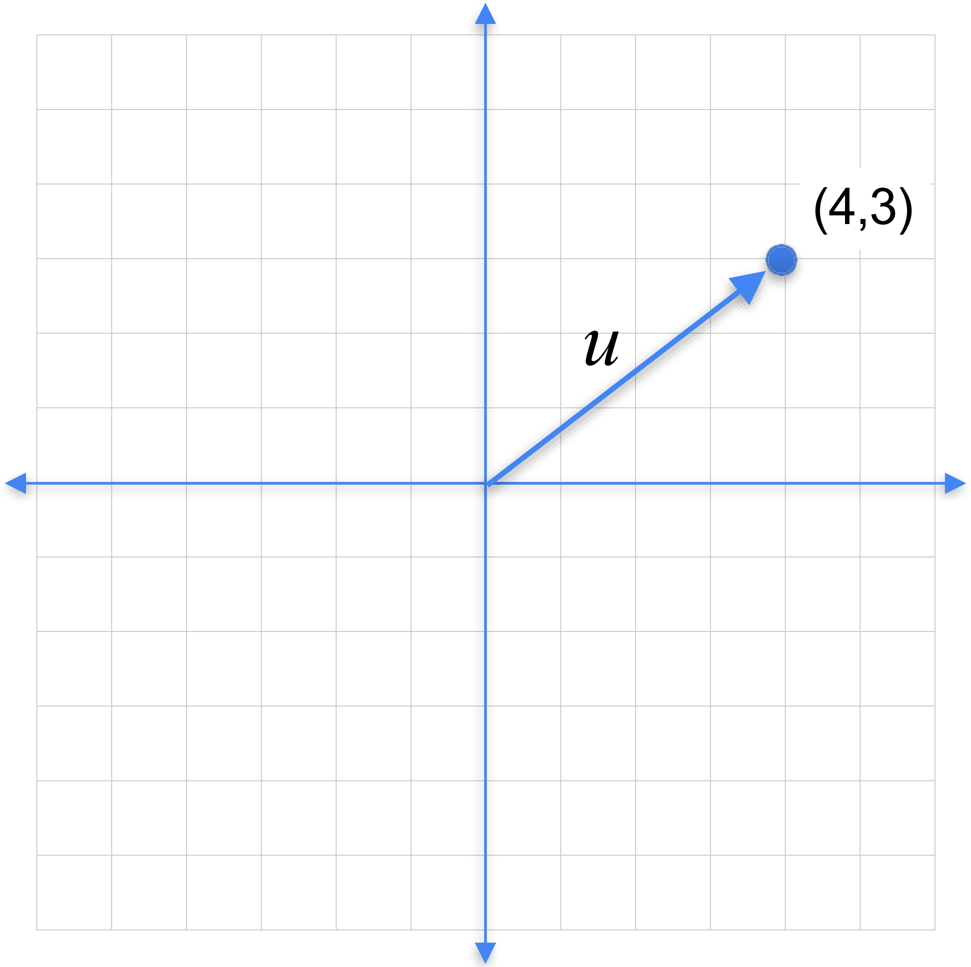
$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

点积（dot product）

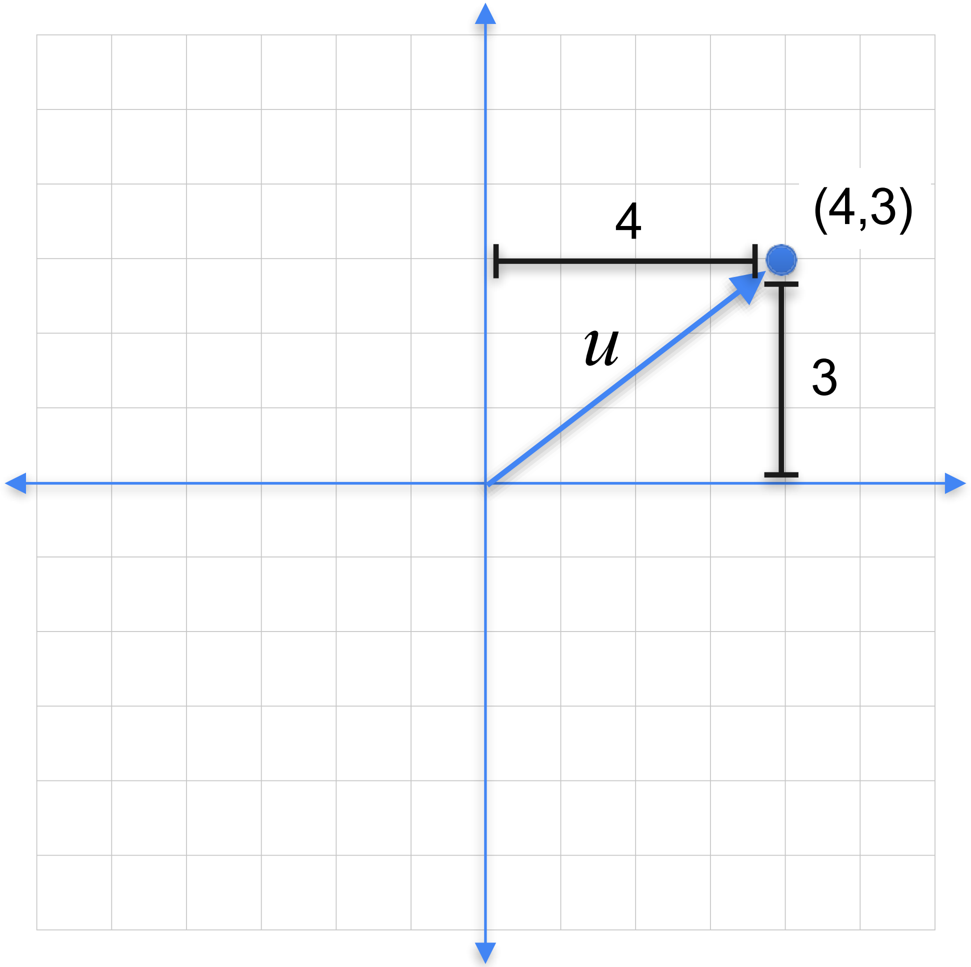
向量$\vec{a} = [a_1, a_2, \cdots, a_n]$和$\vec{b} = [b_1, b_2, \cdots, b_n]$的点积定义为:

$$\vec{a} \cdot \vec{b} = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

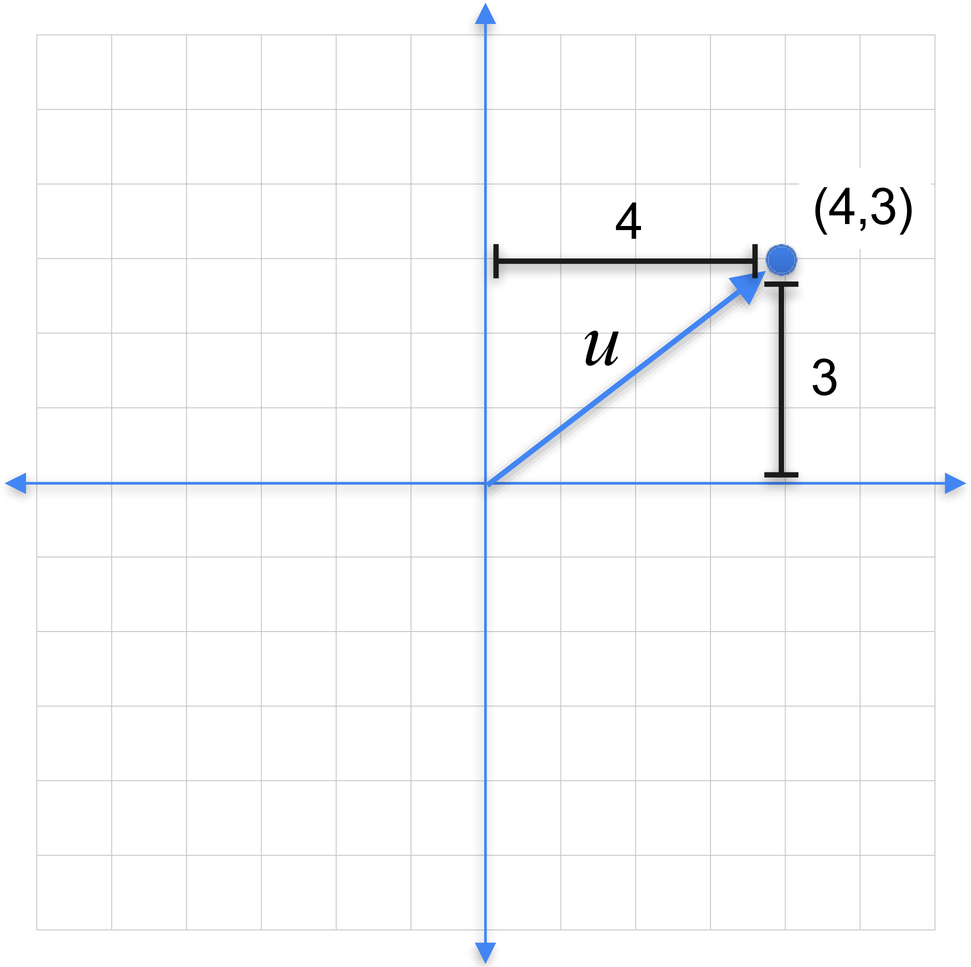
用点积计算范数



用点积计算范数

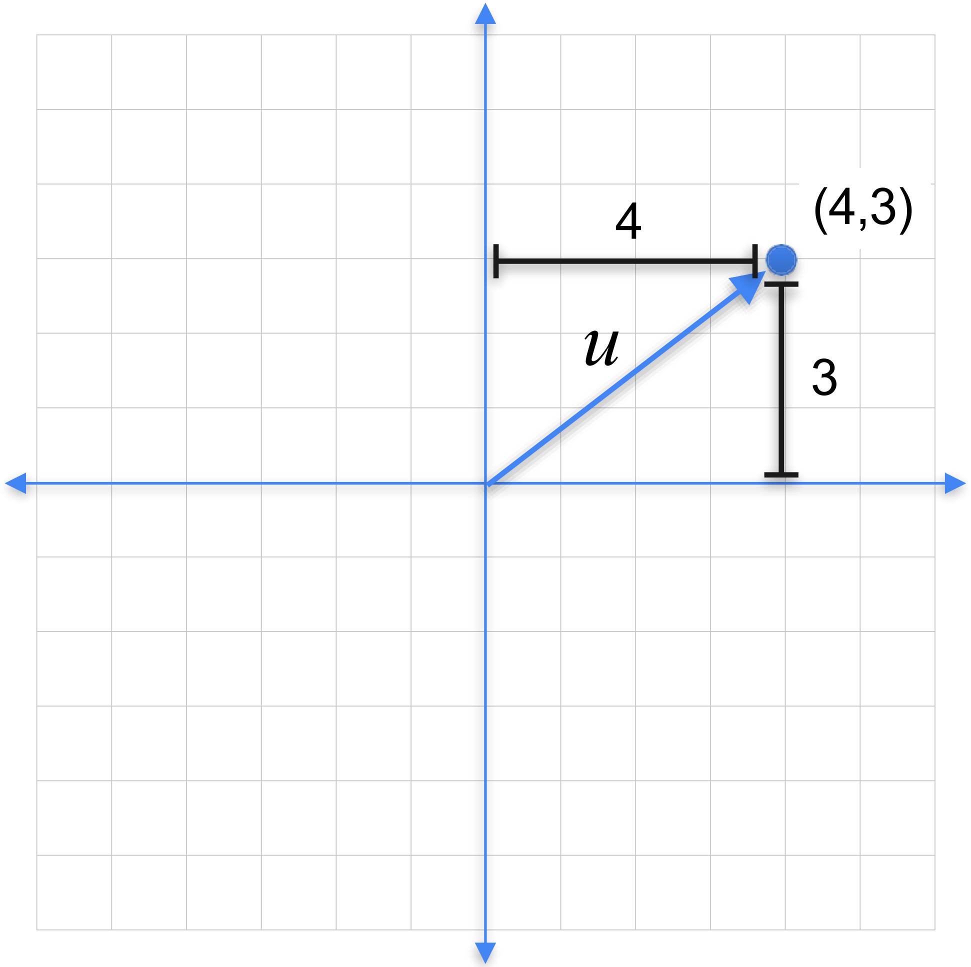


用点积计算范数



$$\sqrt{4^2+3^2} = \sqrt{25} = 5$$

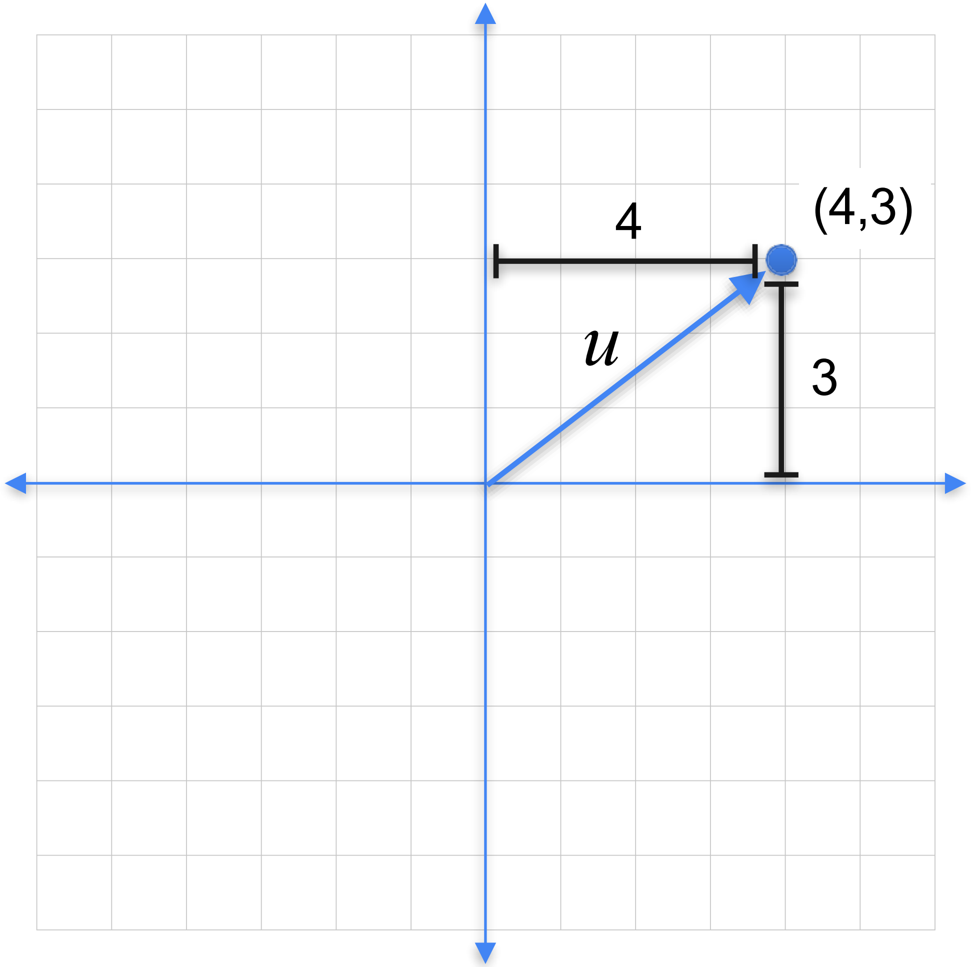
用点积计算范数



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

用点积计算范数

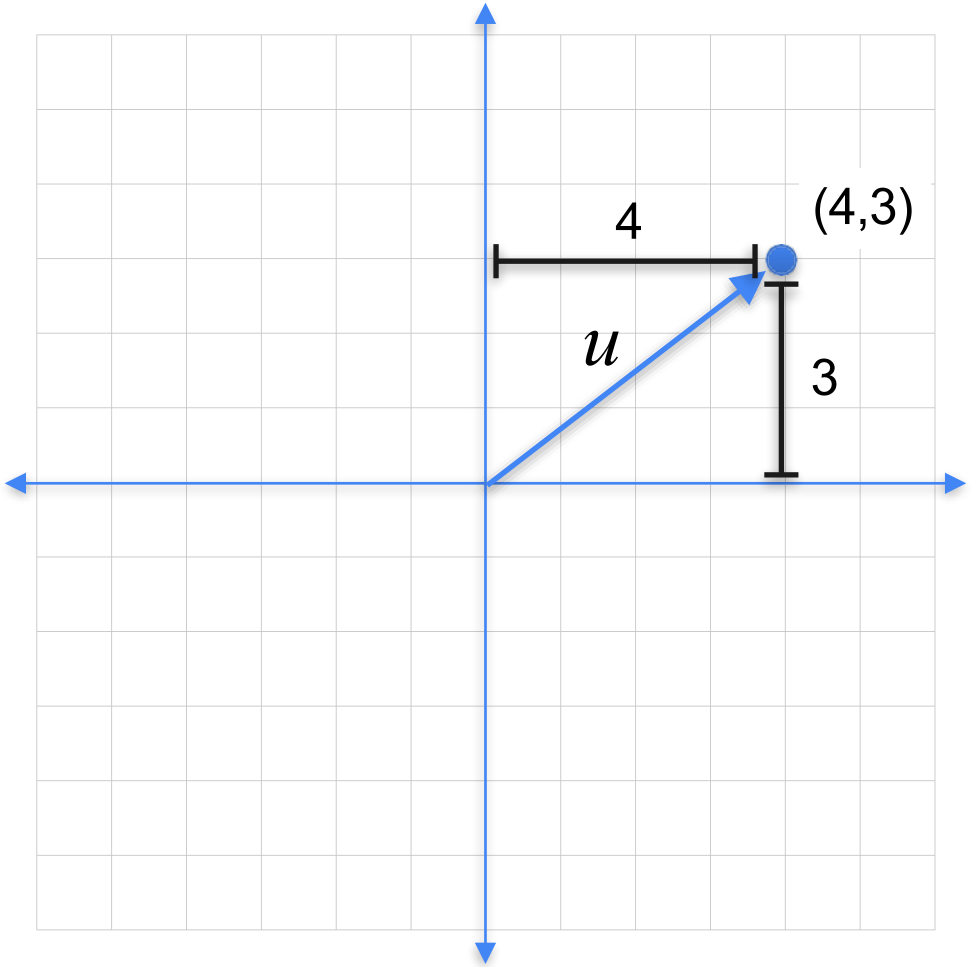


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

4	3	4 / 3	=	25

$$L2 - norm = \sqrt{dot\ \ product(u,u)}$$

用点积计算范数



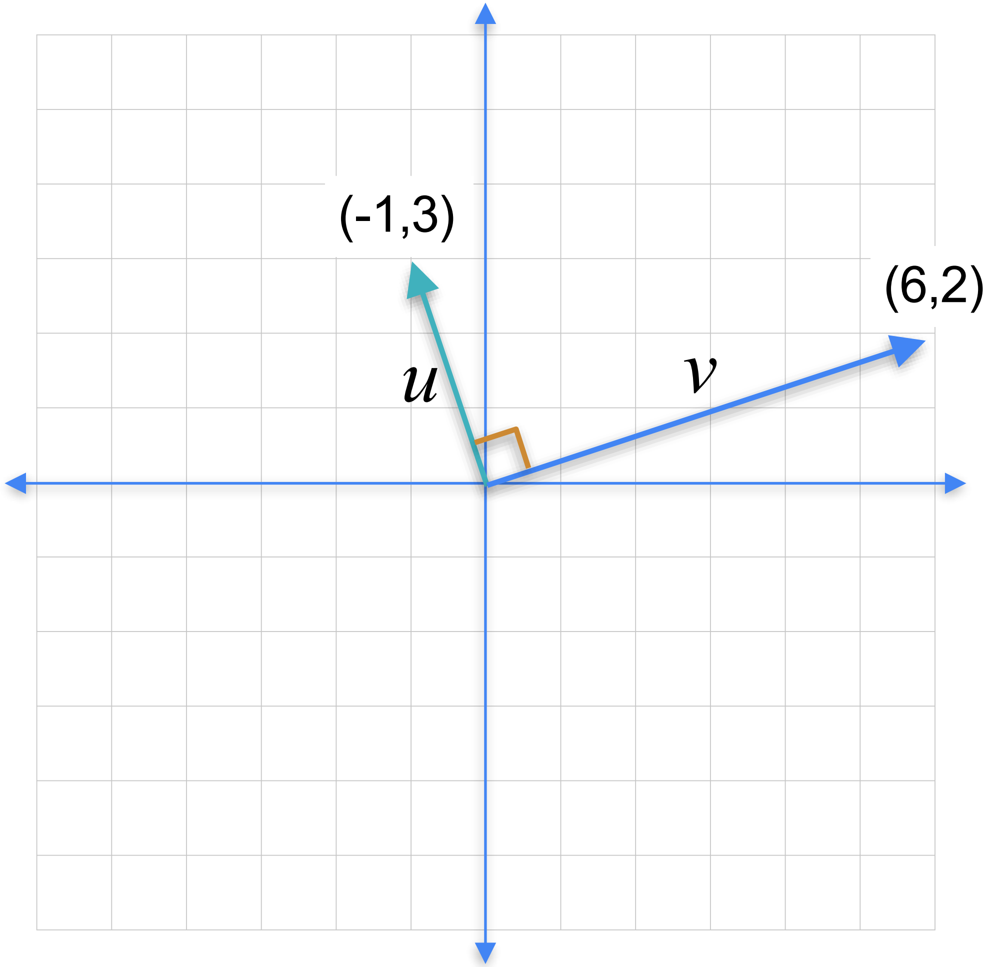
$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

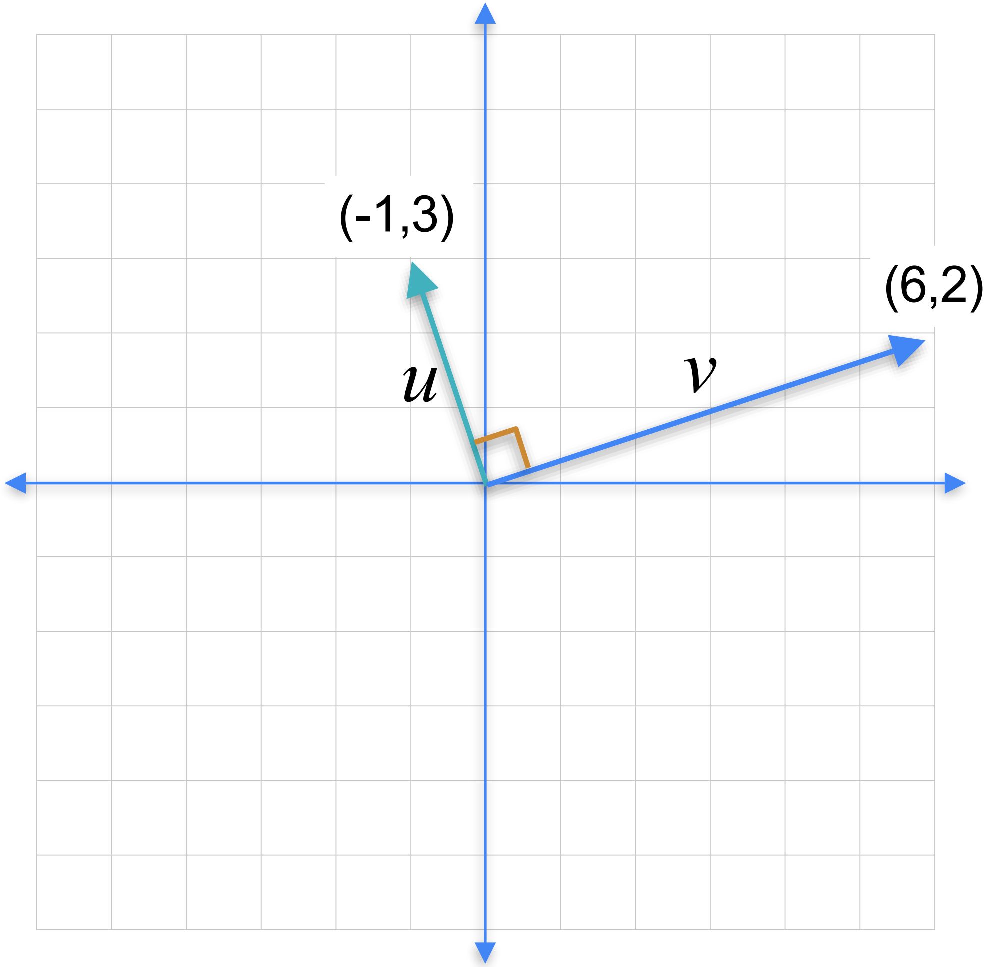
$$L2 - norm = \sqrt{dot\ \ product(u, u)}$$

$$|u|_2 = \sqrt{\langle u, u \rangle}$$

正交向量点积为0

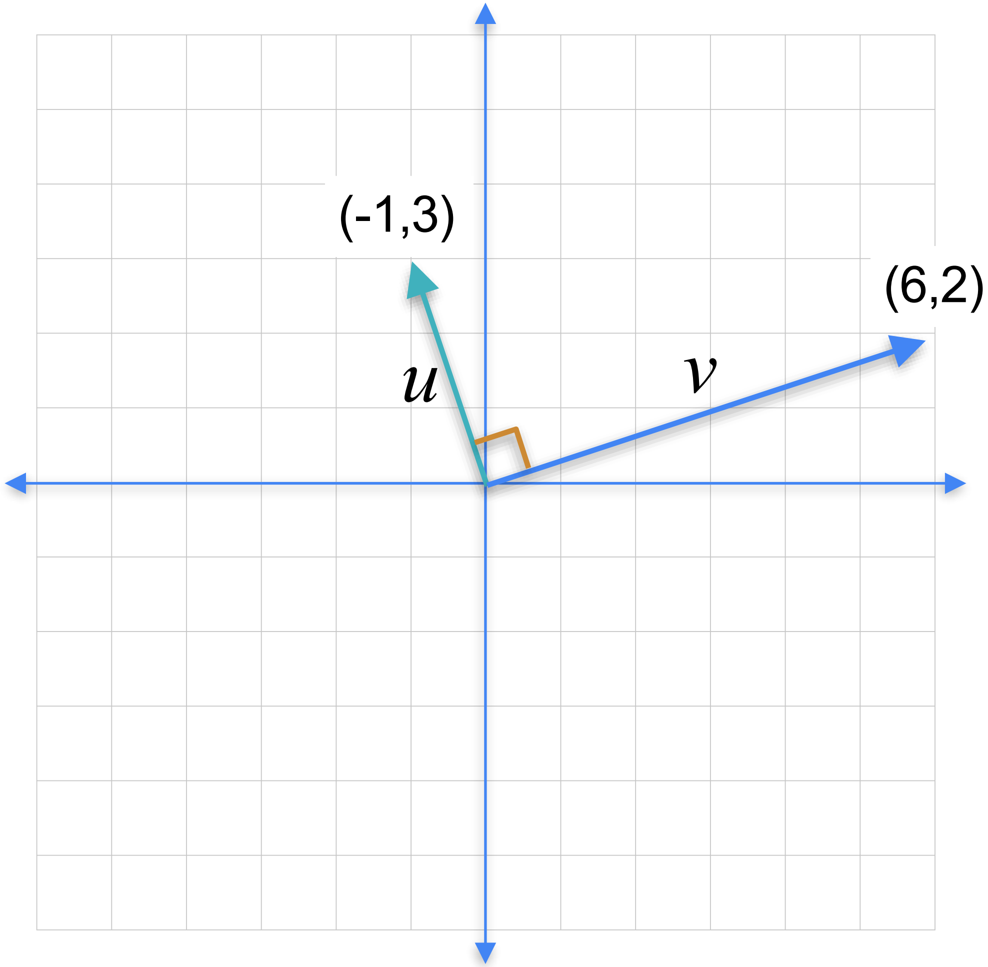


正交向量点积为0



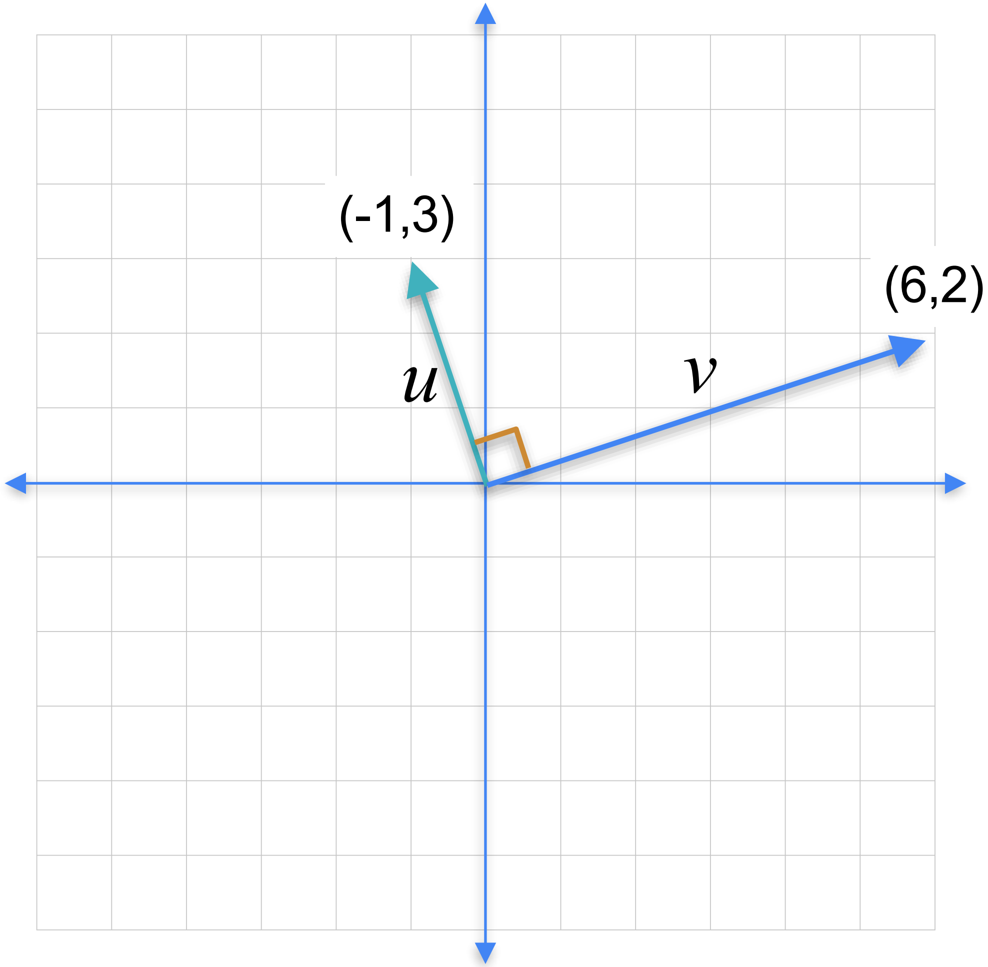
6	2

正交向量点积为0



6	2	-1
		3

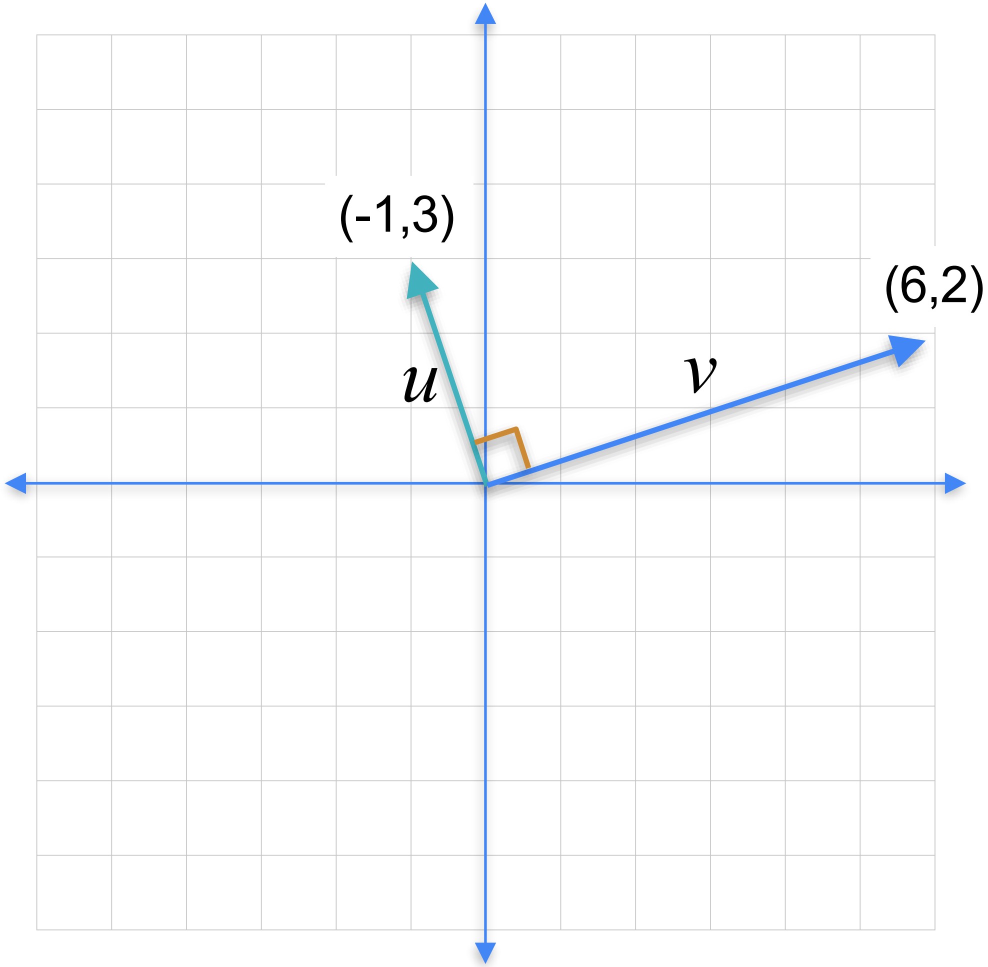
正交向量点积为0



6	2	-1
		3

= 0

正交向量点积为0

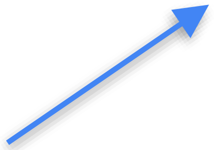


6	2	-1 / 3	=	0

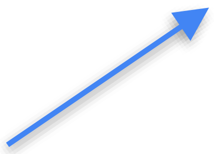
$$\langle u, v \rangle = 0$$

点积

点积

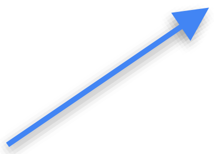


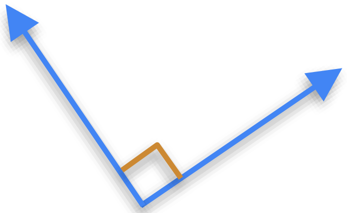
点积



$$\langle u, u \rangle = |u|^2$$

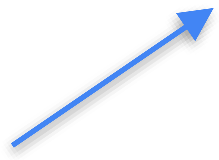
点积



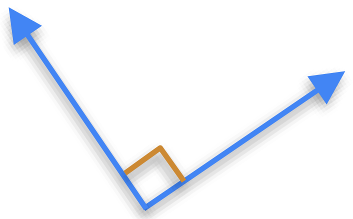


$$\langle u, u\rangle = |u|^2$$

点积



$$\langle u, u \rangle = |u|^2$$



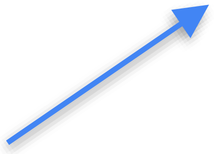
$$\langle u, v \rangle = 0$$

点积

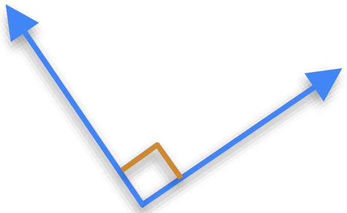
$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

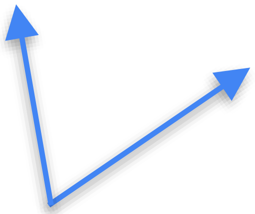
点积



$$\langle u, u \rangle = |u|^2$$



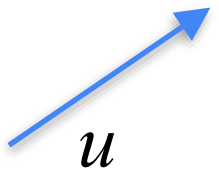
$$\langle u, v \rangle = 0$$



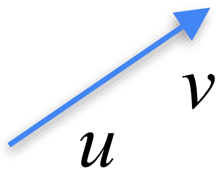
$$\langle u, v \rangle = ?$$

点积

点积

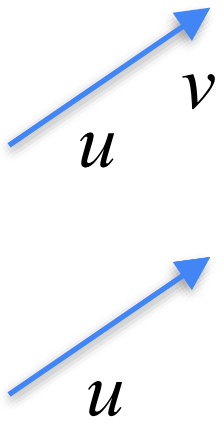


点积



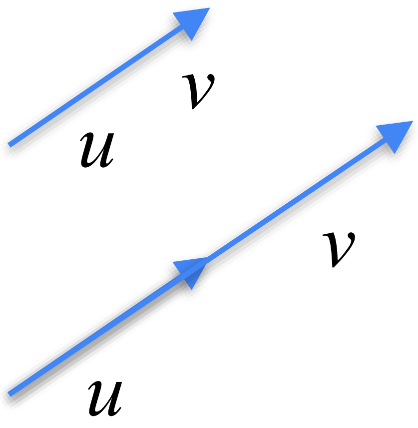
$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

点积



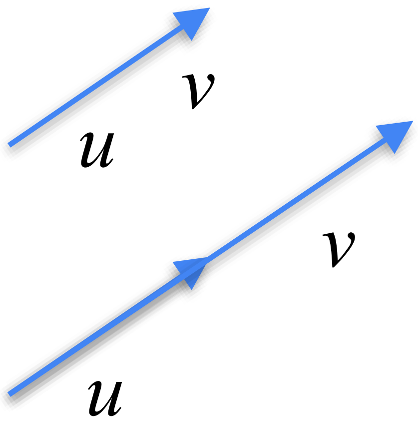
$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

点积



$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

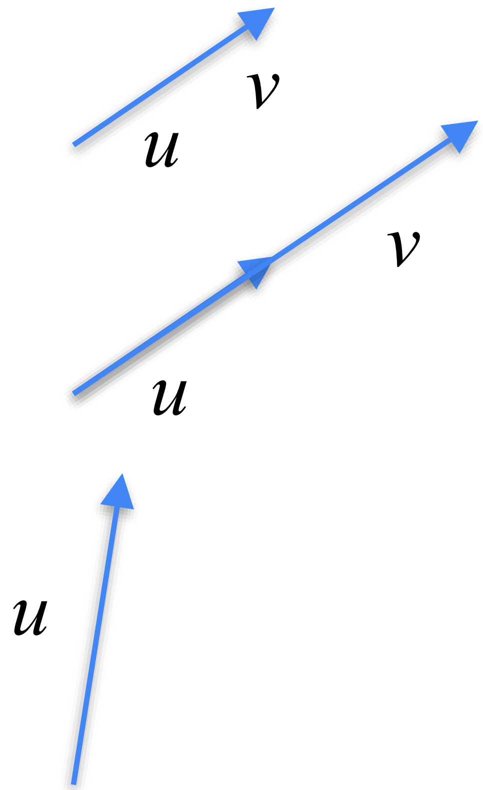
点积



$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v\rangle = |u| \cdot |v|$$

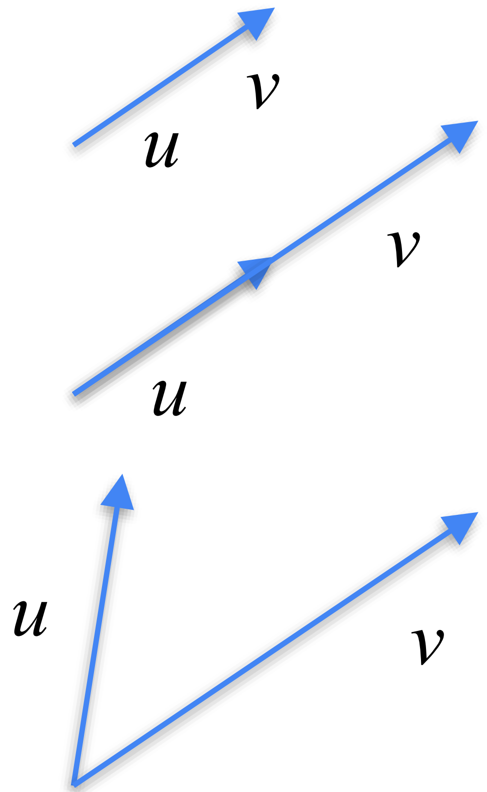
点积



$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v\rangle = |u| \cdot |v|$$

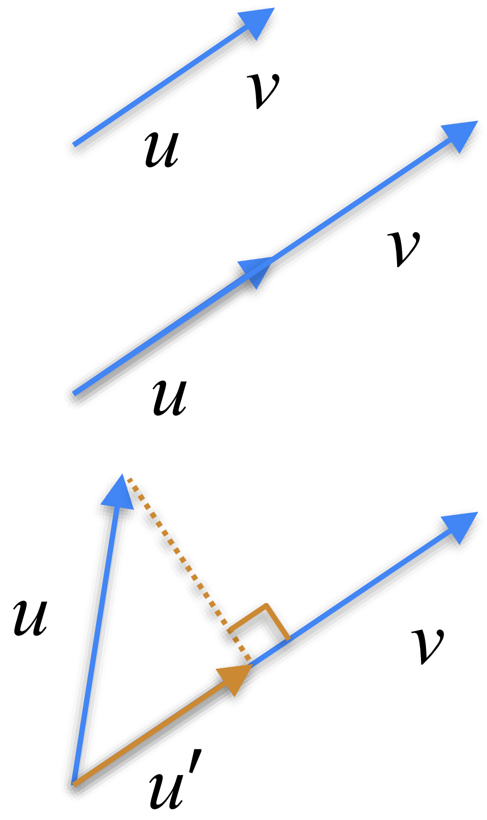
点积



$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v\rangle = |u| \cdot |v|$$

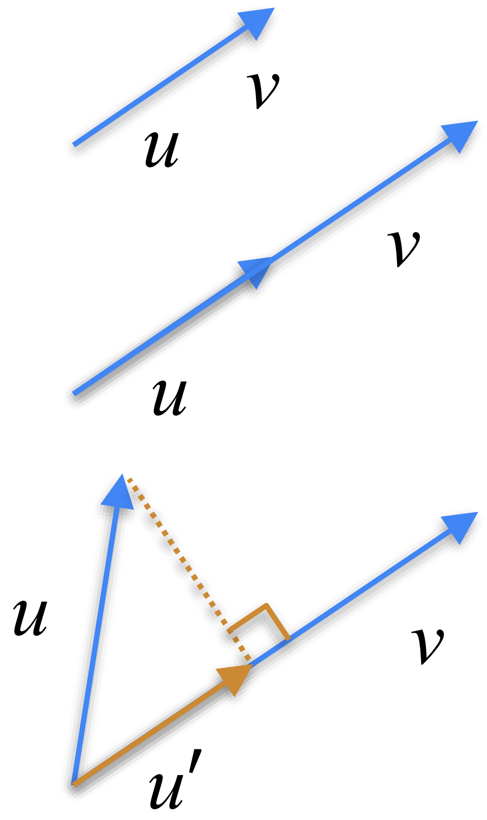
点积



$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v\rangle = |u| \cdot |v|$$

点积

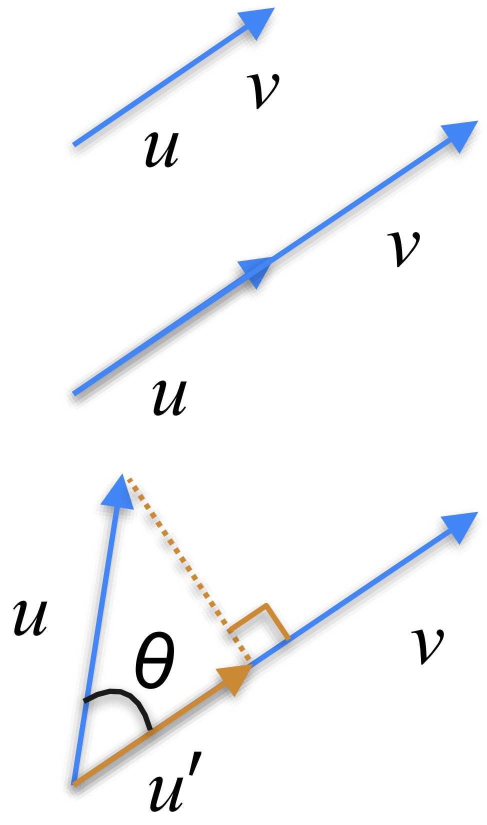


$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v\rangle = |u| \cdot |v|$$

$$\langle u, v\rangle = |u'| \cdot |v|$$

点积

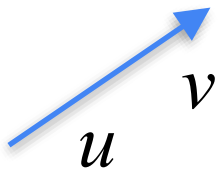


$$\langle u, u\rangle = |u|^2 = |u| \cdot |u|$$

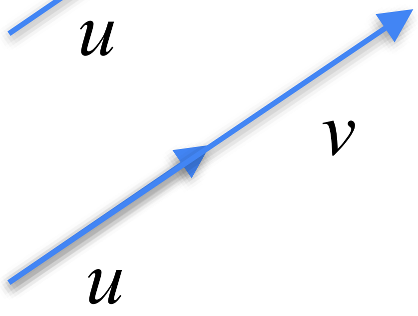
$$\langle u, v\rangle = |u| \cdot |v|$$

$$\langle u, v\rangle = |u'| \cdot |v|$$

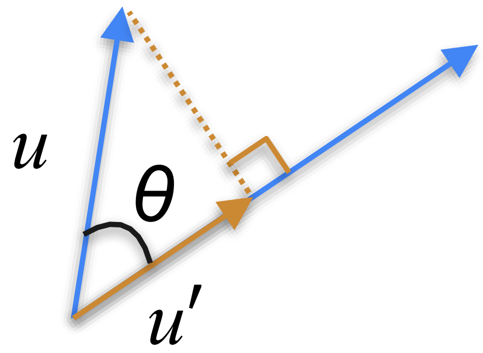
点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



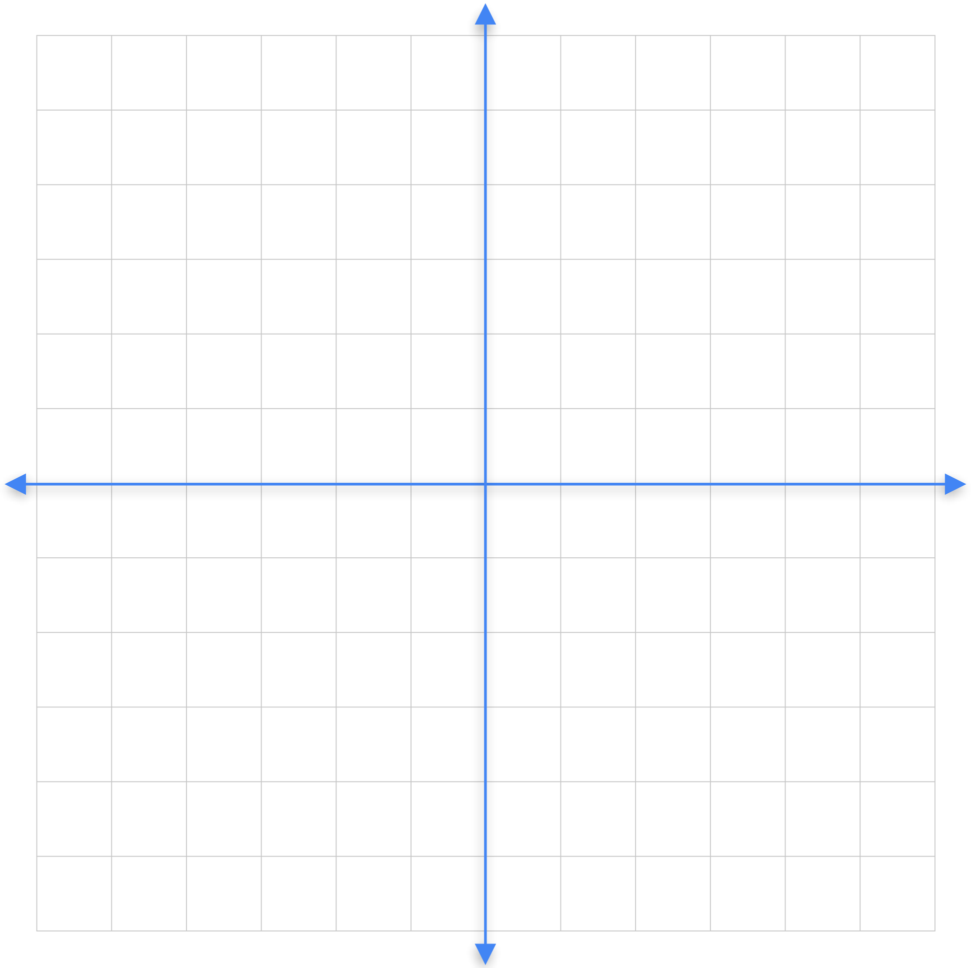
$$\langle u, v \rangle = |u| \cdot |v|$$



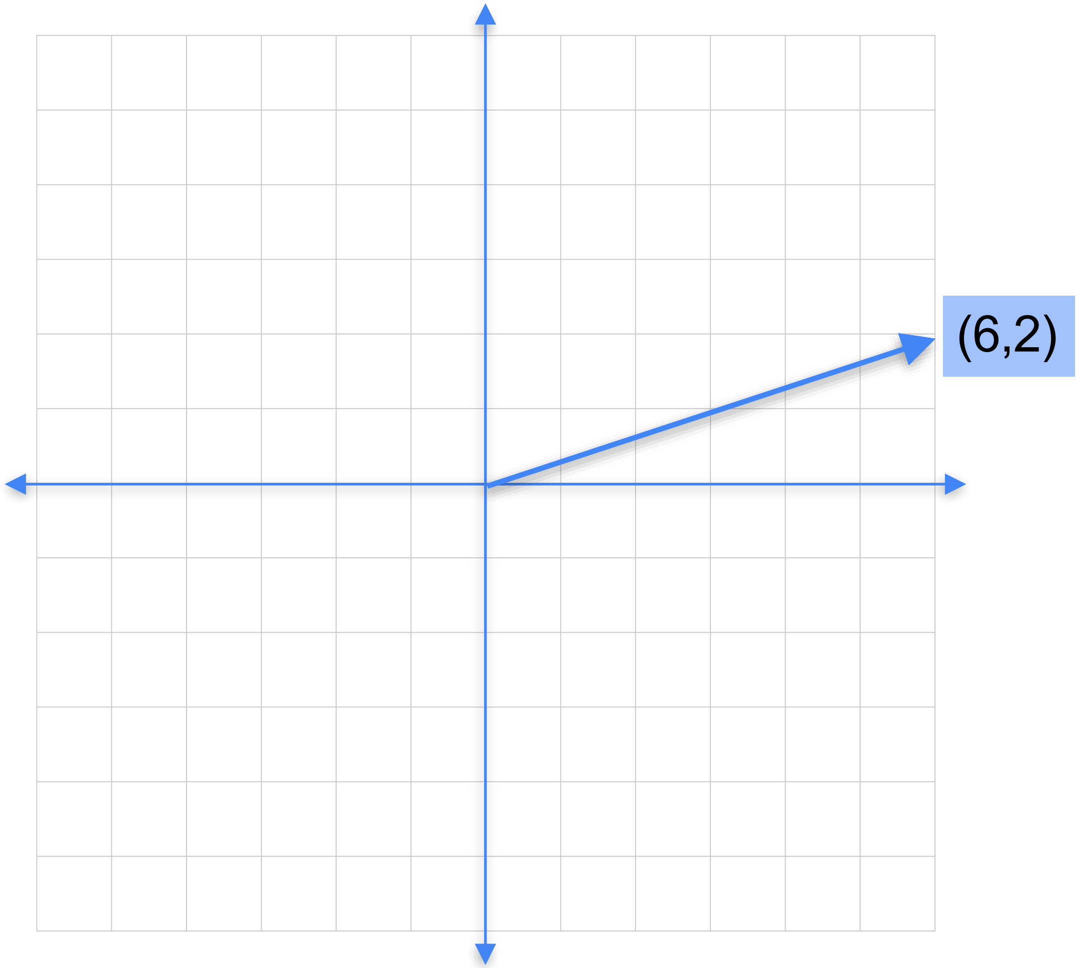
$$\langle u, v \rangle = |u'| \cdot |v|$$

$$= |u||v|\cos(\theta)$$

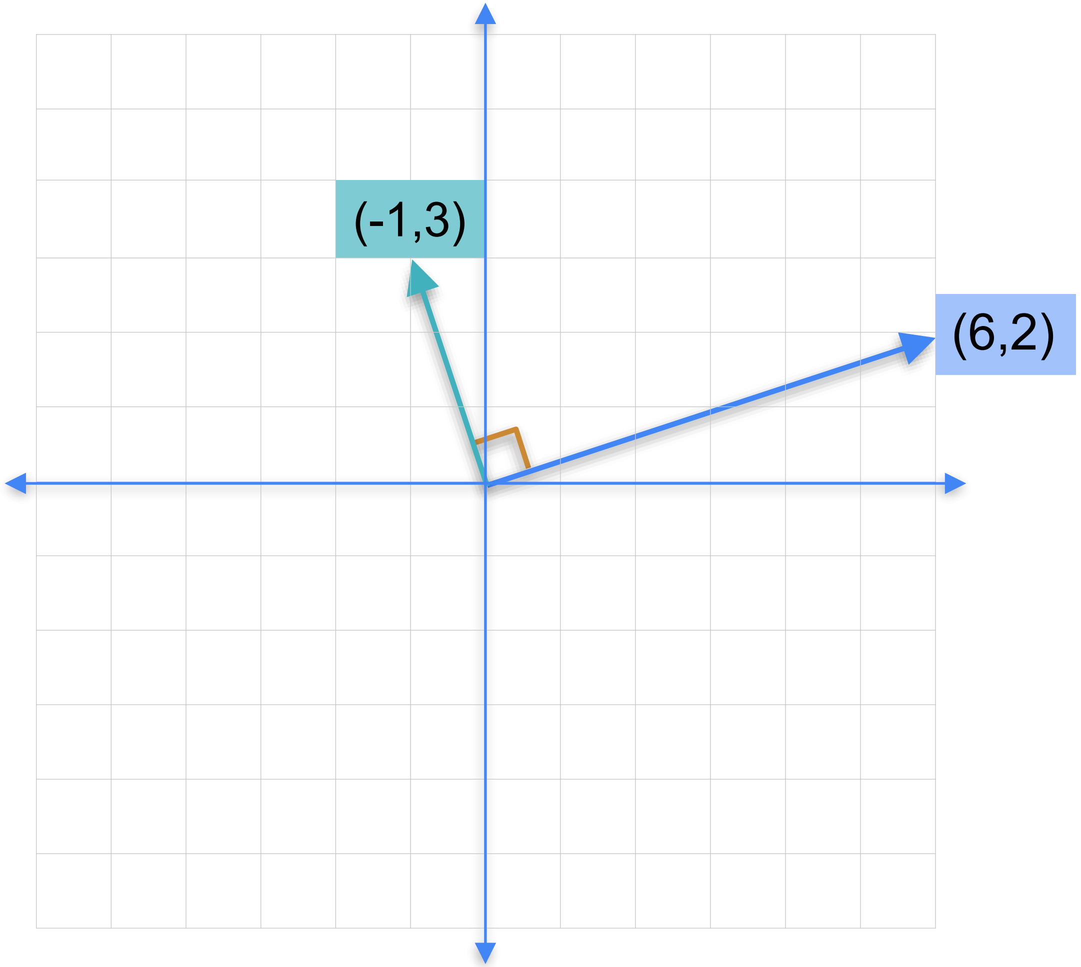
点积



点积



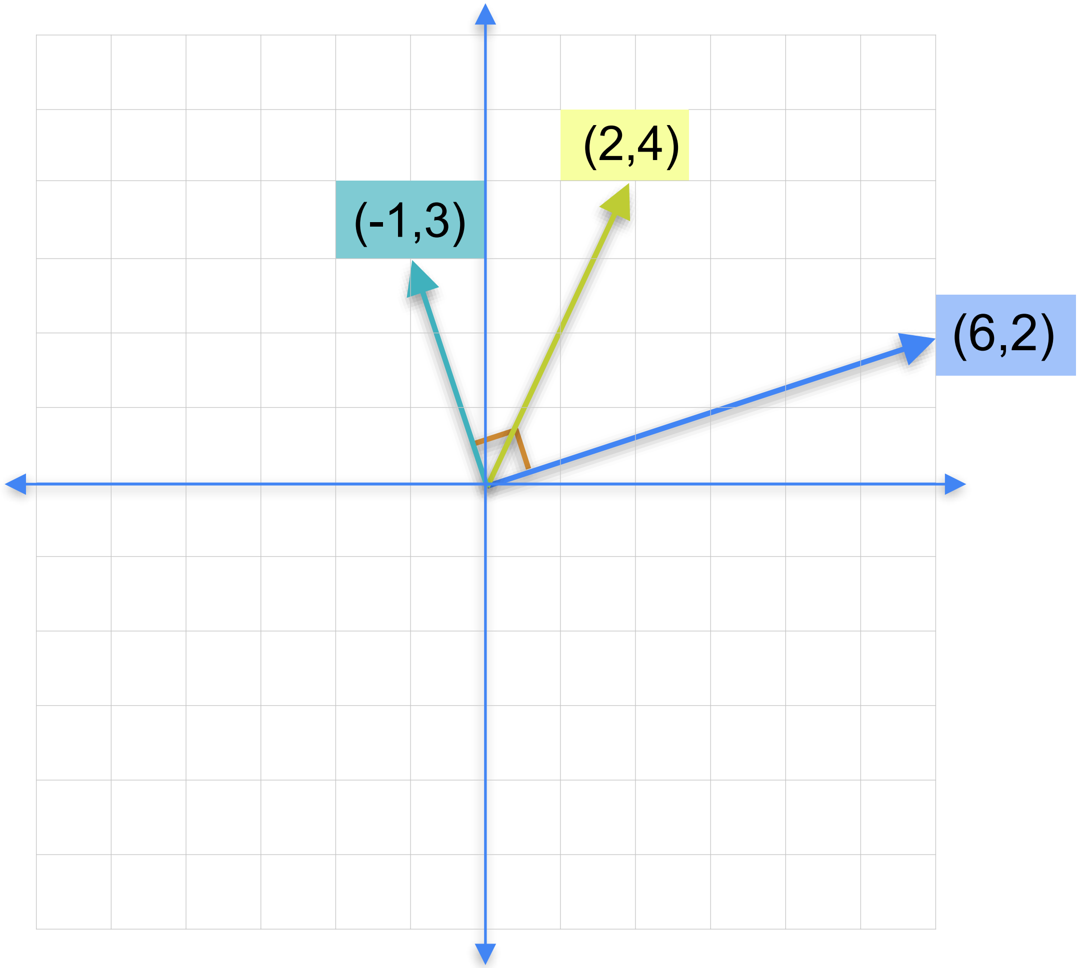
点积



$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix}=0$$

1.向量及其属性

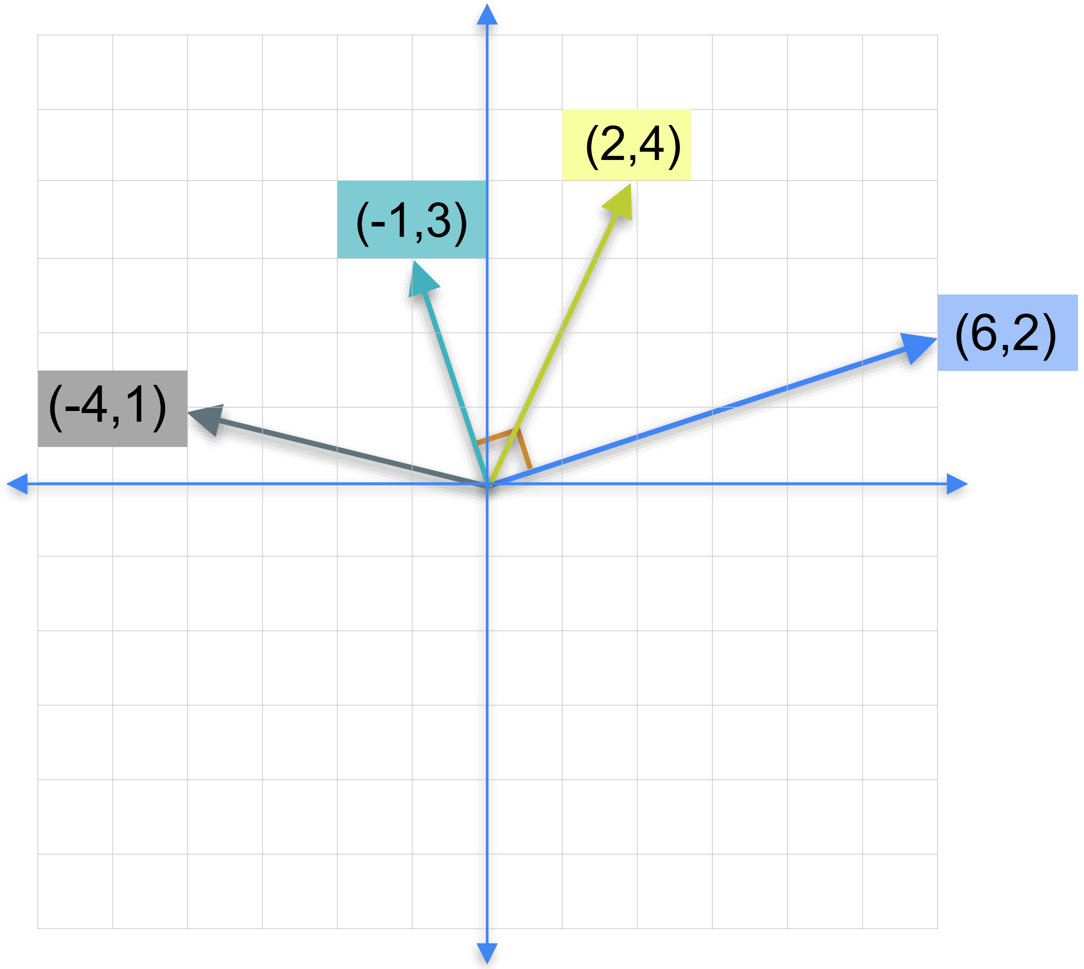
点积



$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix}=20$$

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix}=0$$

点积

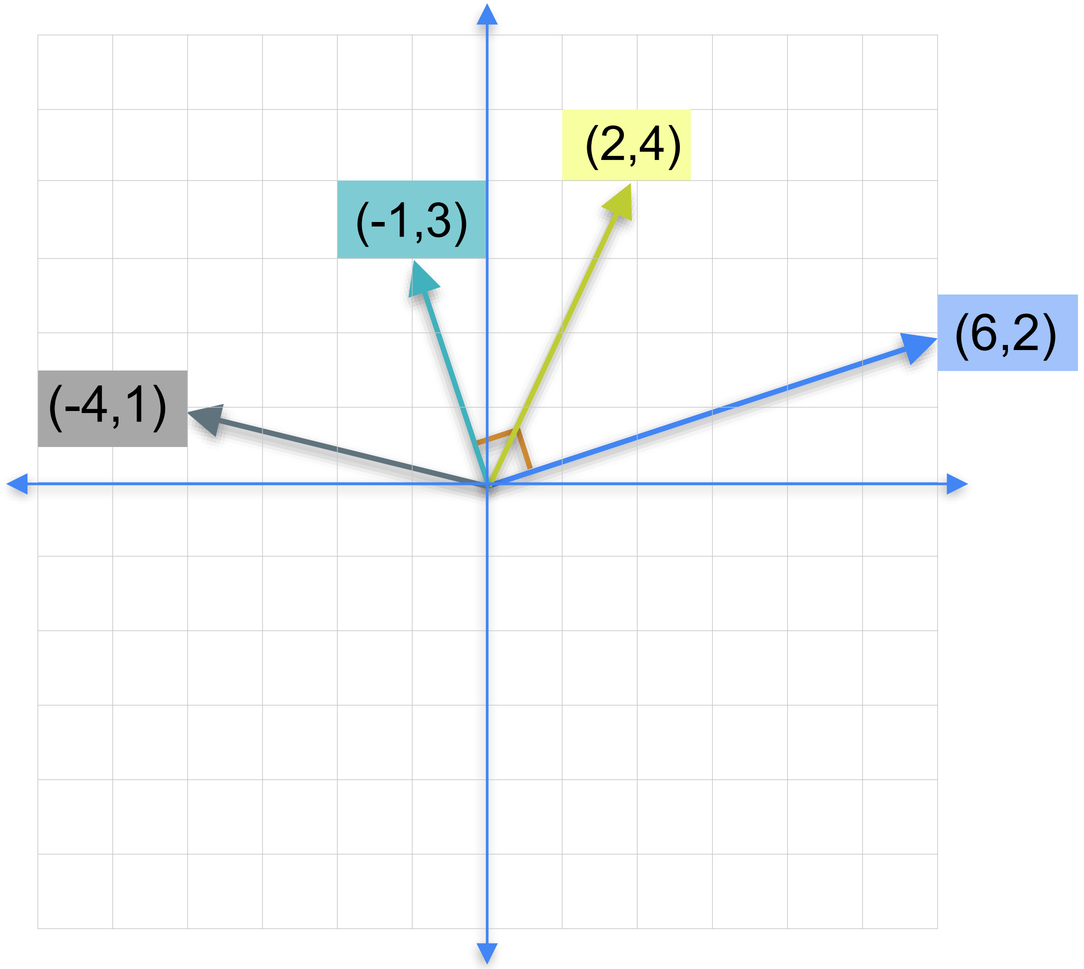


$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix} = 20$$

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix} = 0$$

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-4\\1\end{bmatrix} = -22$$

点积



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 20$$

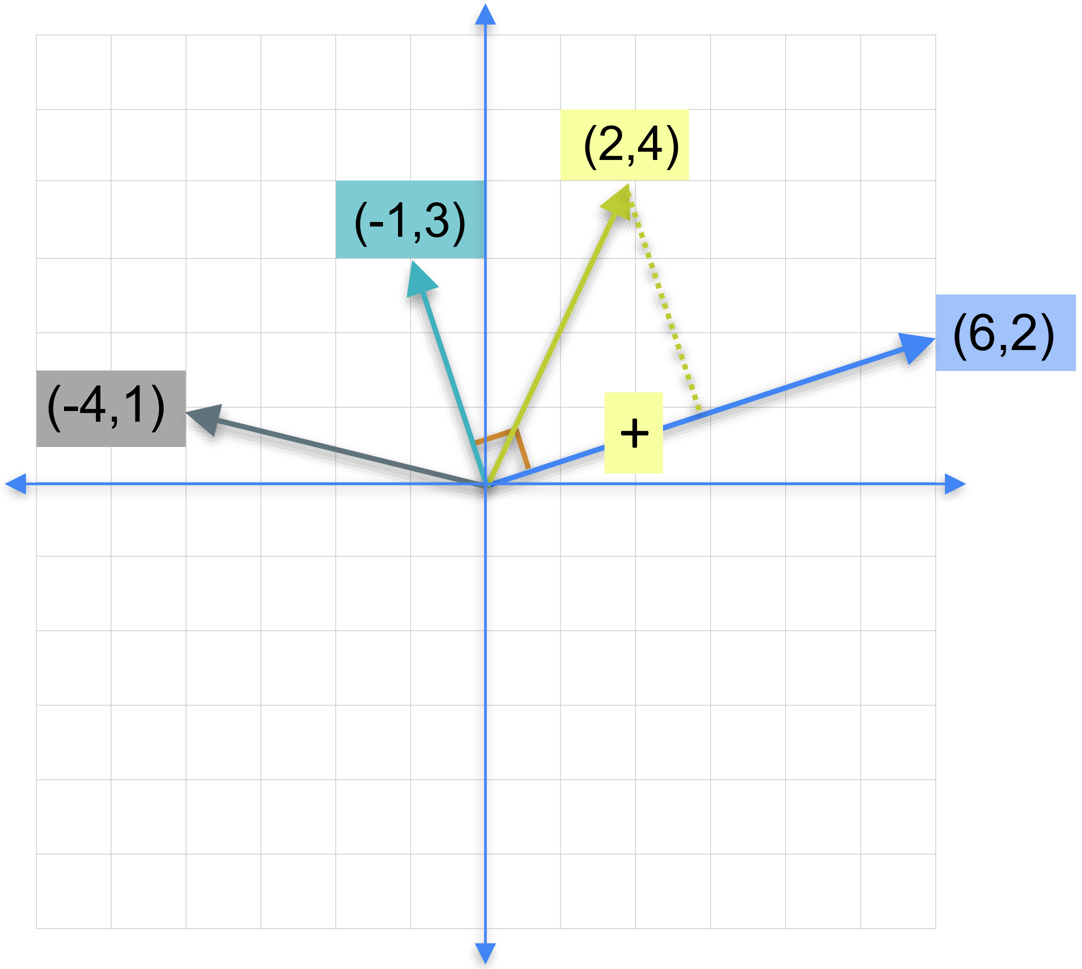
Positive

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = -22$$

1.向量及其属性

点积



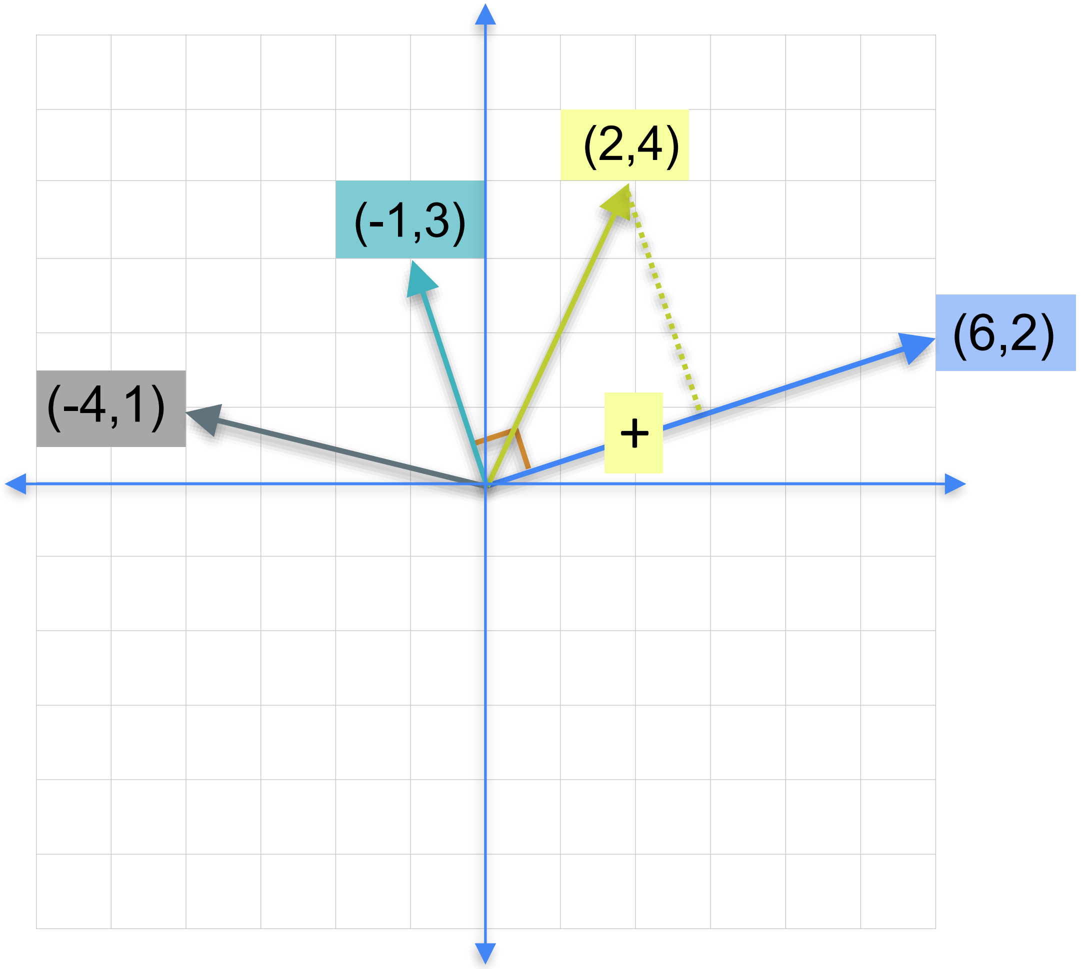
$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix} = 20$$

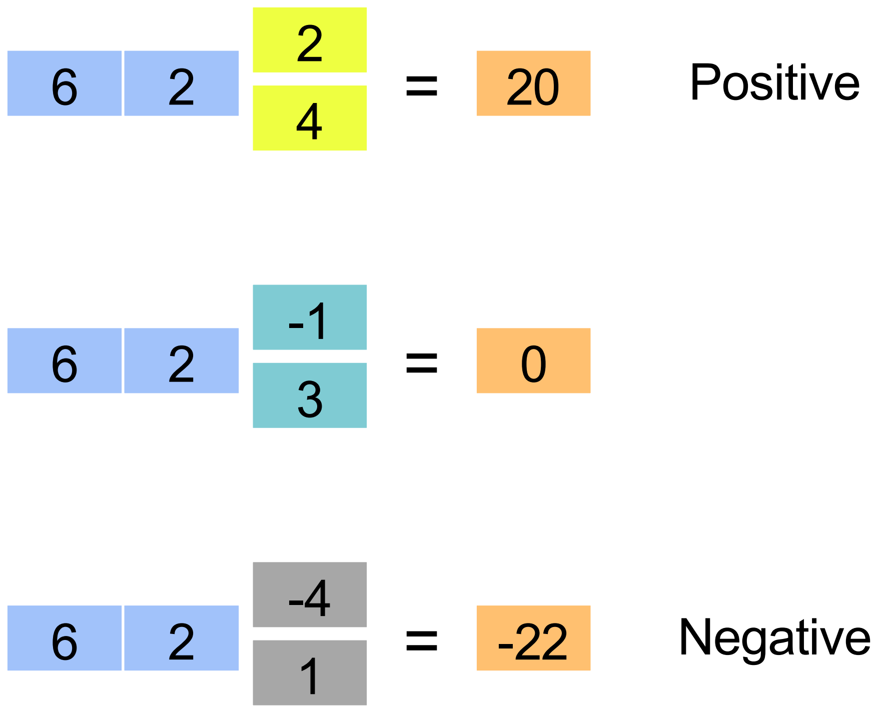
Positive

$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix} = 0$$

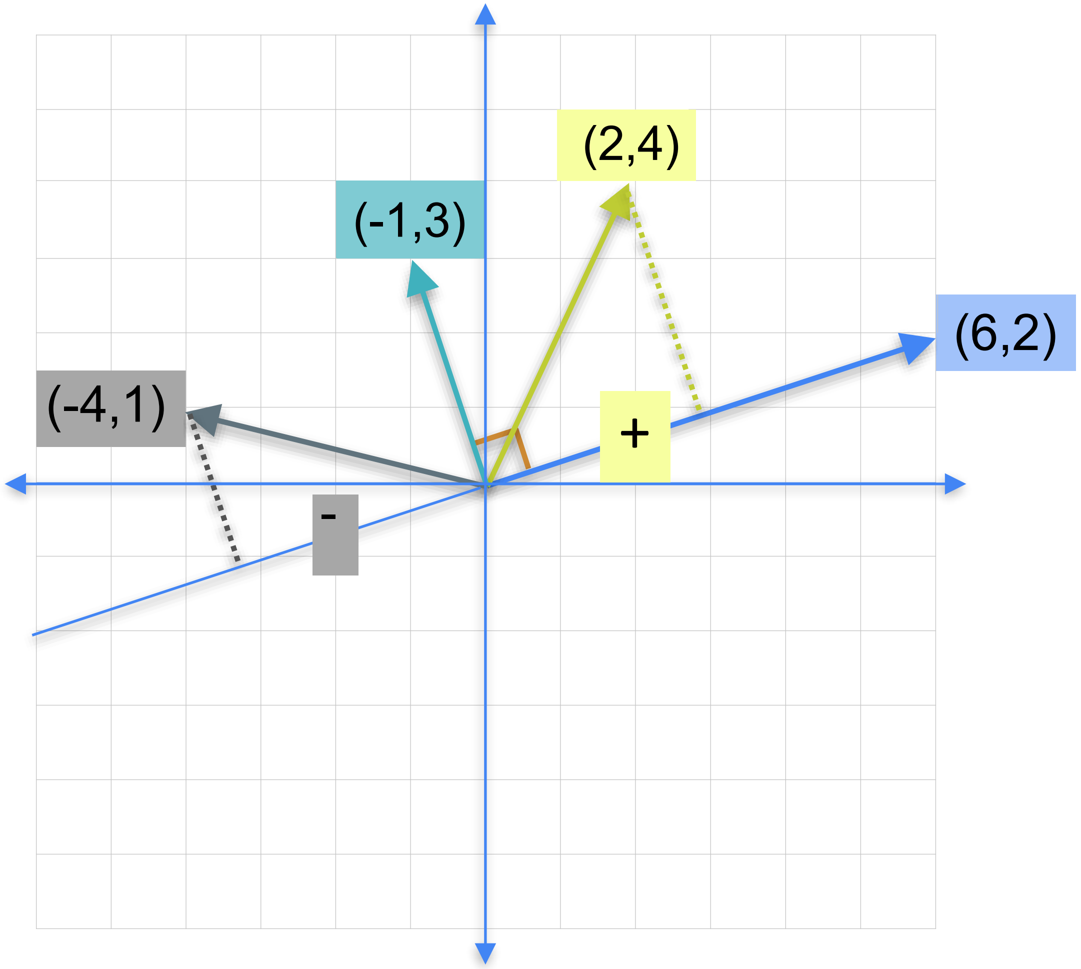
$$\begin{bmatrix}6 & 2\end{bmatrix}\begin{bmatrix}-4\\1\end{bmatrix} = -22$$

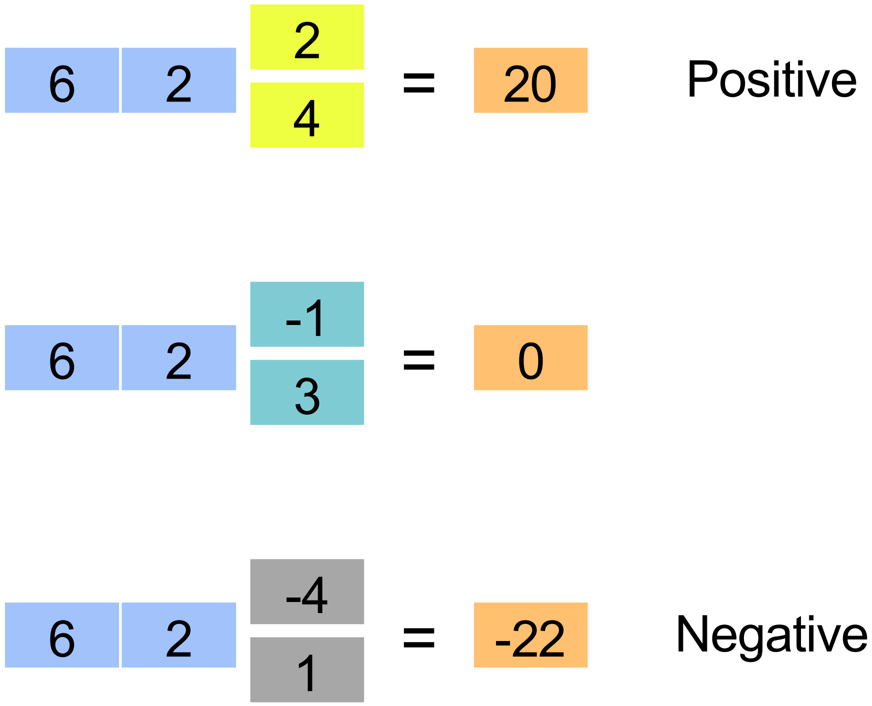
点积



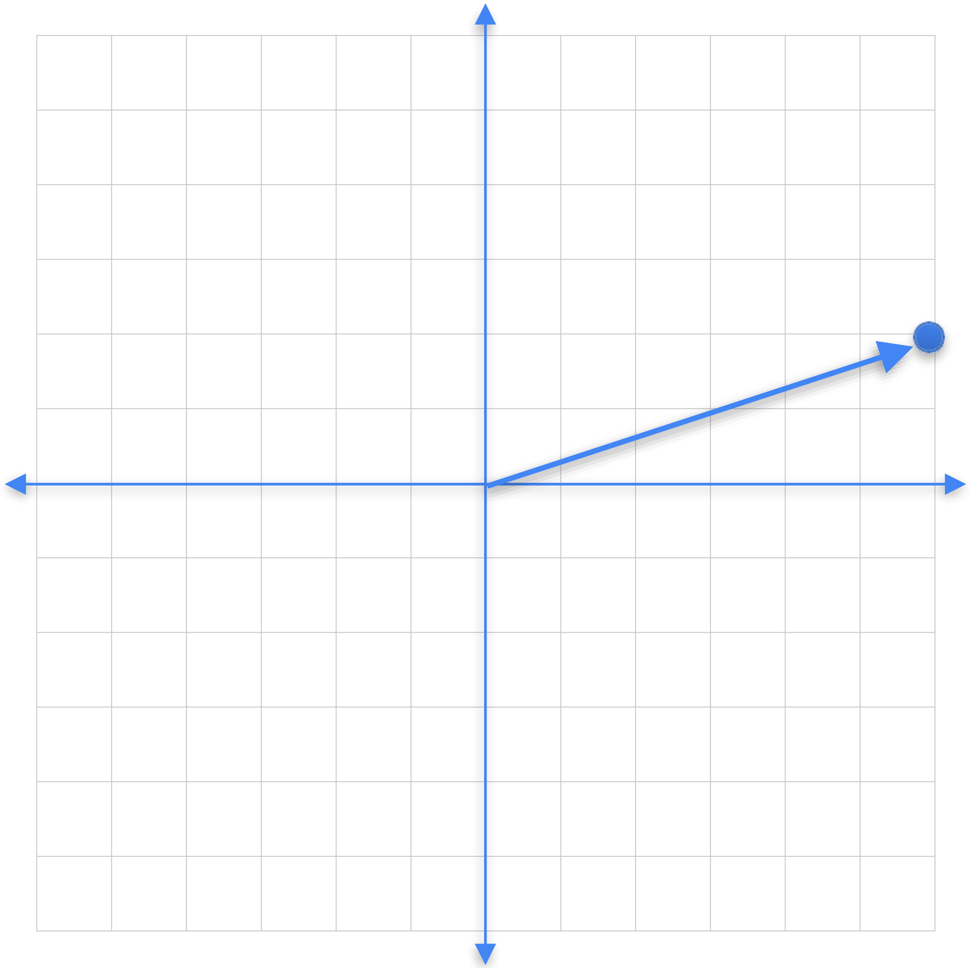


点积





点积



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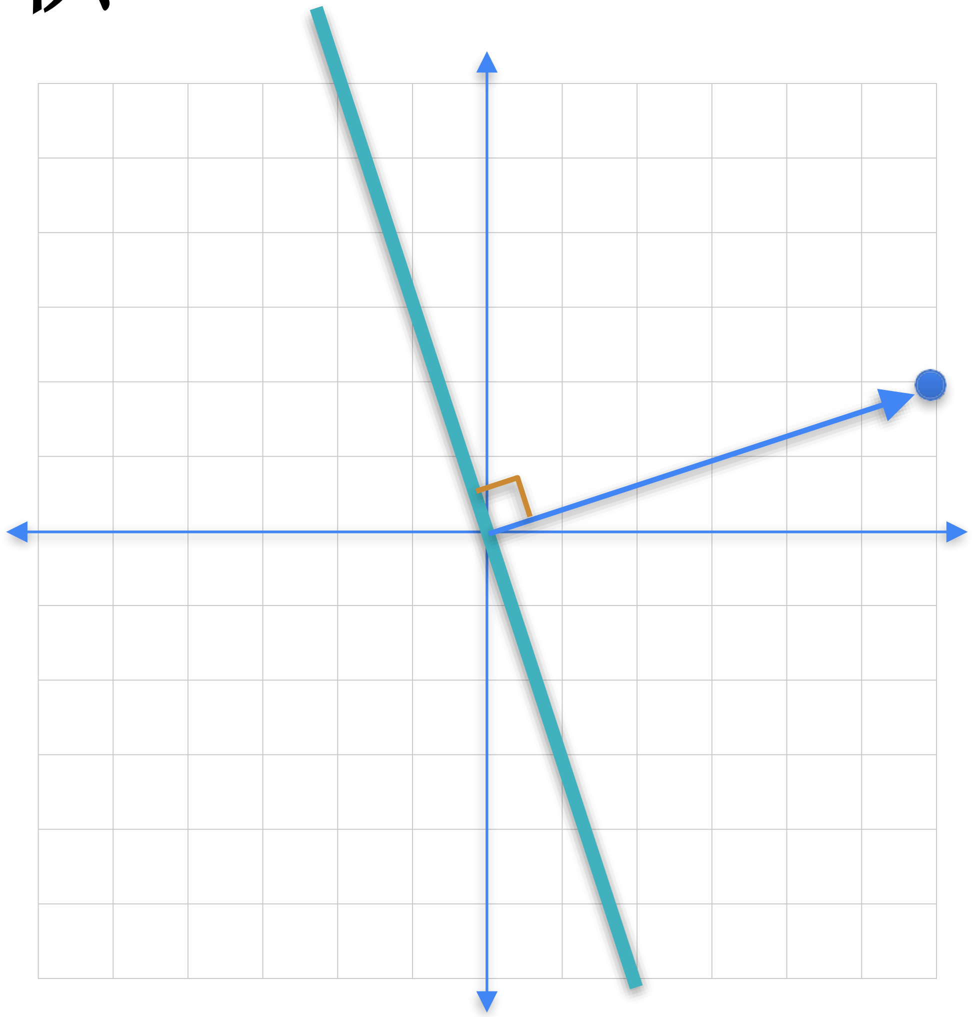
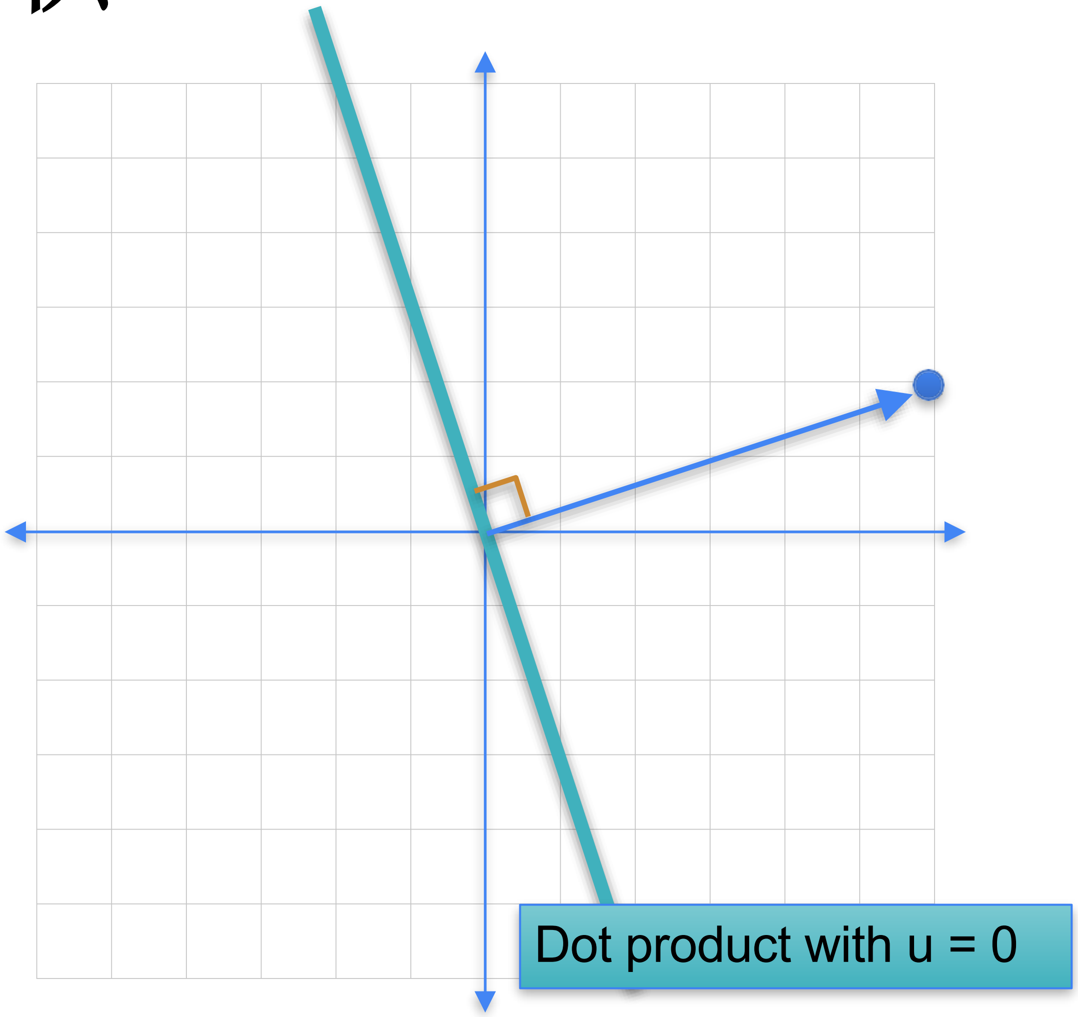
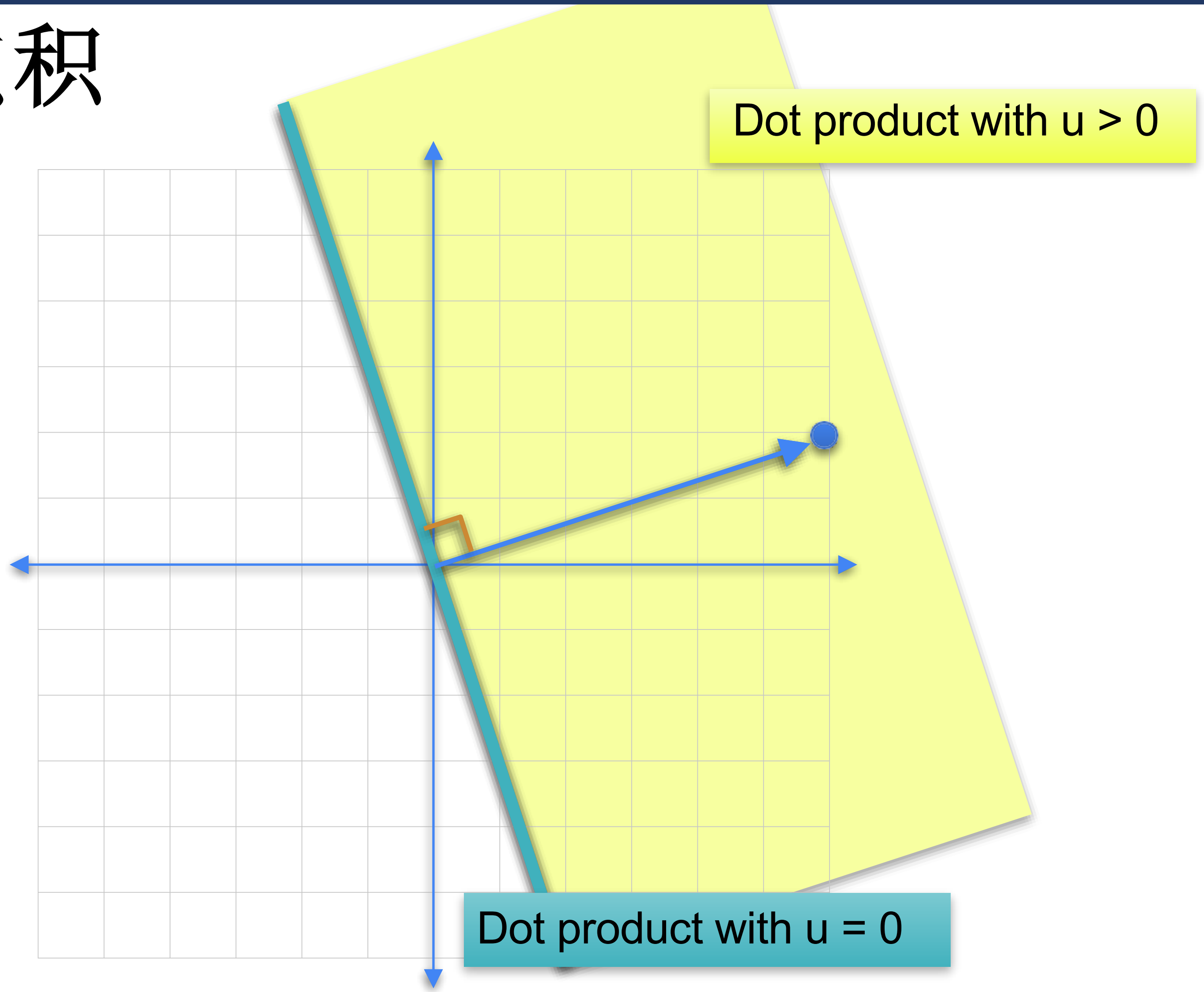


点积



$$\langle u, v \rangle = 0$$

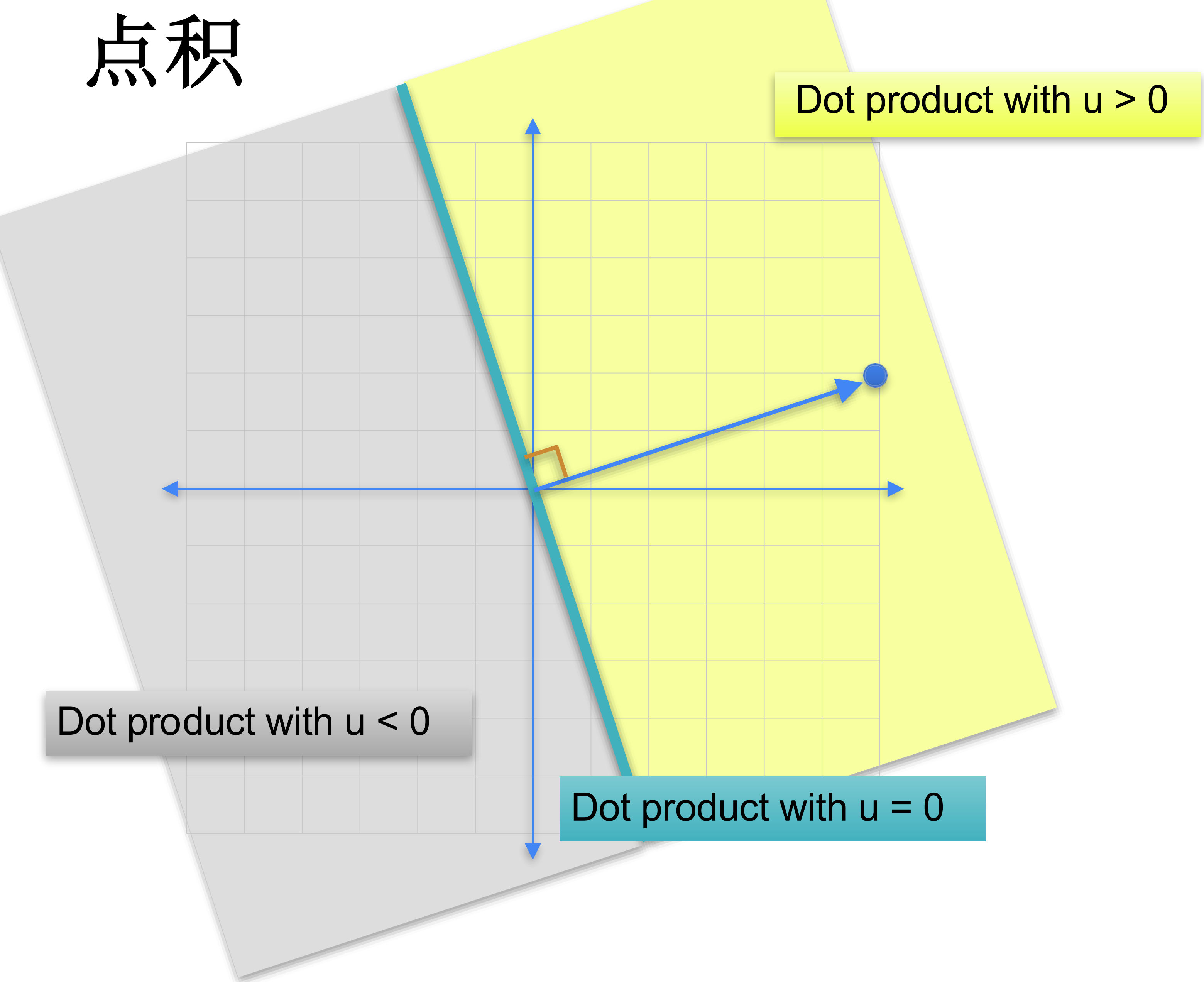
点积



$$\langle u, v\rangle > 0$$

$$\langle u, v\rangle = 0$$

点积



$\langle u, v \rangle > 0$

$\langle u, v \rangle = 0$

$\langle u, v \rangle < 0$

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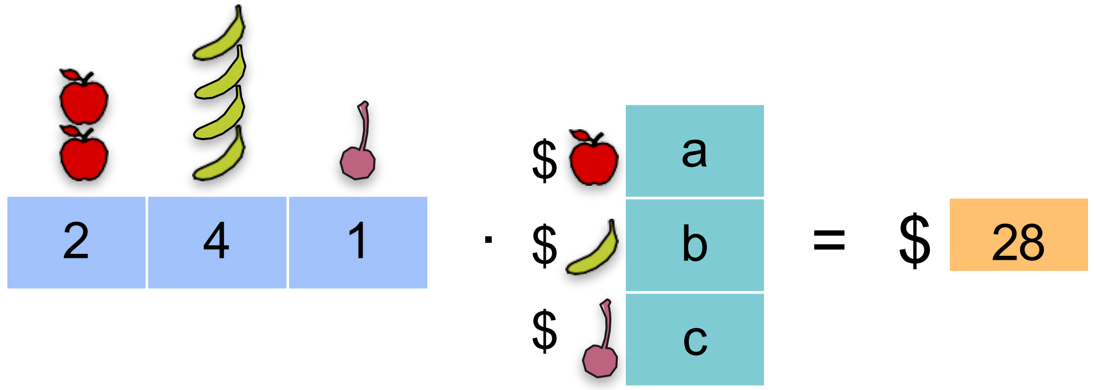
用点积表示线性方程组

$$2a + 4b + c = 28$$



用点积表示线性方程组

a + b + c = 10	a + 2b + c = 15	a + b + 2c = 12

用点积表示线性方程组

a + b + c = 10

a + 2b + c = 15

a + b + 2c = 12

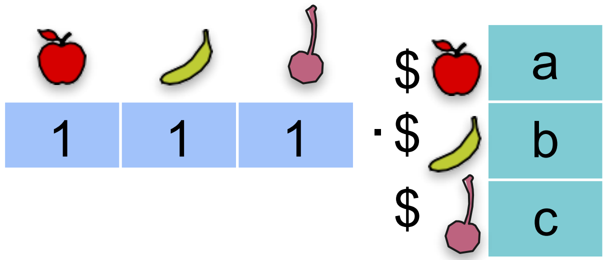
1	1	1

用点积表示线性方程组

a + b + c = 10

a + 2b + c = 15

a + b + 2c = 12

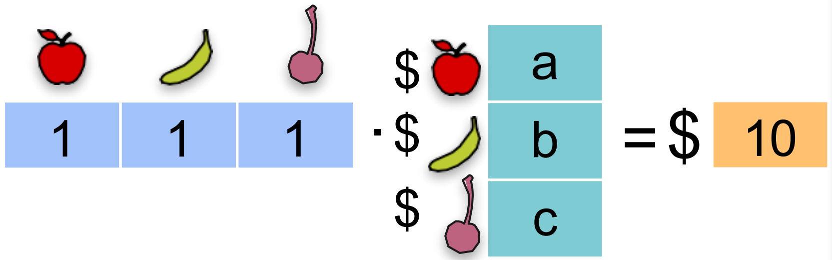


用点积表示线性方程组

$a + b + c = 10$

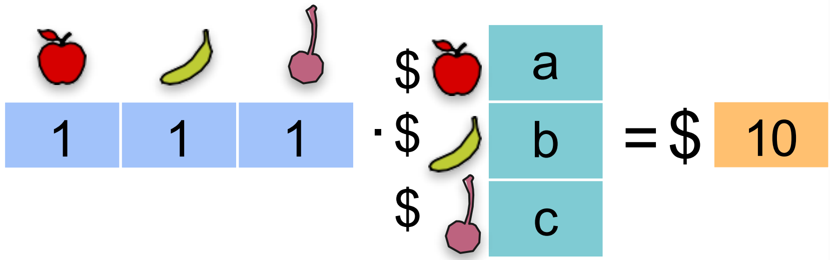
$a + 2b + c = 15$

$a + b + 2c = 12$

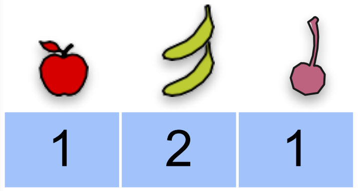


用点积表示线性方程组

$a + b + c = 10$



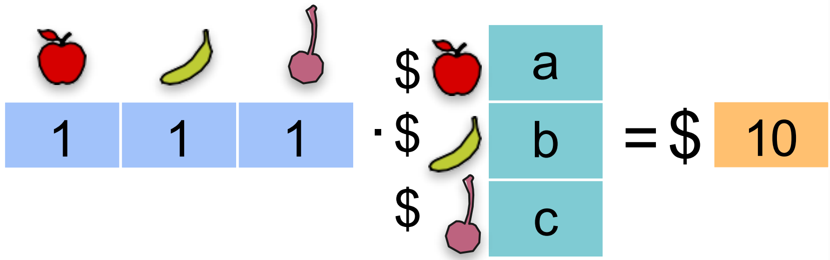
$a + 2b + c = 15$



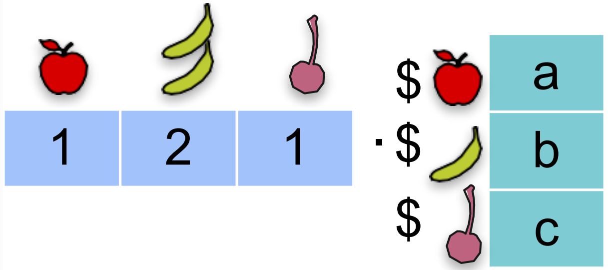
$a + b + 2c = 12$

用点积表示线性方程组

a + b + c = 10



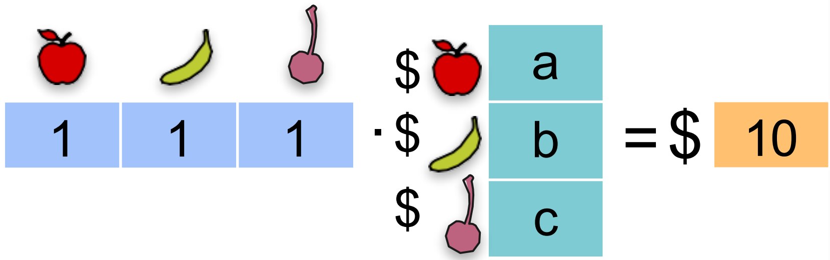
a + 2b + c = 15



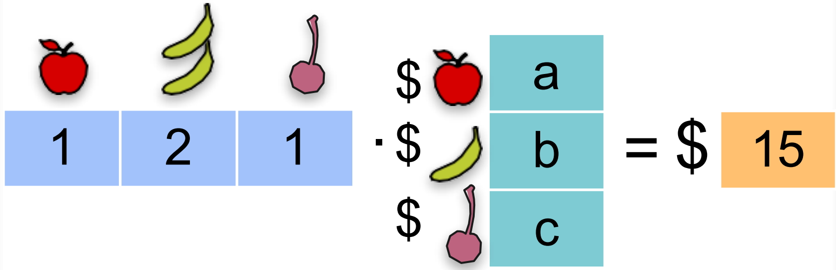
a + b + 2c = 12

用点积表示线性方程组

a + b + c = 10



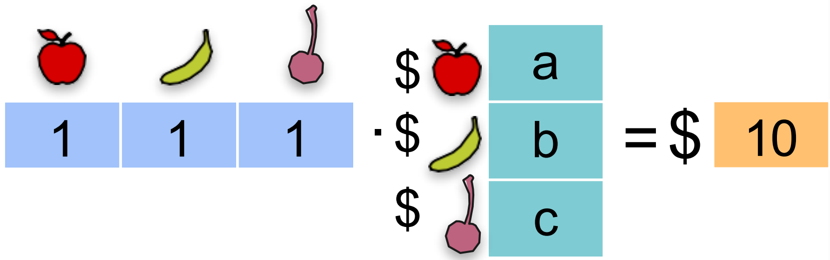
a + 2b + c = 15



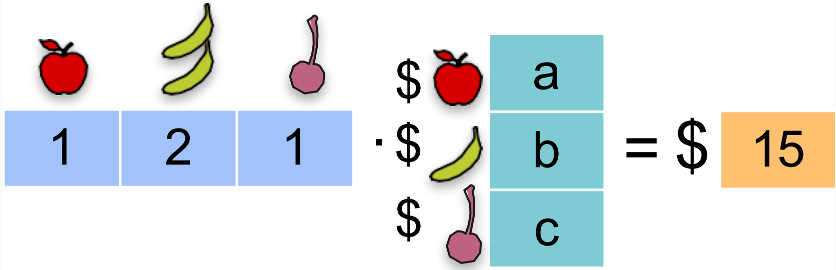
a + b + 2c = 12

用点积表示线性方程组

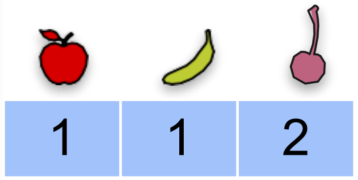
$a + b + c = 10$



$a + 2b + c = 15$

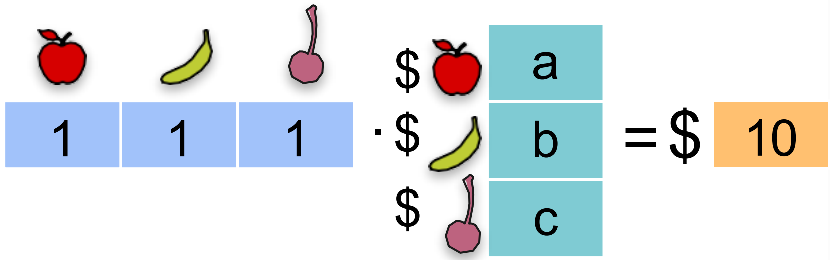


$a + b + 2c = 12$

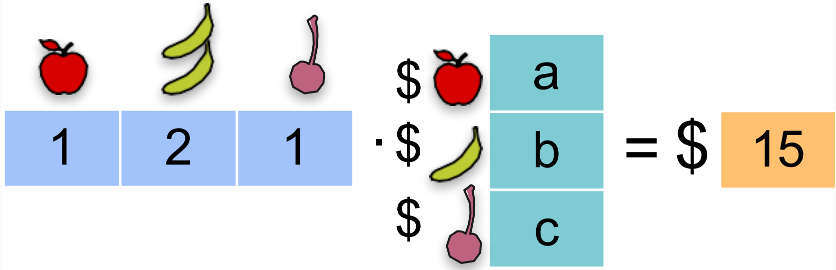


用点积表示线性方程组

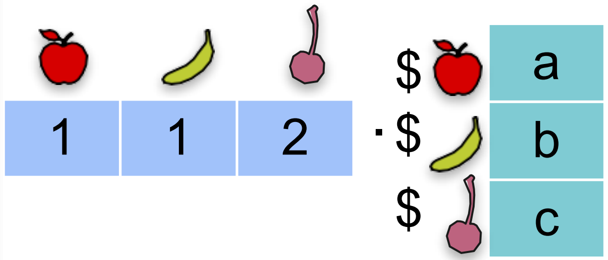
$a + b + c = 10$



$a + 2b + c = 15$

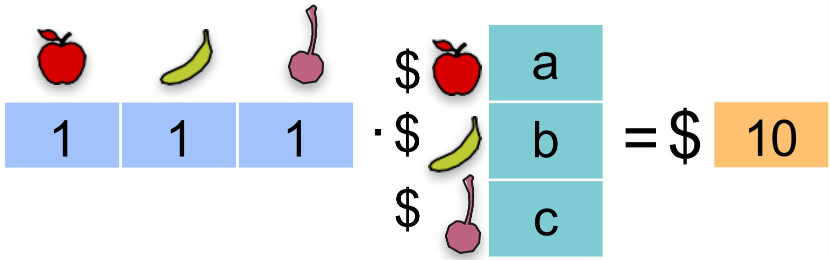


$a + b + 2c = 12$

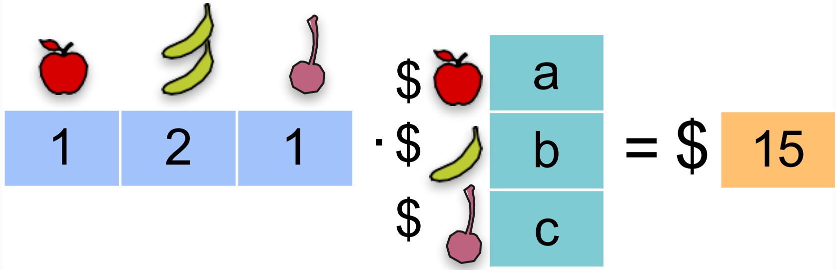


用点积表示线性方程组

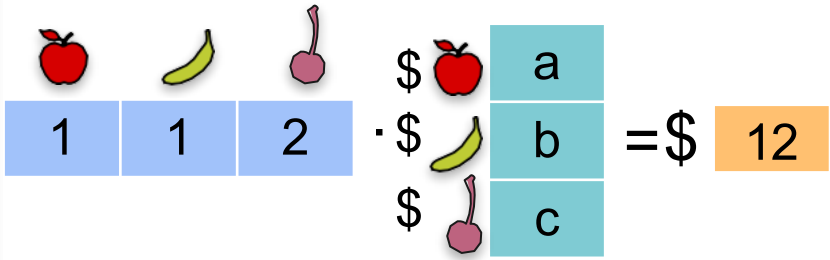
a + b + c = 10



a + 2b + c = 15

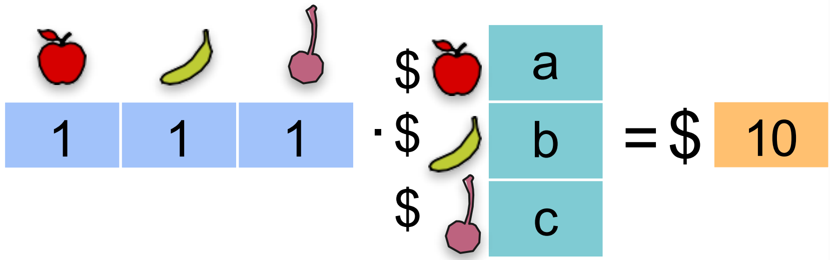


a + b + 2c = 12

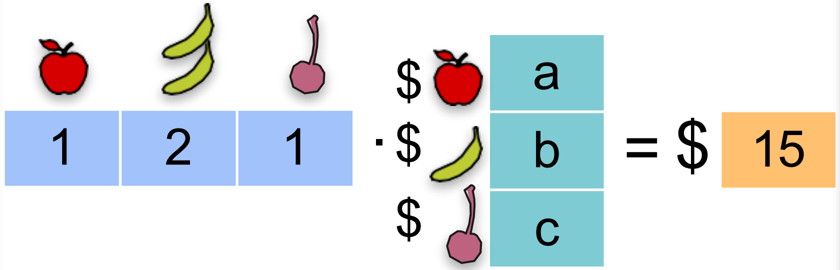


用点积表示线性方程组

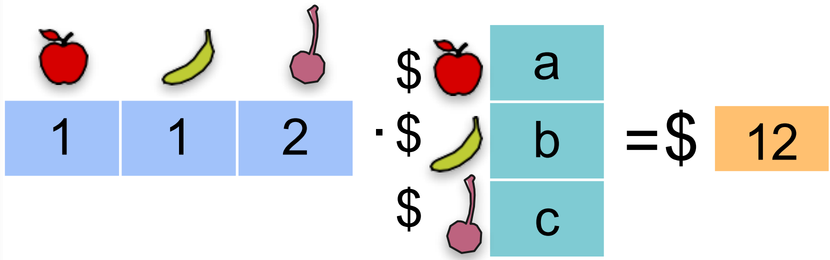
a + b + c = 10



a + 2b + c = 15



a + b + 2c = 12



用点积表示线性方程组

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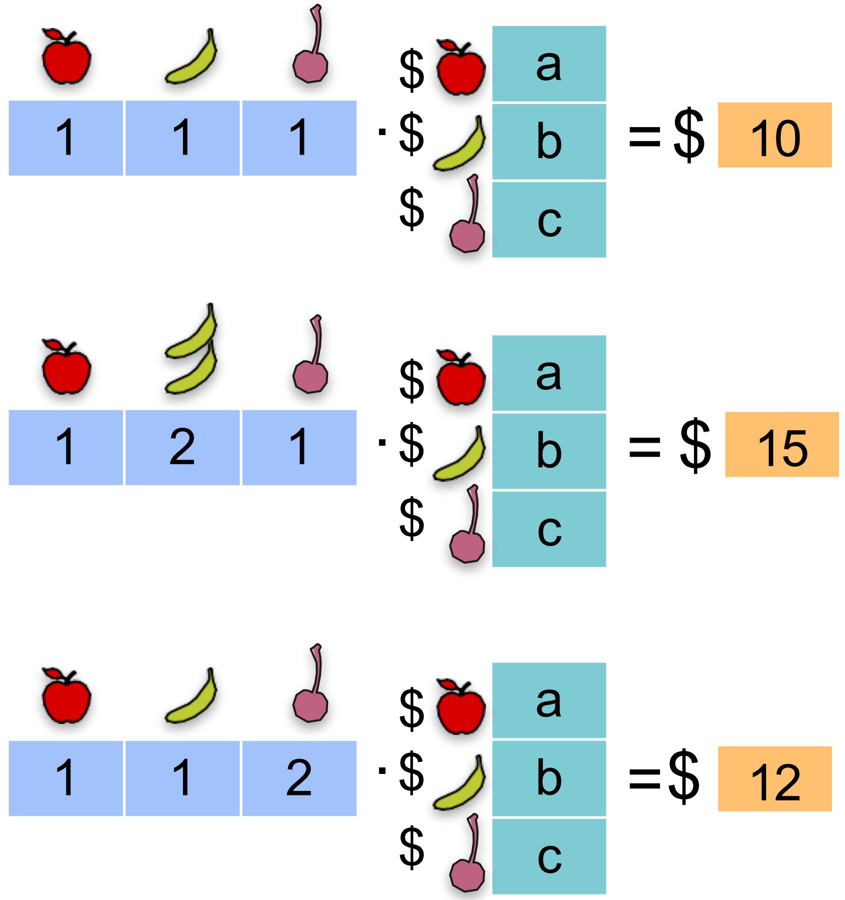
a + b + 2c = 12



用点积表示线性方程组

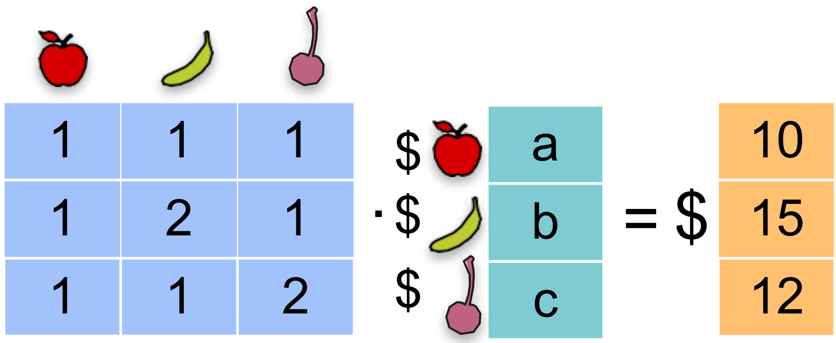
线性方程组

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

矩阵和向量相乘



用点积表示线性方程组

线性方程组

a + b + c = 10

a + 2b + c = 15

a + b + 2c = 12

矩阵和向量相乘

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

矩阵

矩阵

数学上，一个m×n的矩阵（matrix）是一个有m行（row）n列（column）元素的**矩形阵列**。

行和列相同的元素称为矩阵**主对角线**上的元素，如a_{11}，a_{22} ，a_{33}等。

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \cdots & \cdots & & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

特殊矩阵

行数与列数相同的矩阵称为**方块矩阵**，简称**方阵**。

如果一个方阵**只有主对角线上的元素不是0**，其它都是0，那么称其为**对角矩阵**（diagonal matrix）。

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

主对角线元素全部为1的对角矩阵称为**单位矩阵**（identity matrix），一般用I_n表示。

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

特殊矩阵

如果主对角线**上方的元素都是0**，那么称为**下三角矩阵**（lower triangular matrix）。

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

如果主对角线**下方的元素都是0**，那么称为**上三角矩阵**（upper triangular matrix）。

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

矩阵的基本运算

矩阵的最基本运算包括矩阵加（减）法，数乘和转置运算。
加（减）法：

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 5 \\ 7 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 & 1+5 \\ 1+7 & 0+5 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 8 & 5 & 0 \end{bmatrix}$$

数乘：

$$2 \cdot \begin{bmatrix} 1 & 8 & -3 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 & 2\cdot 8 & 2\cdot(-3) \\ 2\cdot 4 & 2\cdot(-2) & 2\cdot 5 \end{bmatrix} = \begin{bmatrix} 2 & 16 & -6 \\ 8 & -4 & 10 \end{bmatrix}$$

转置：

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & 7 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 2 & -6 \\ 3 & 7 \end{bmatrix}$$

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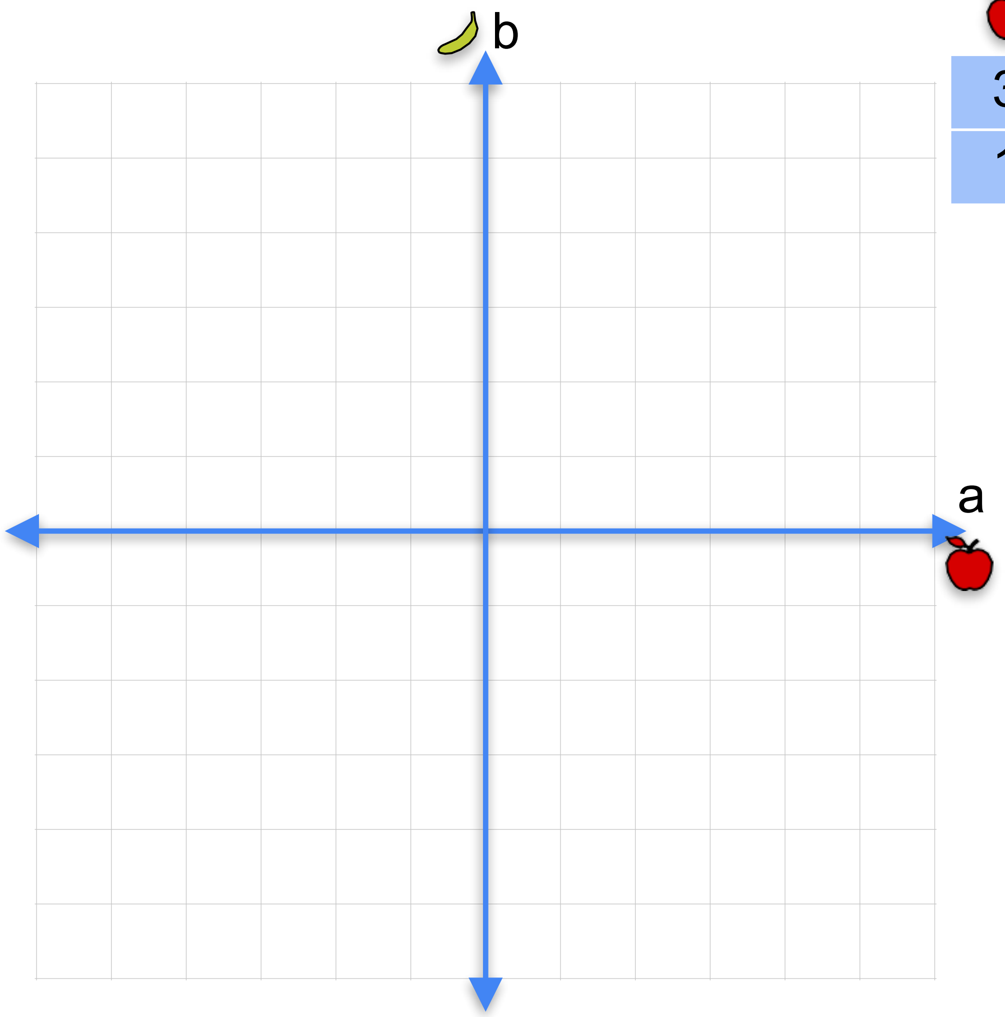
矩阵是线性变换

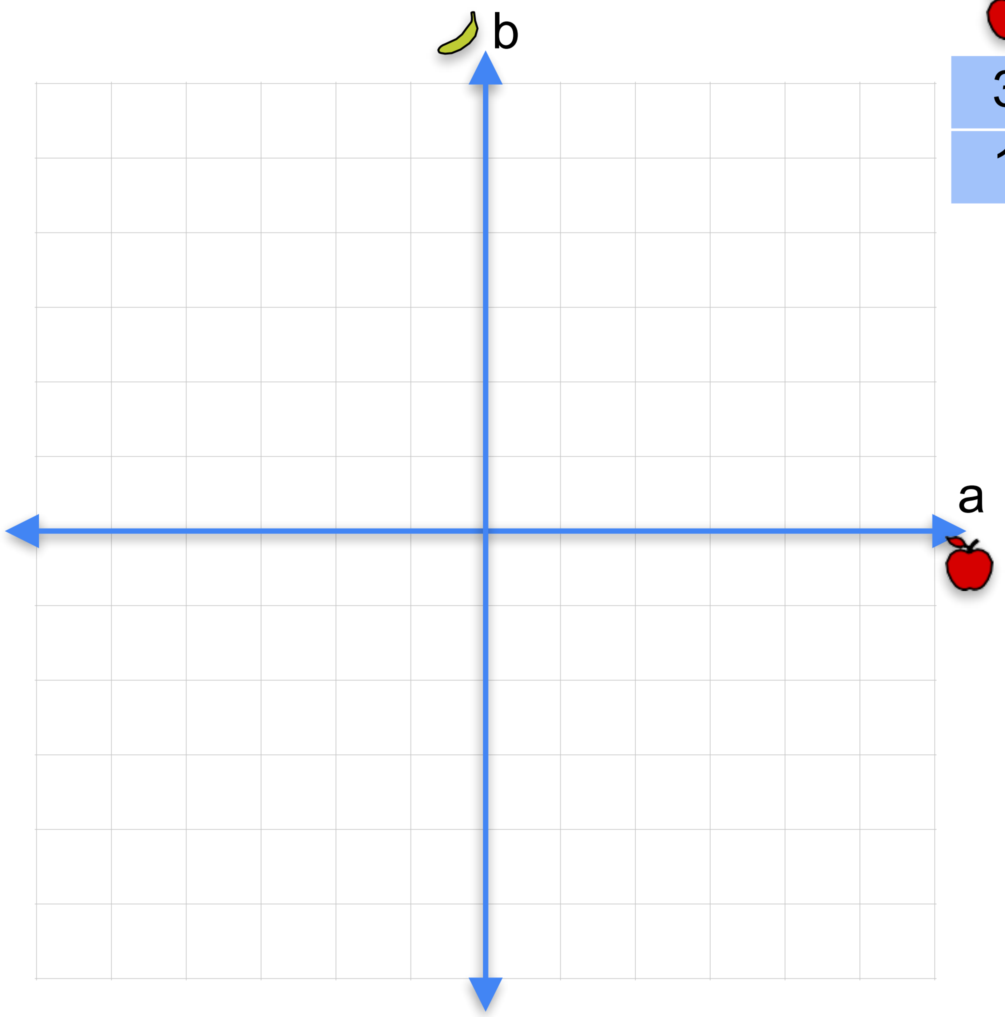
	
3	1
1	2

矩阵是线性变换

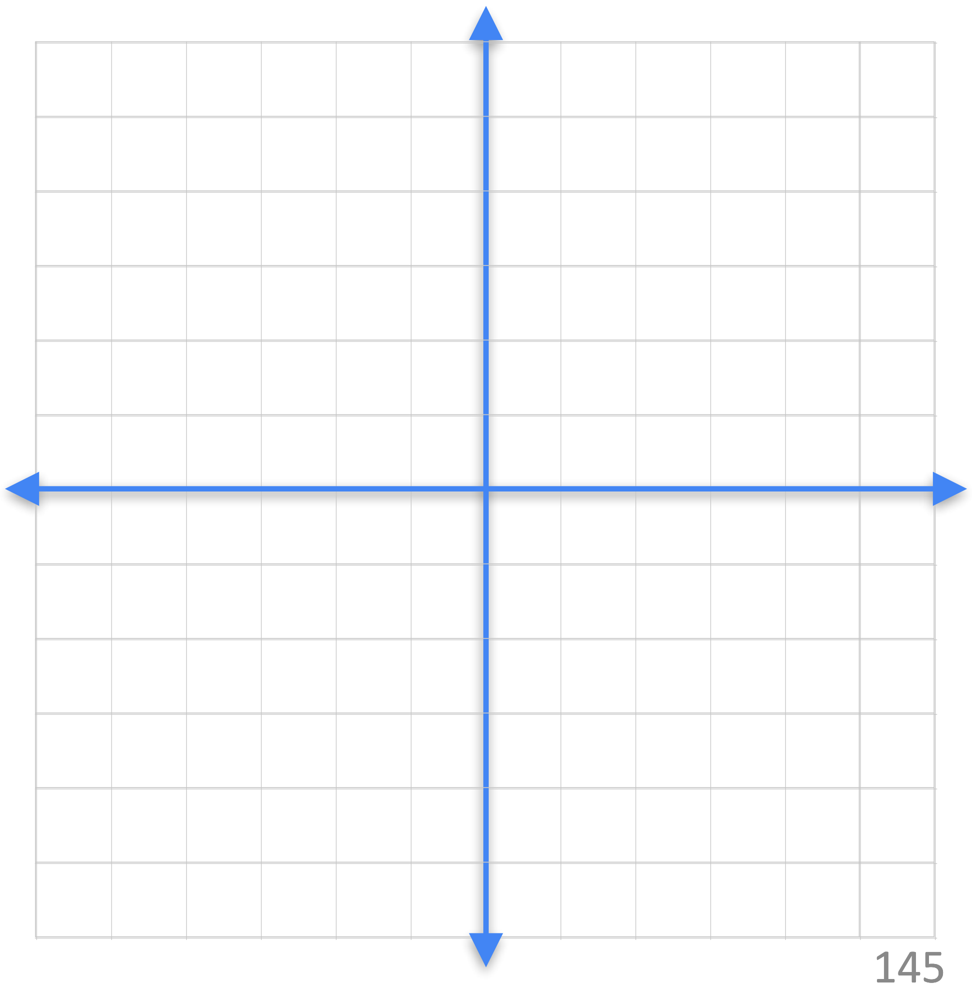


🍎	🍌
3	1
1	2

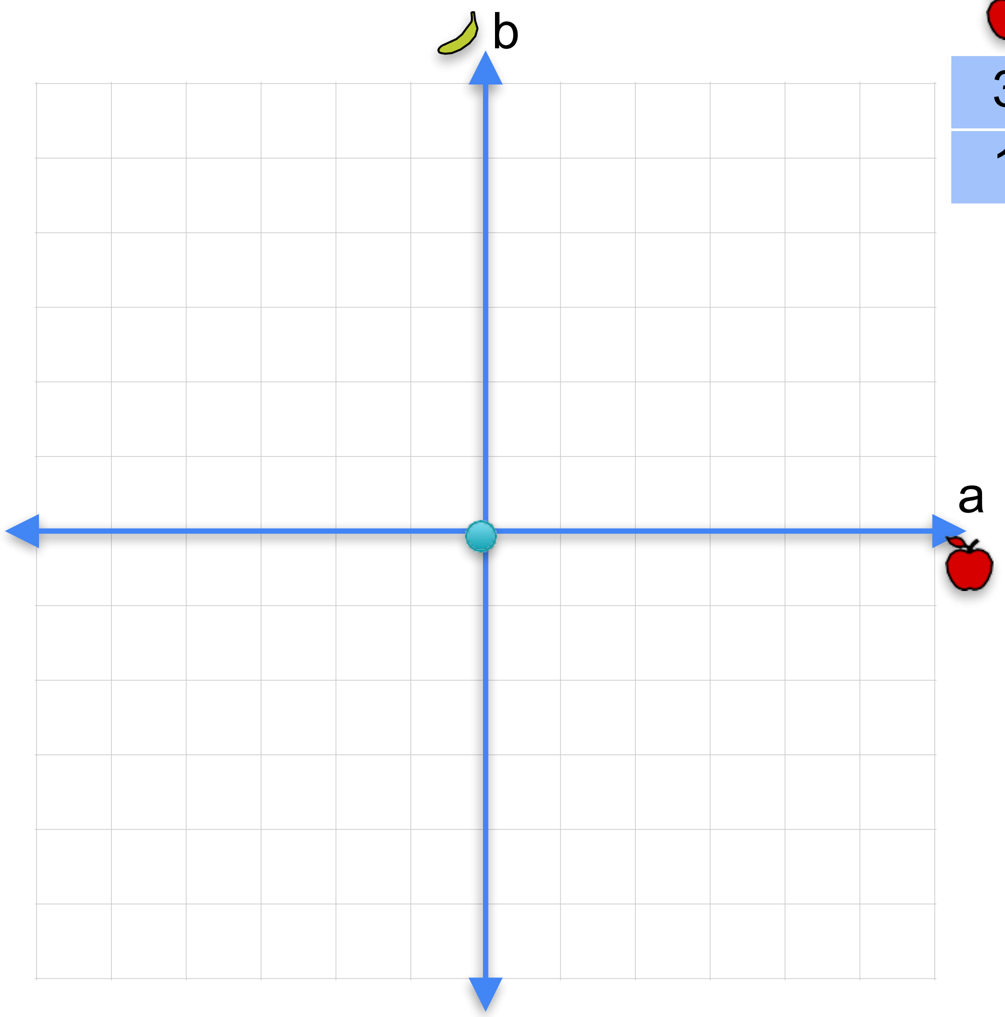
矩阵是线性变换



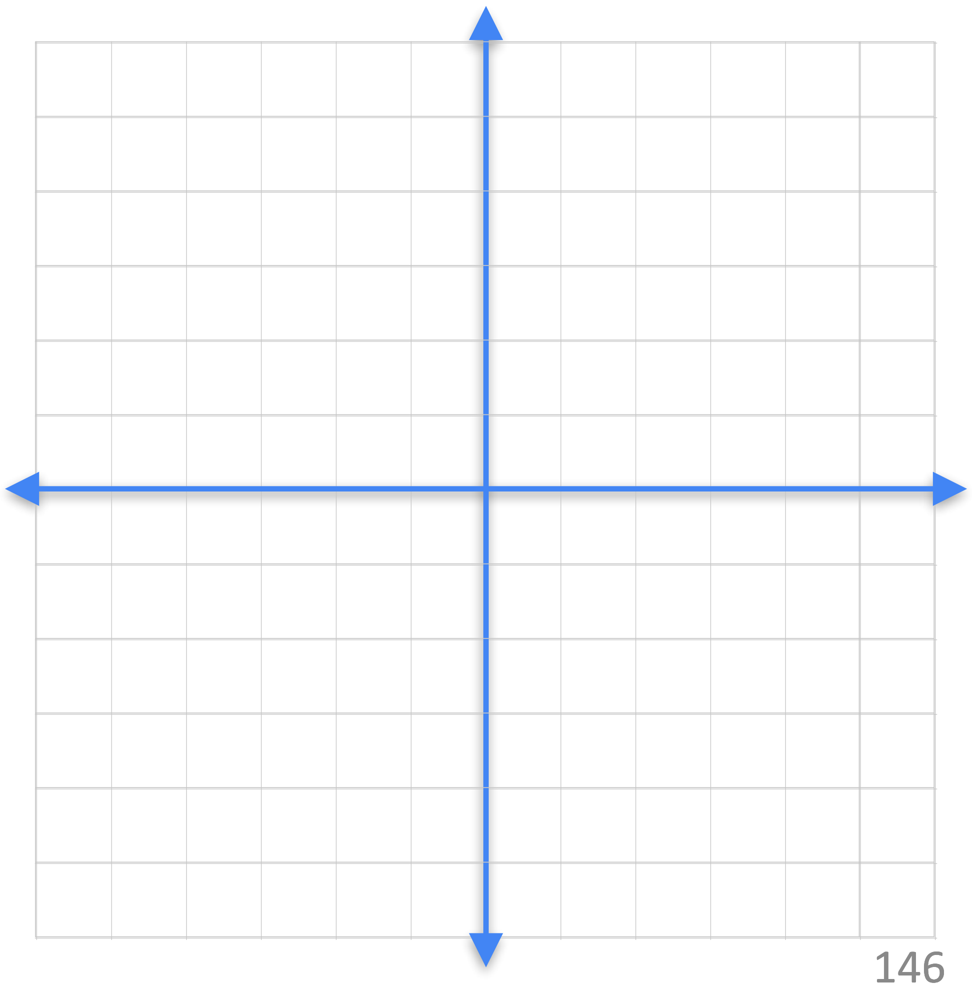
🍎	🍌
3	1
1	2



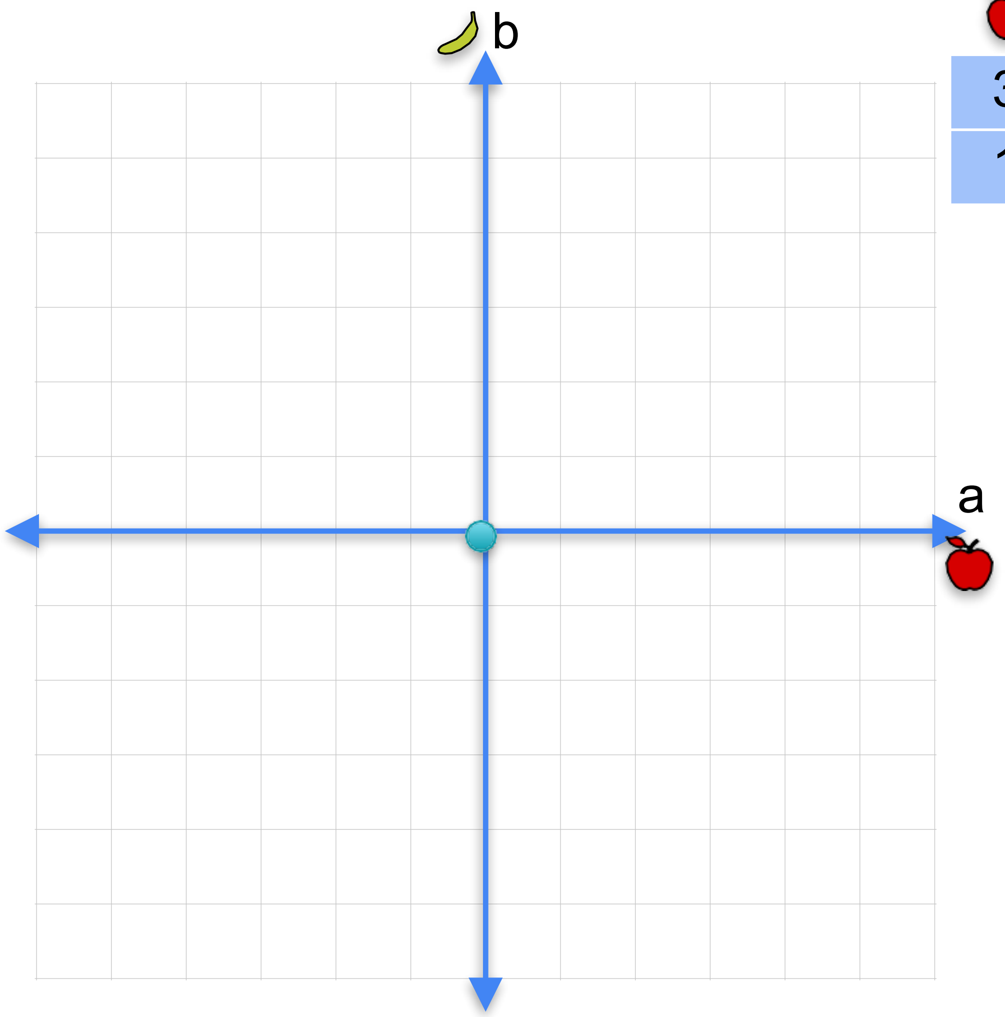
矩阵是线性变换



3	1
1	2



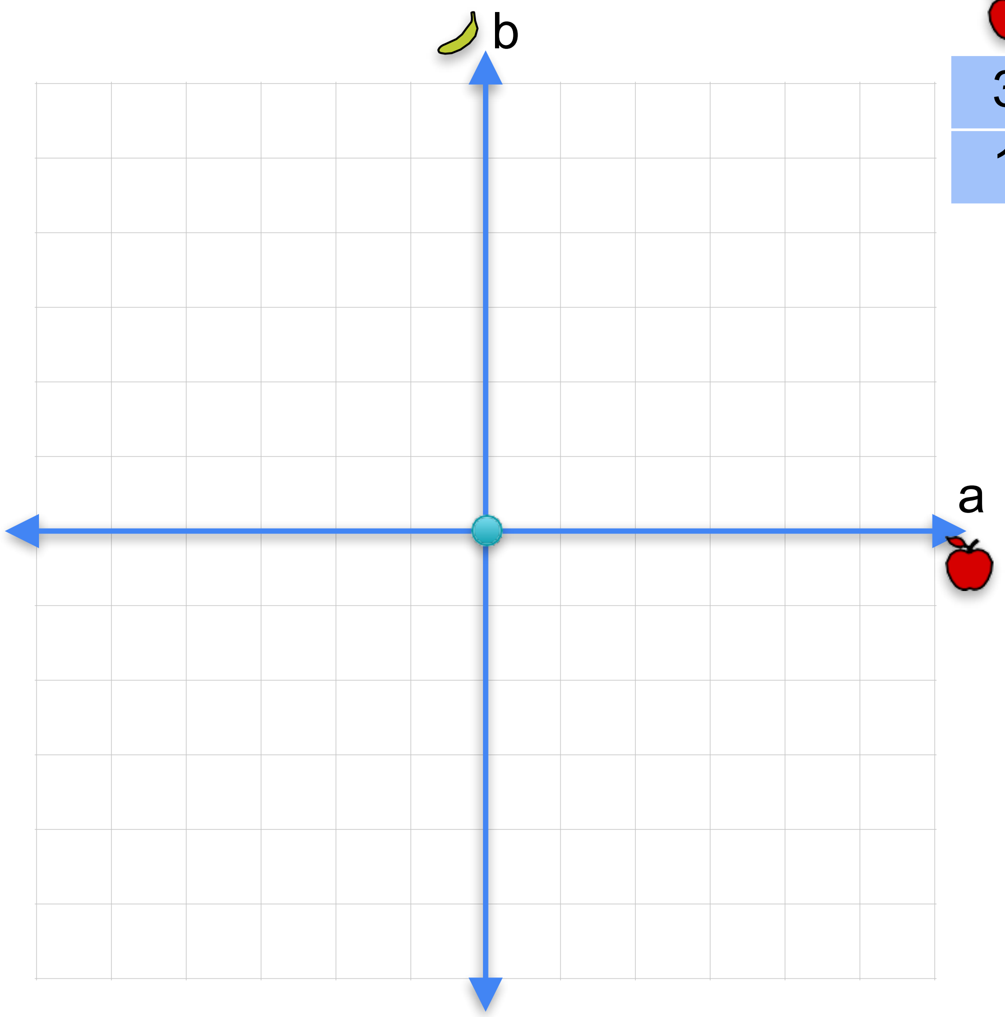
矩阵是线性变换



3	1	0	=	0
1	2	0		0

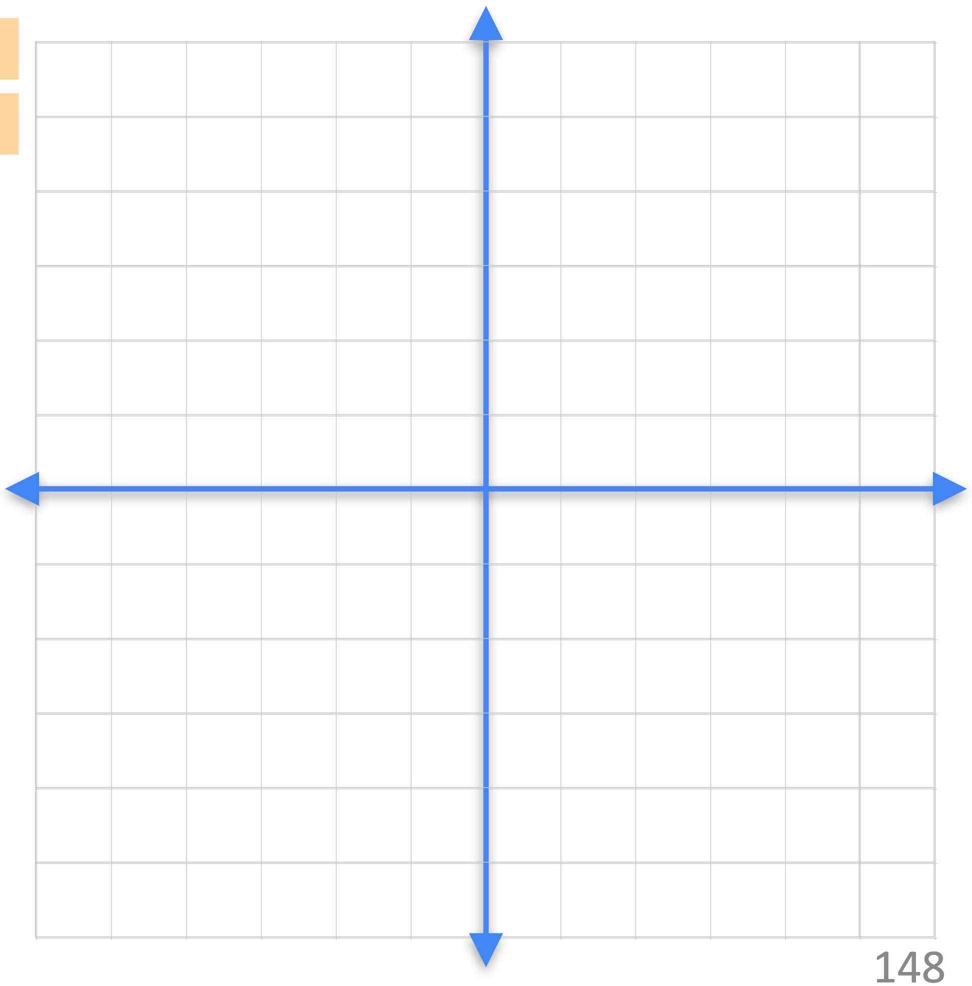
(0,0) → (0,0)

矩阵是线性变换

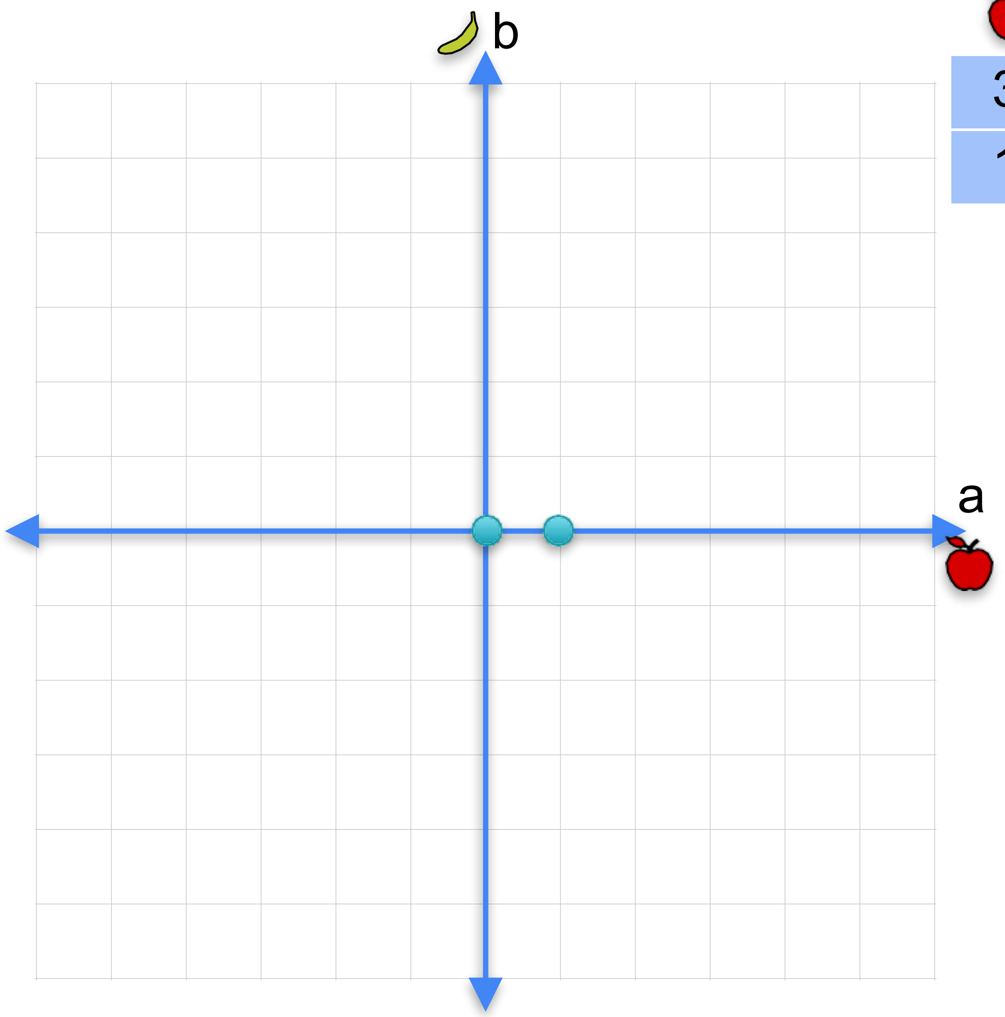


3	1	0	=	0
1	2	0		0

(0,0) → (0,0)

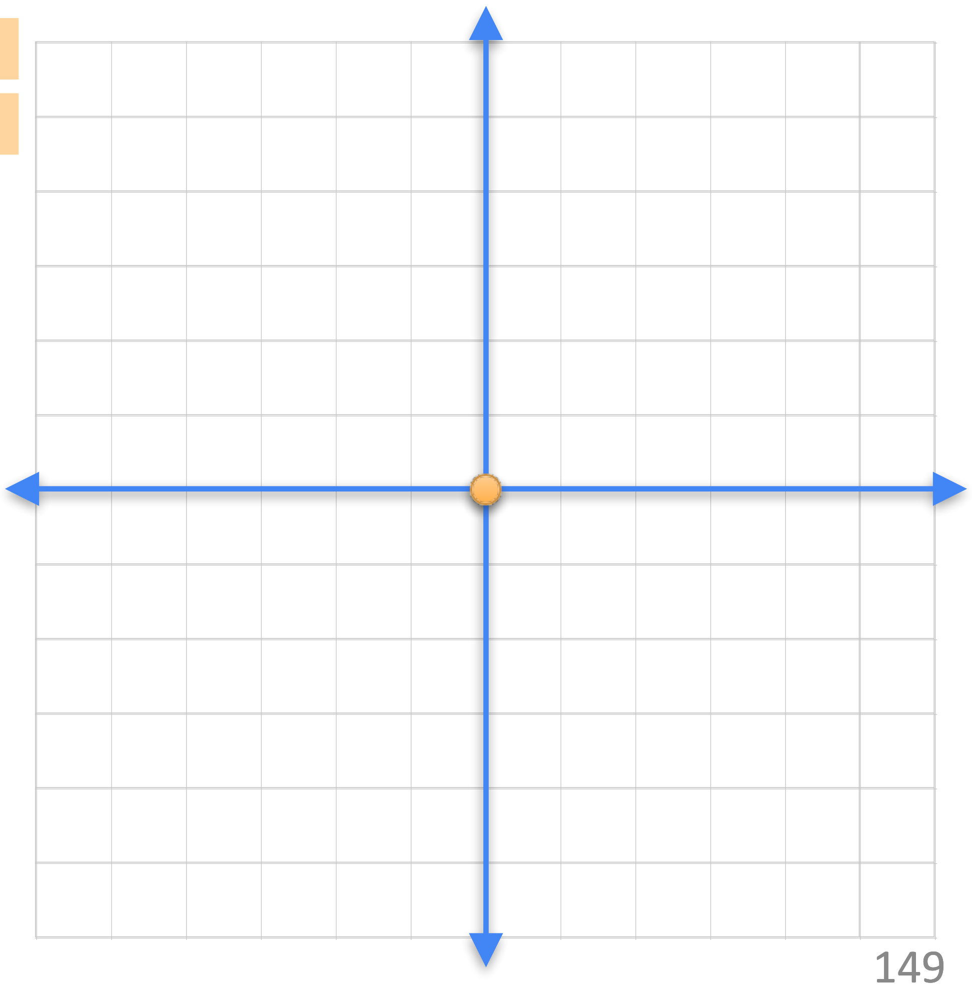


矩阵是线性变换

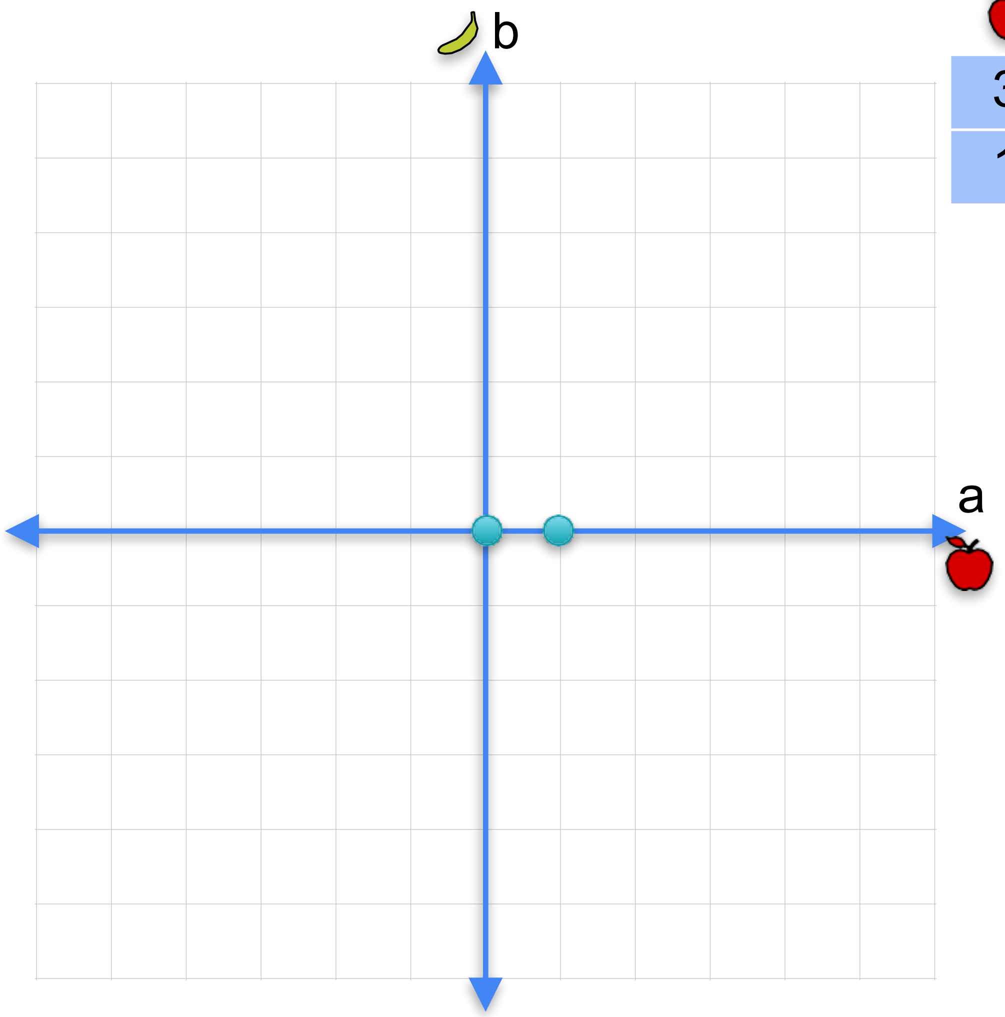


🍎	🍌			
3	1	0	=	0
1	2	0		0

(0,0) → (0,0)



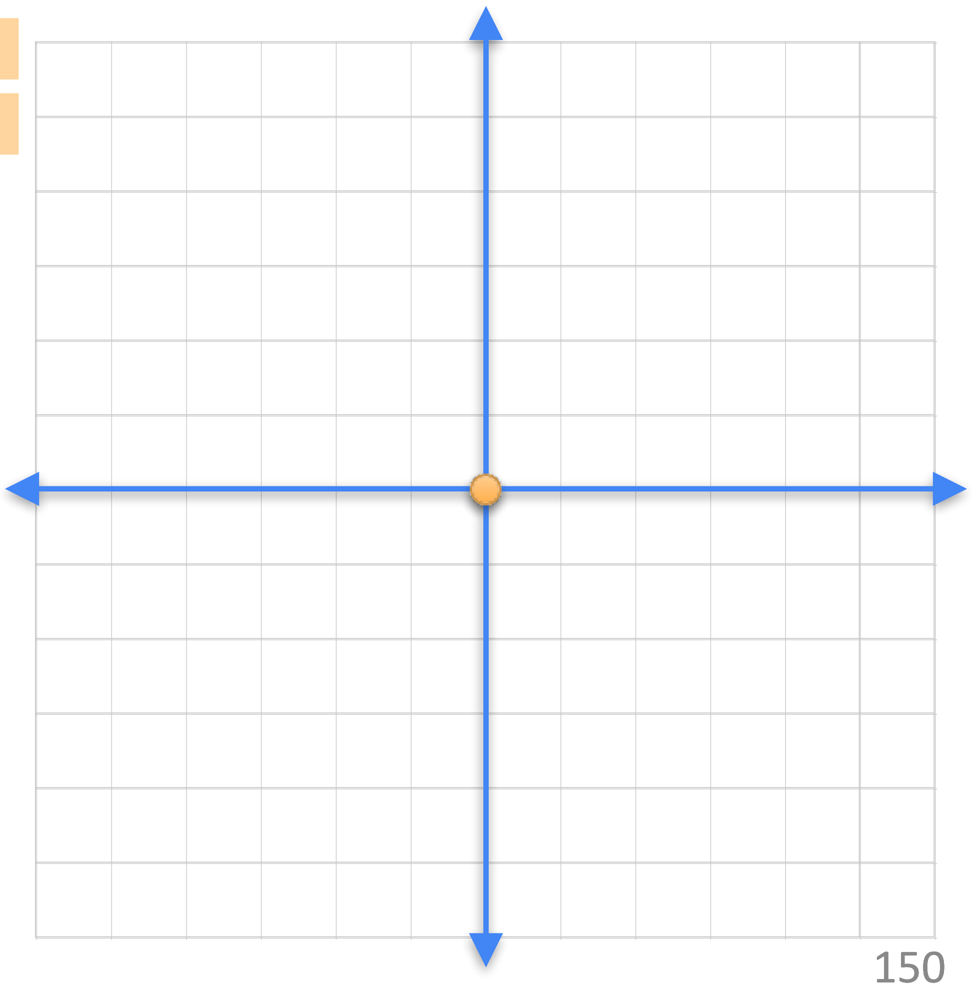
矩阵是线性变换



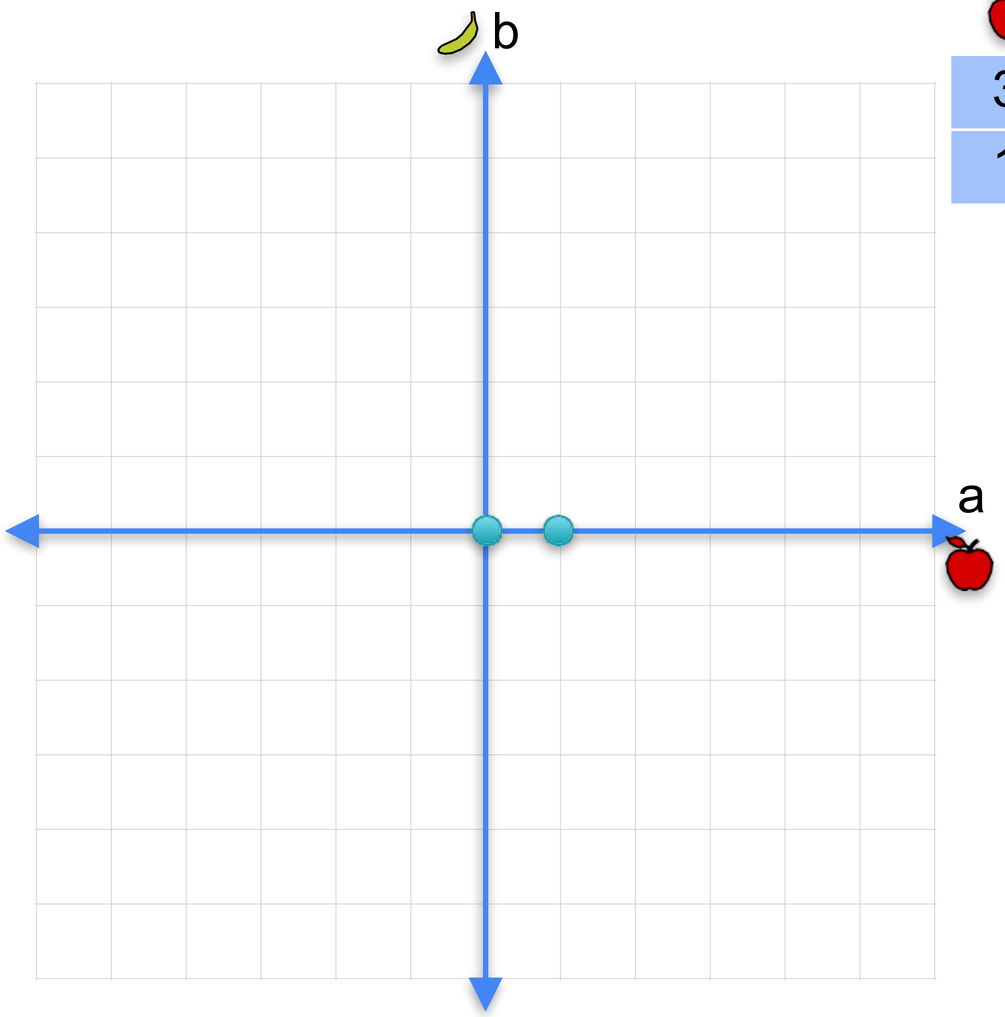
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (3,1)



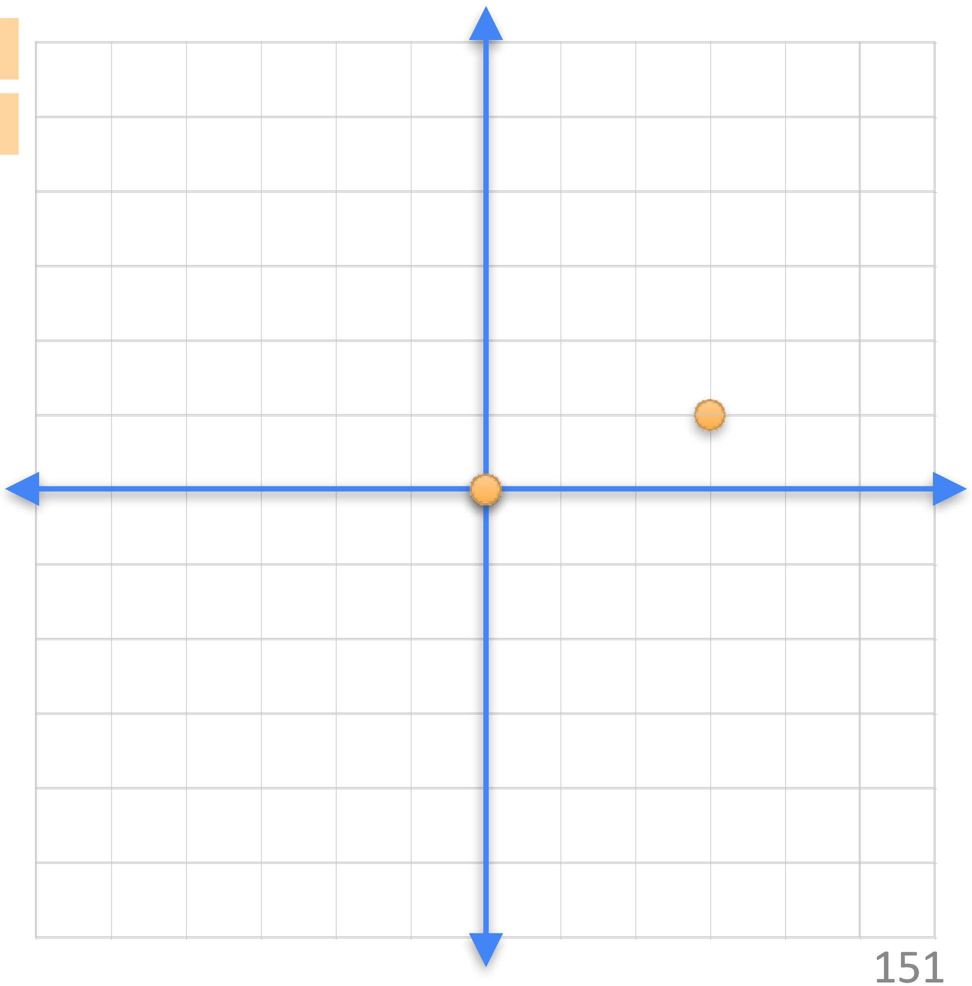
矩阵是线性变换



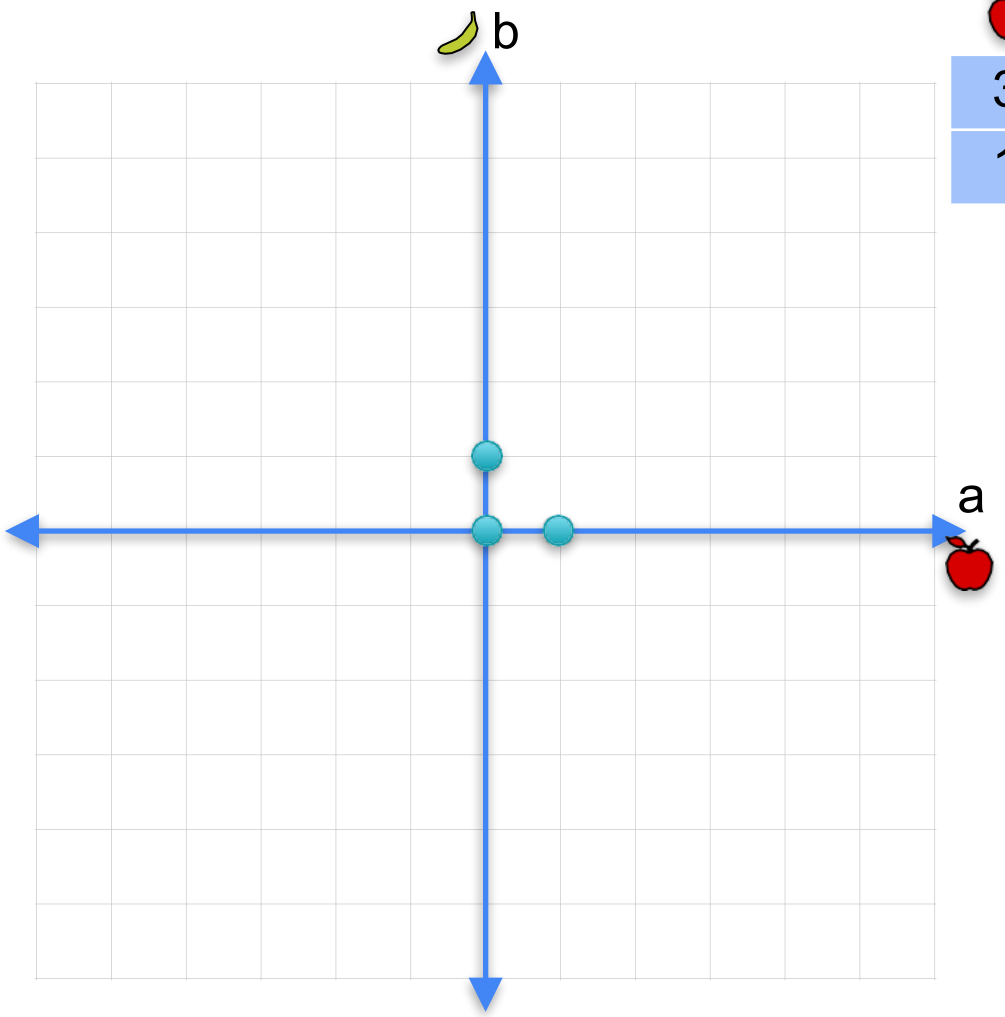
3	1	1	=	3
1	2	0		1

(0,0) → (0,0)

(1,0) → (3,1)



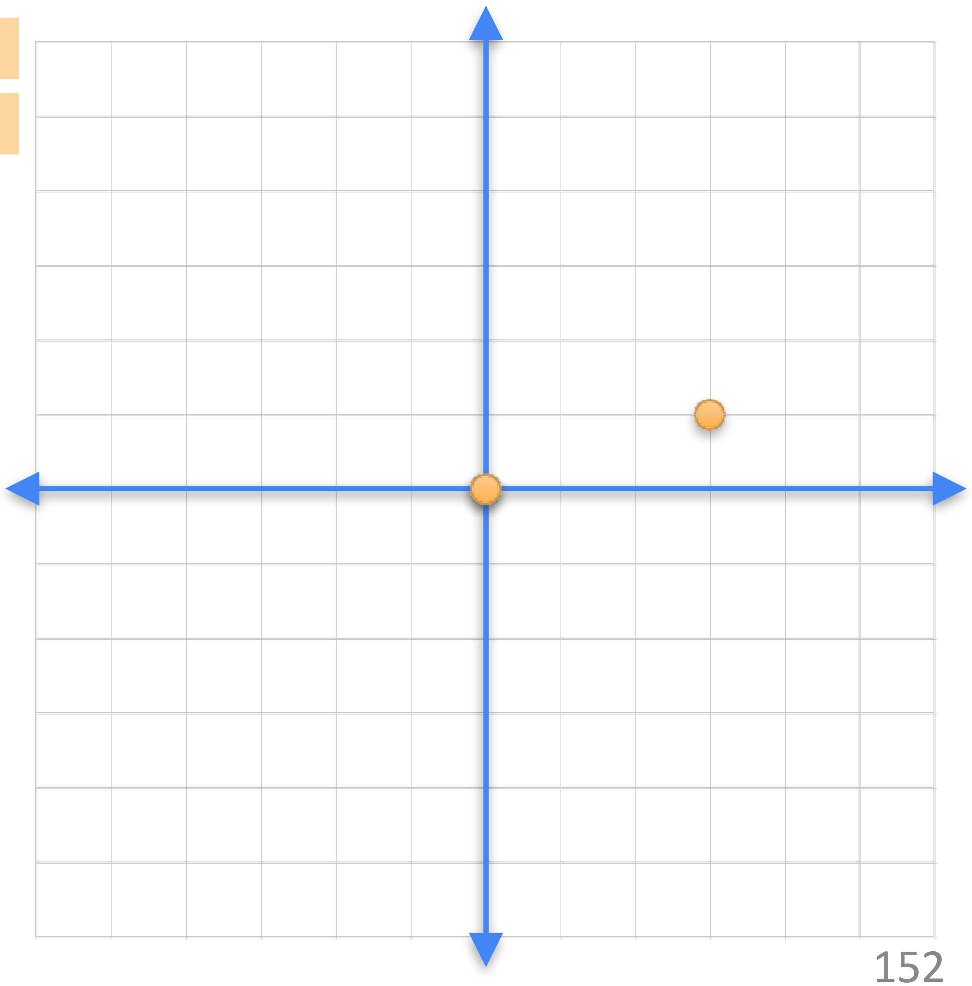
矩阵是线性变换



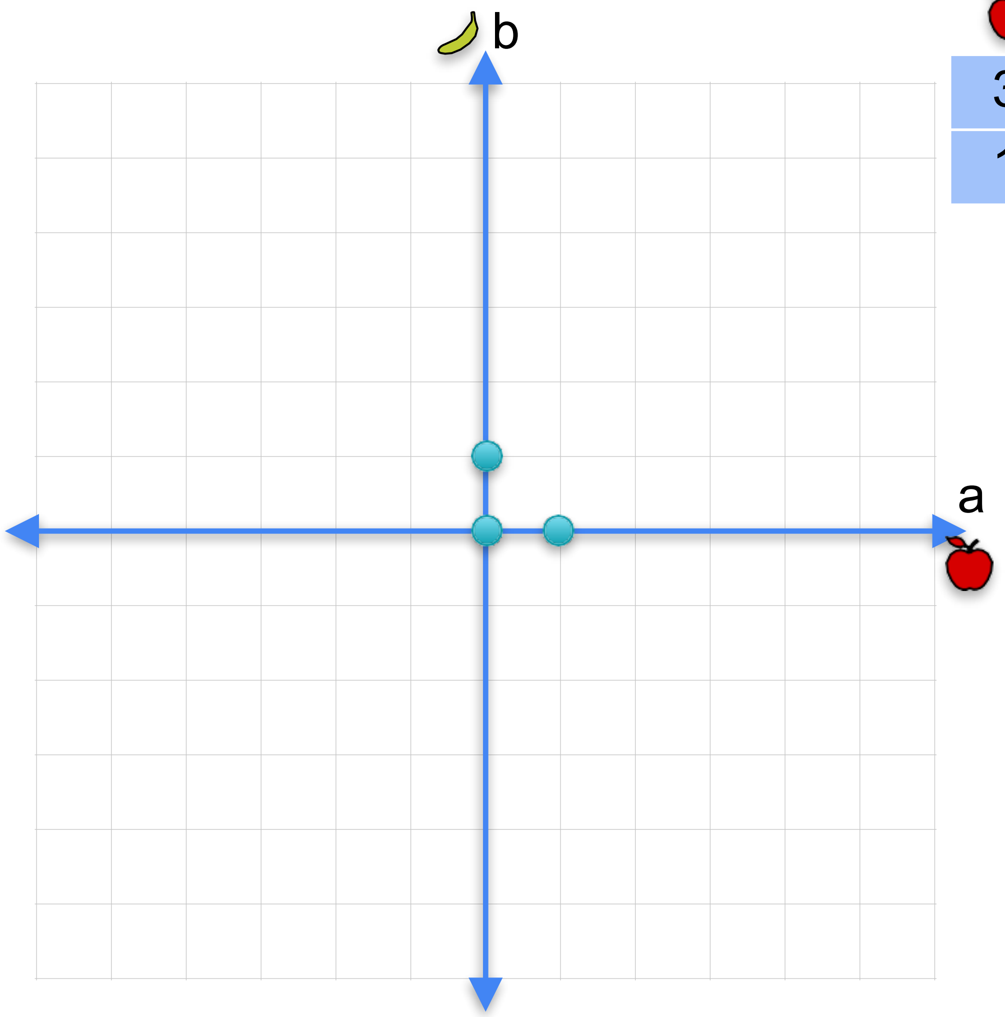
3	1	1	=	3
1	2	0		1

(0,0) → (0,0)

(1,0) → (3,1)



矩阵是线性变换

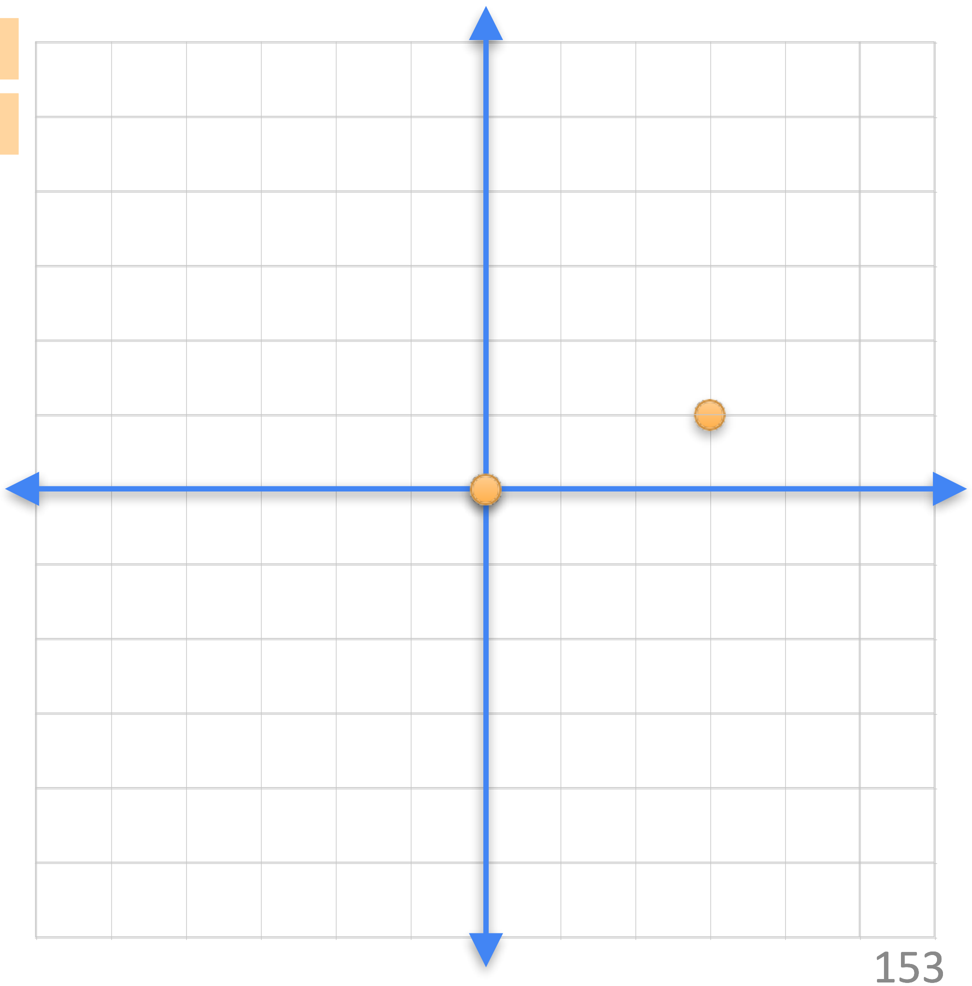


🍎	🍌			
3	1	0	=	1
1	2	1		2

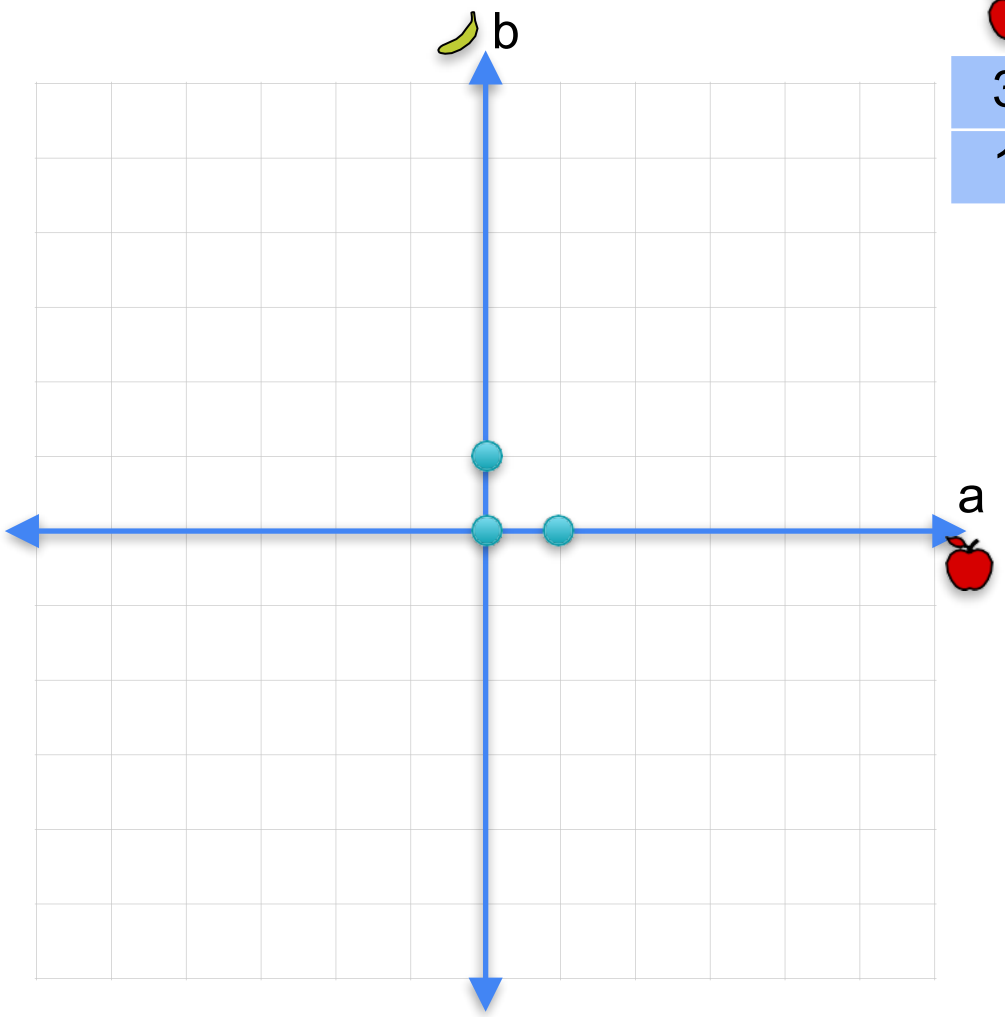
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)



矩阵是线性变换

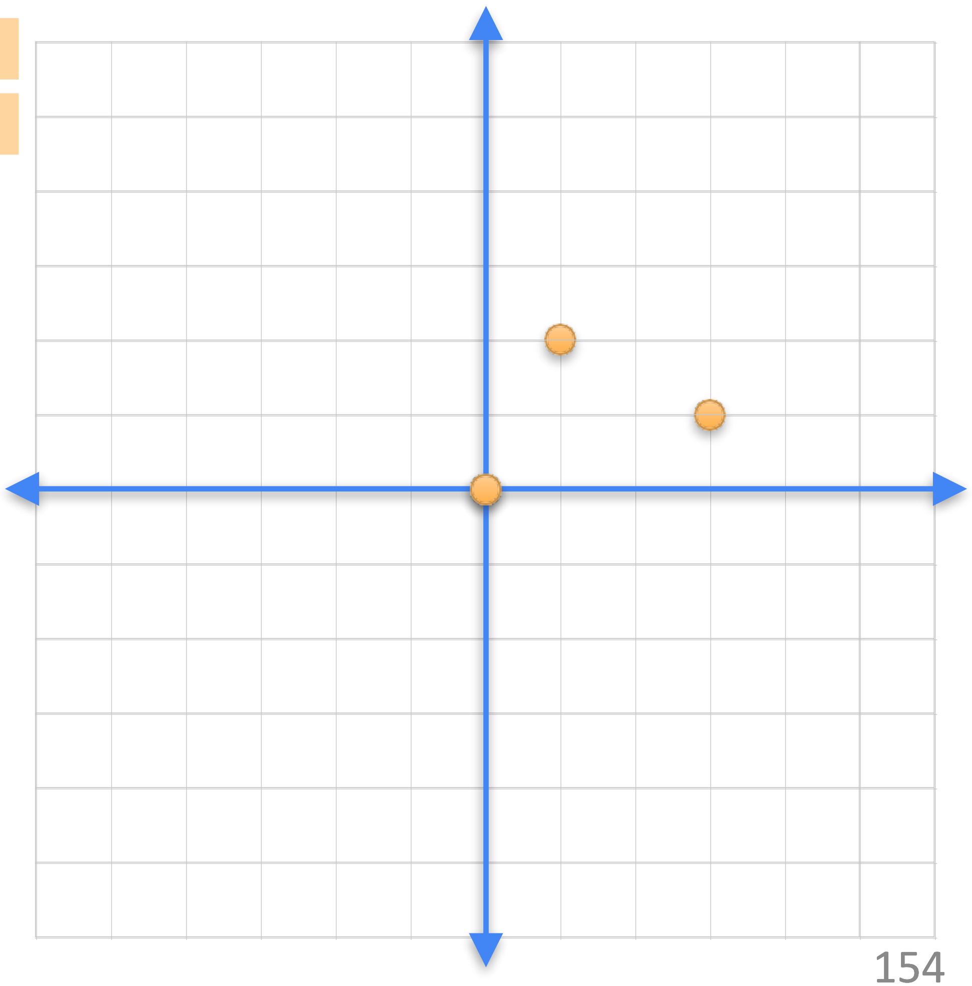


$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}1\\ 2\end{bmatrix}$$

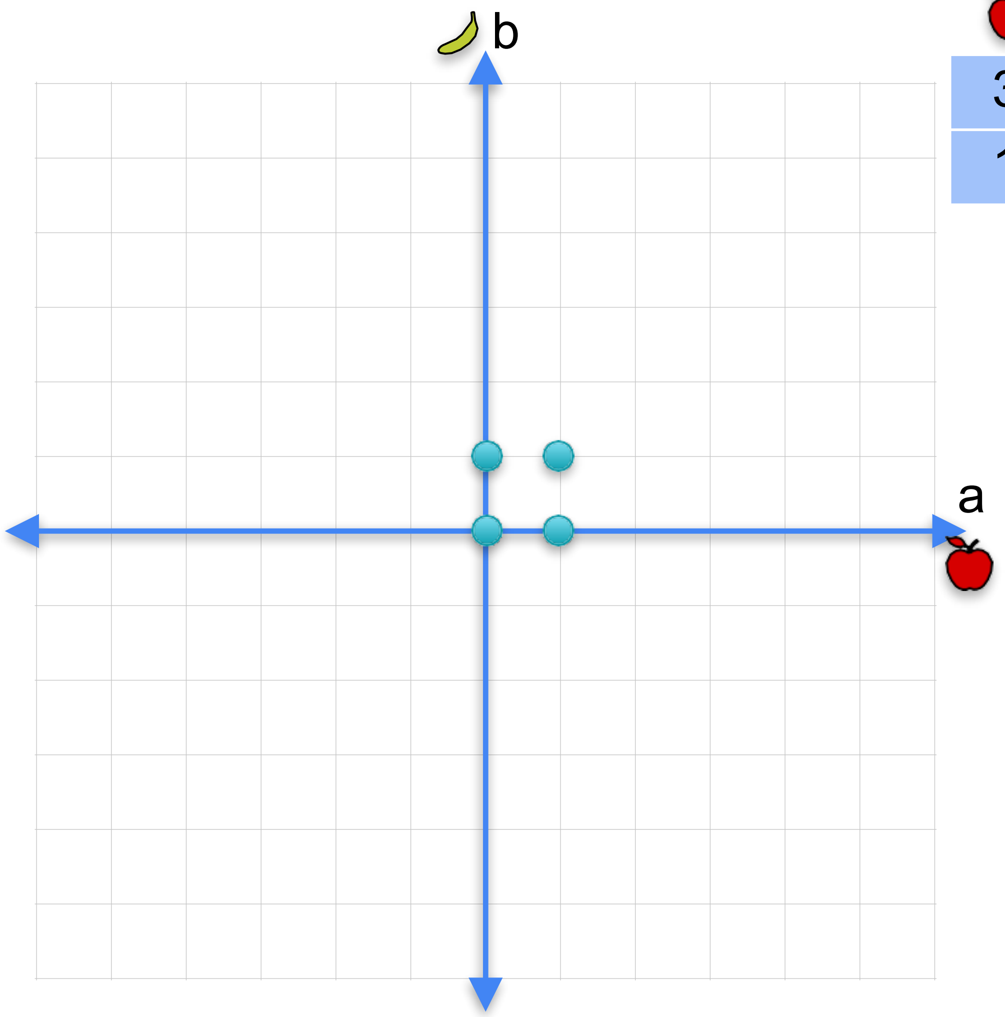
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)



矩阵是线性变换

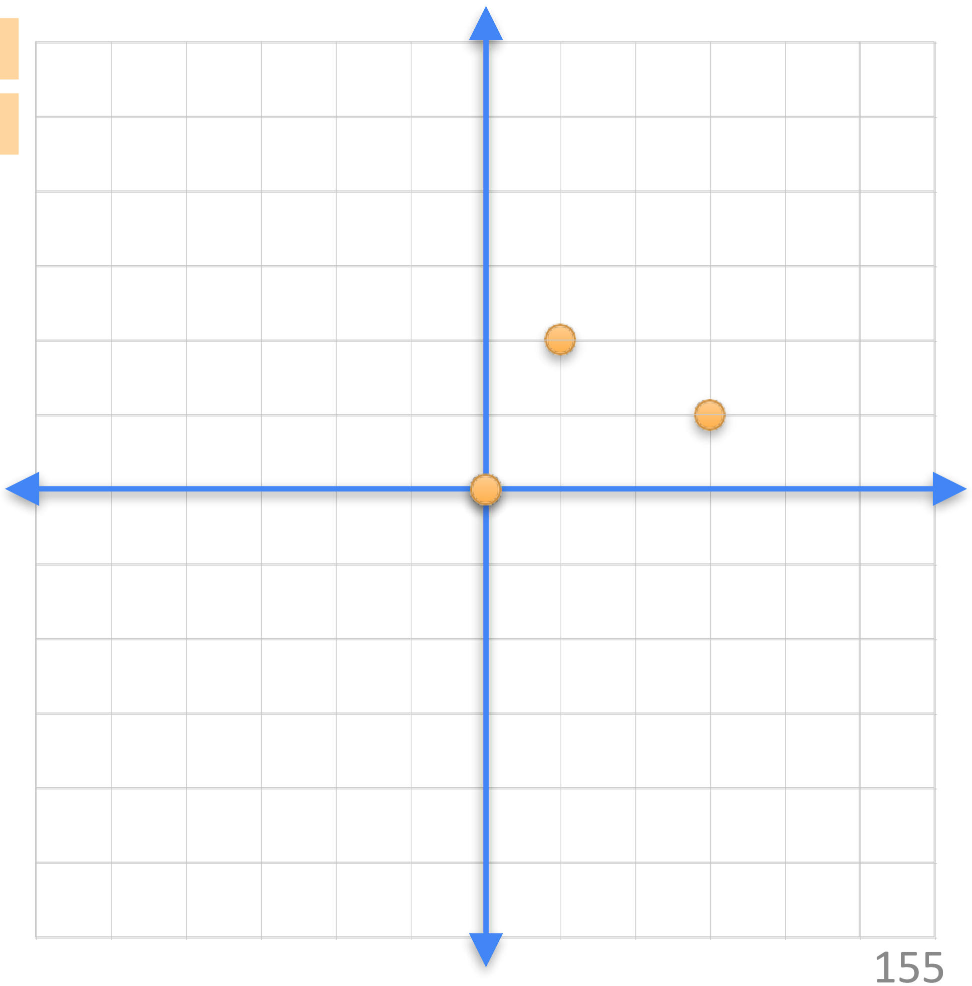


$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

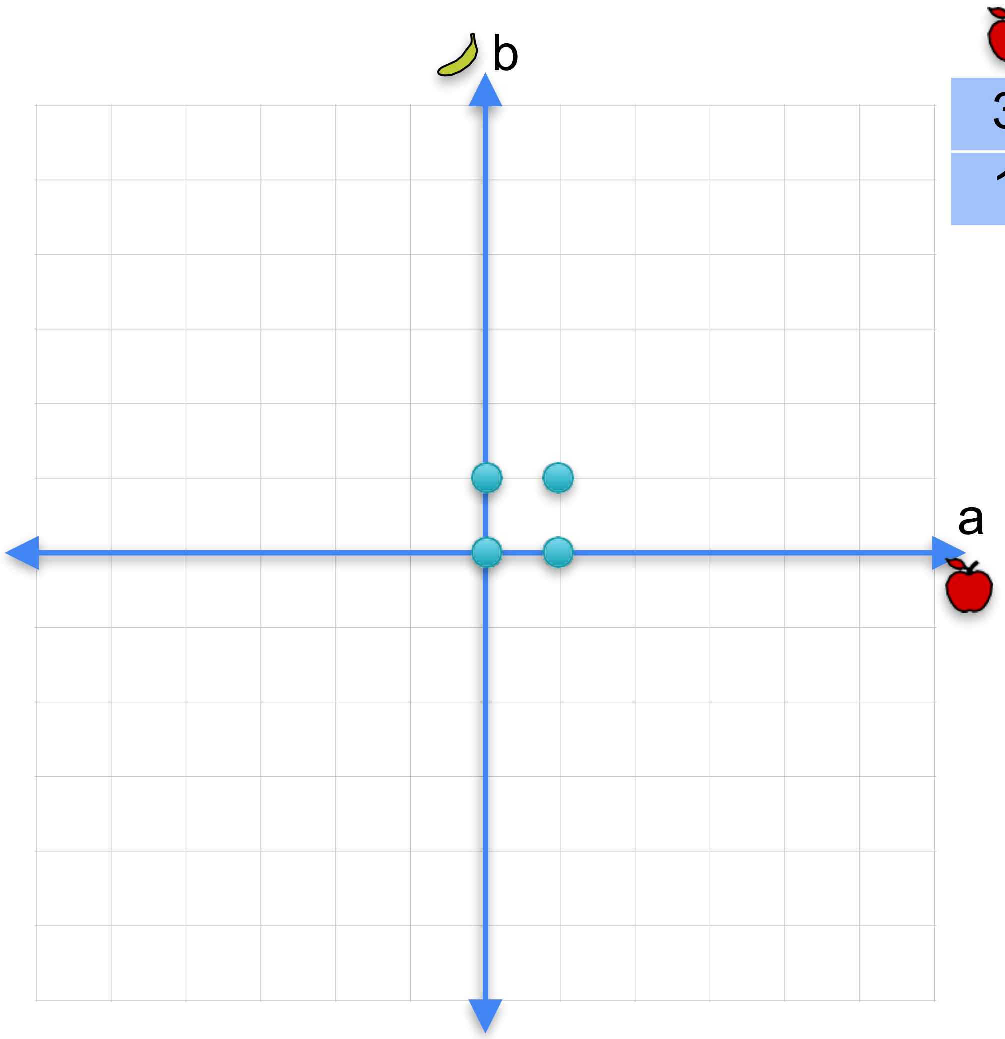
(0,0) → (0,0)

(1,0) → (3,1)

(0,1) → (1,2)



矩阵是线性变换



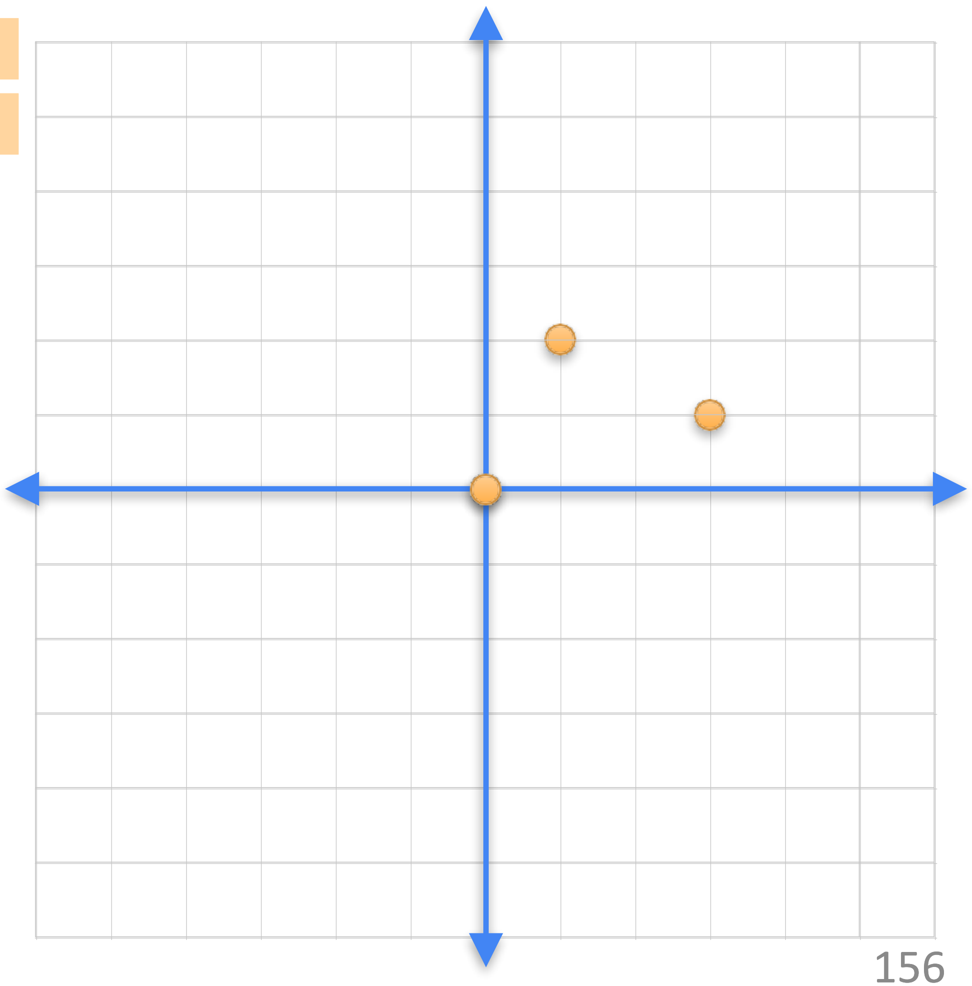
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

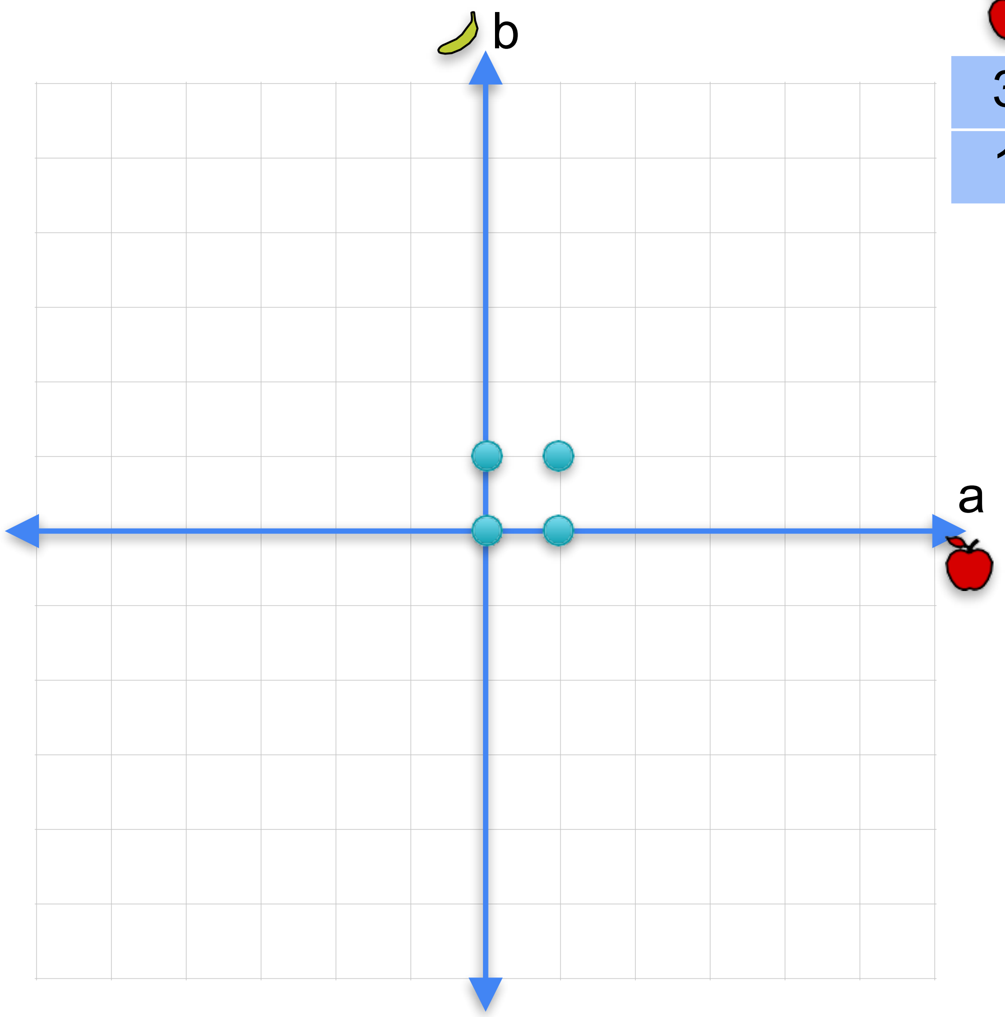
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



矩阵是线性变换



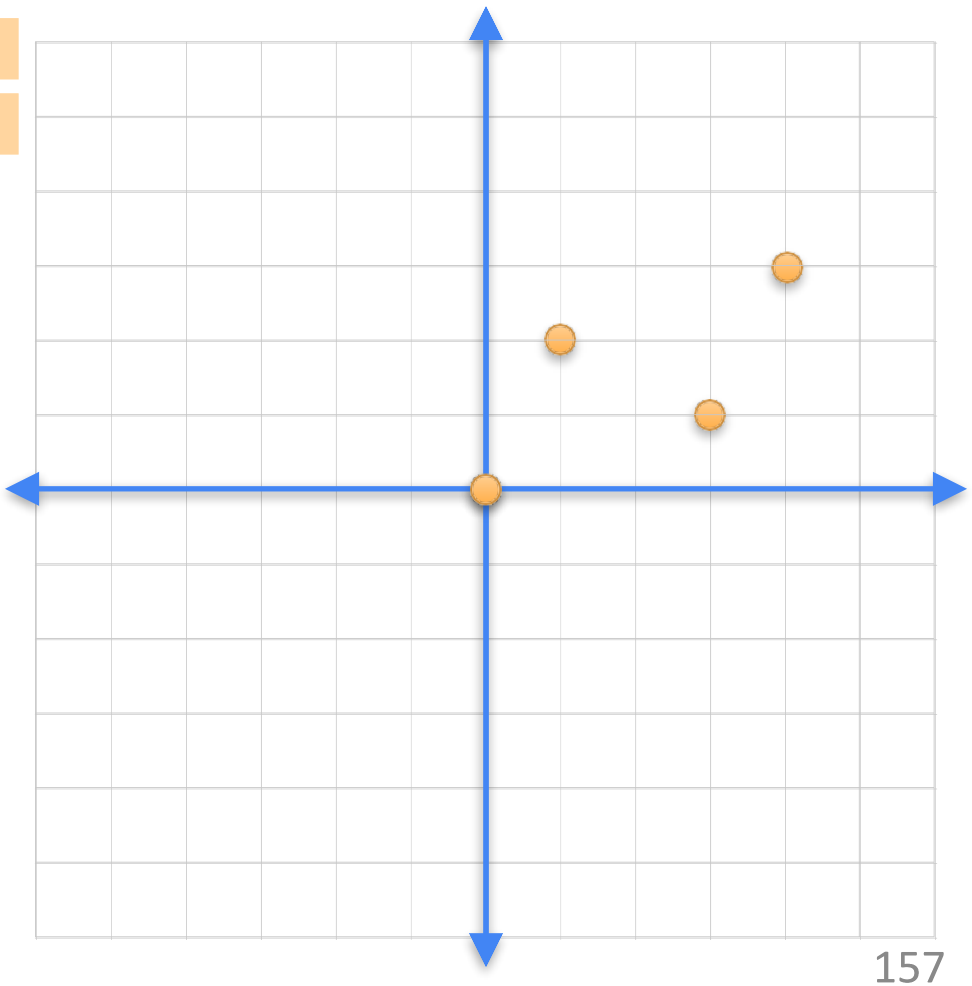
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

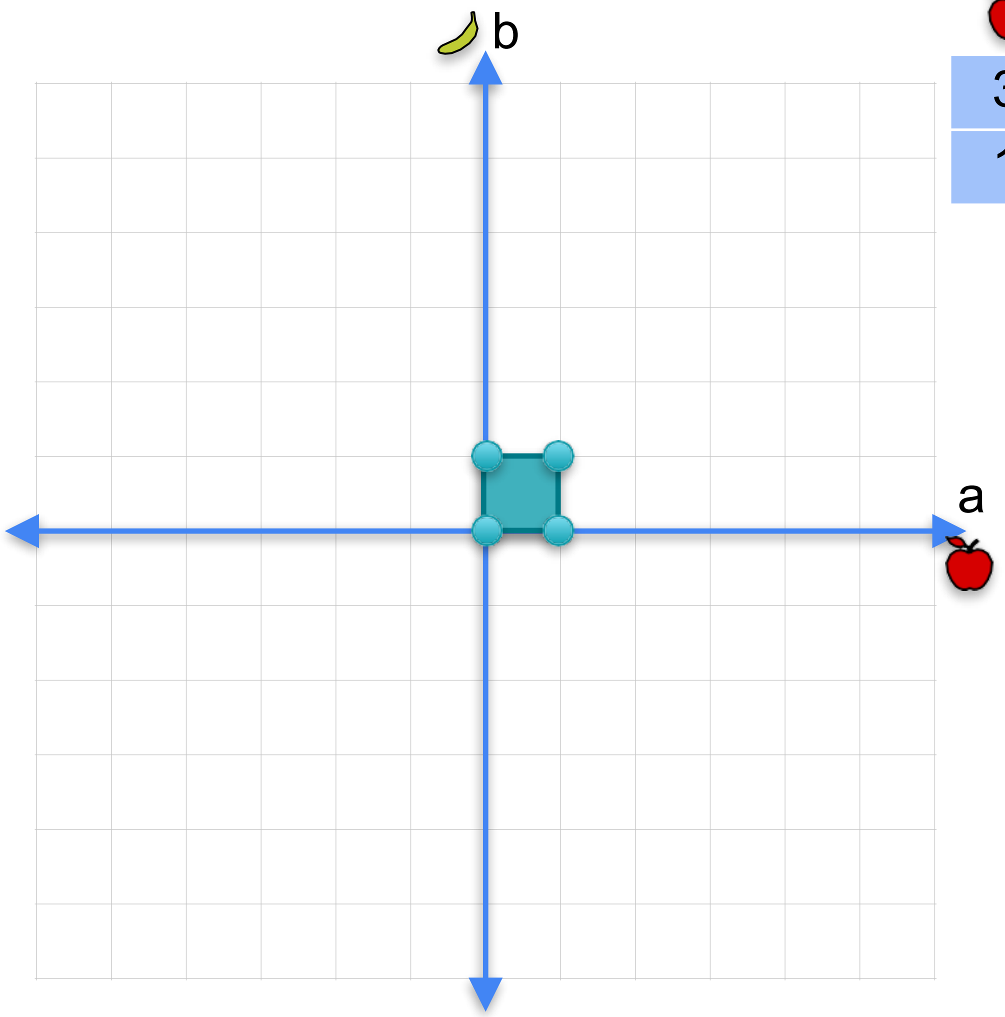
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



矩阵是线性变换



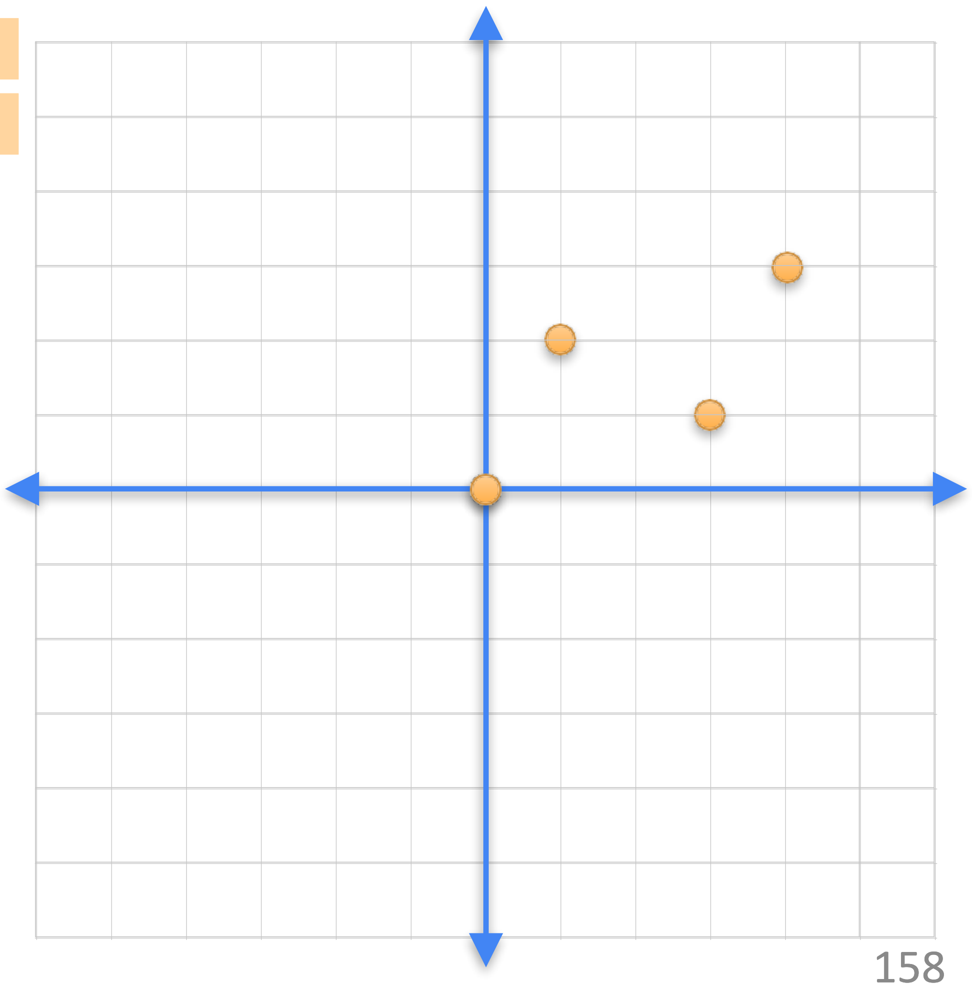
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

(0,0) → (0,0)

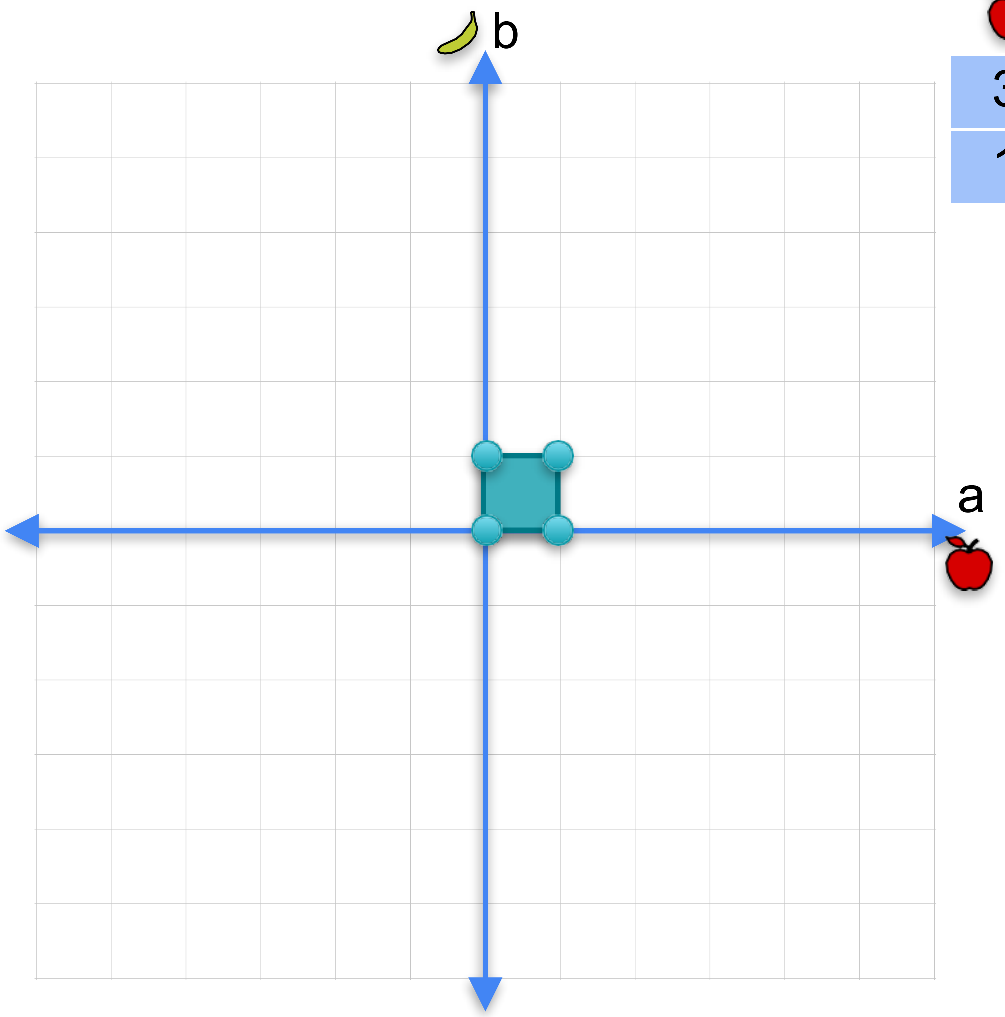
(1,0) → (3,1)

(0,1) → (1,2)

(1,1) → (4,3)



矩阵是线性变换



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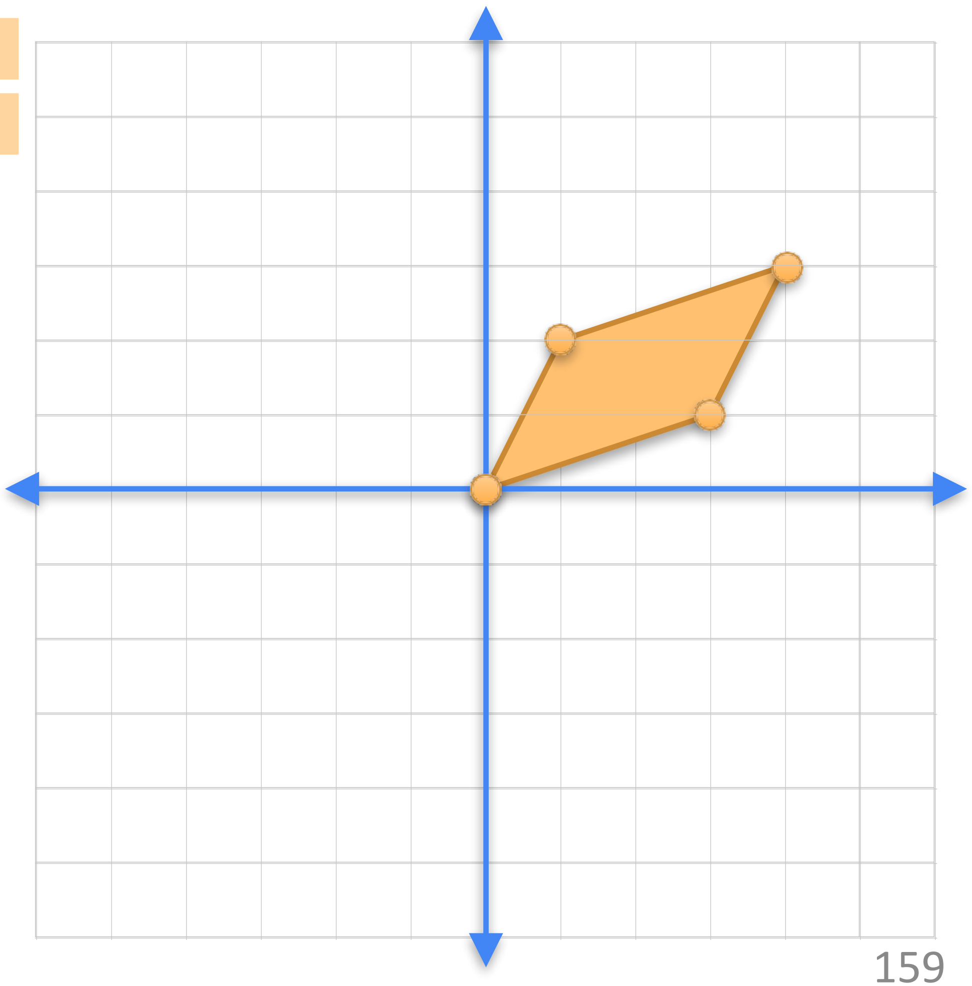
3	1	1	=	4
1	2	1		3

(0,0) → (0,0)

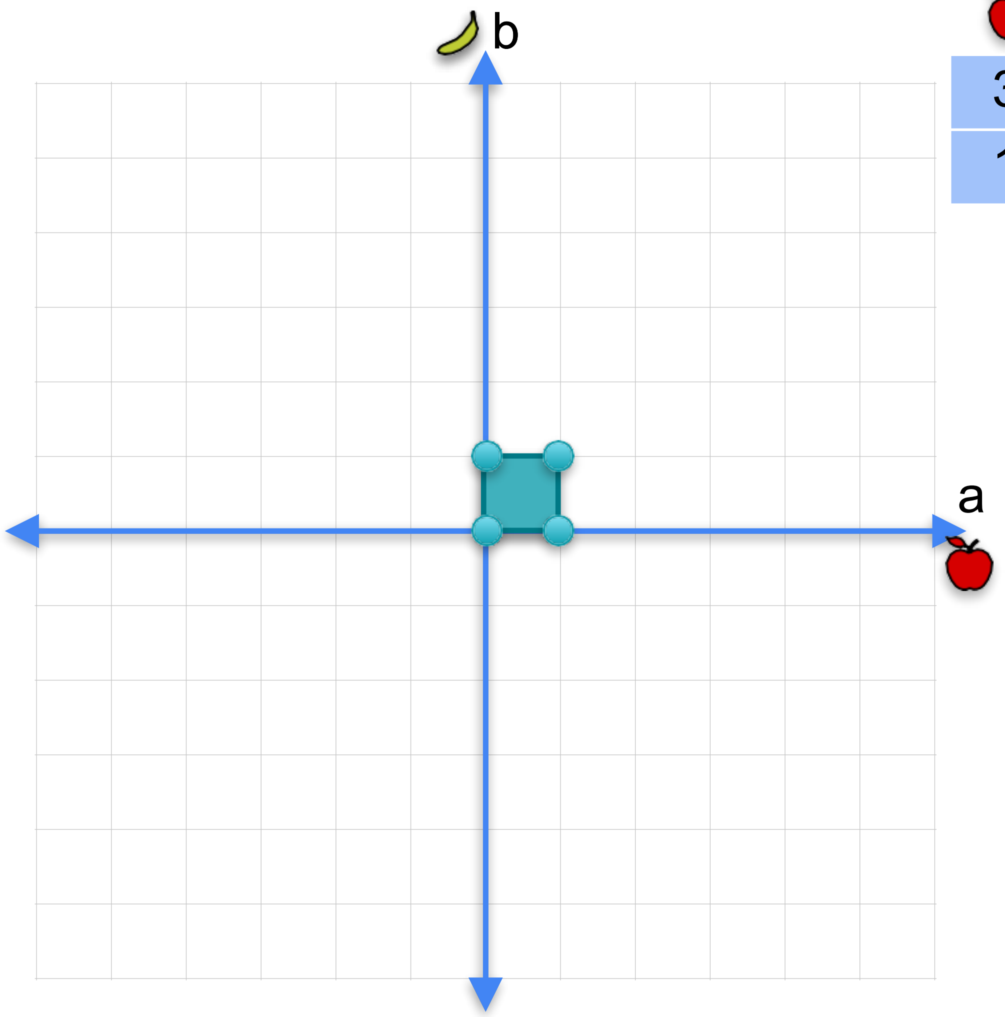
(1,0) → (3,1)

(0,1) → (1,2)

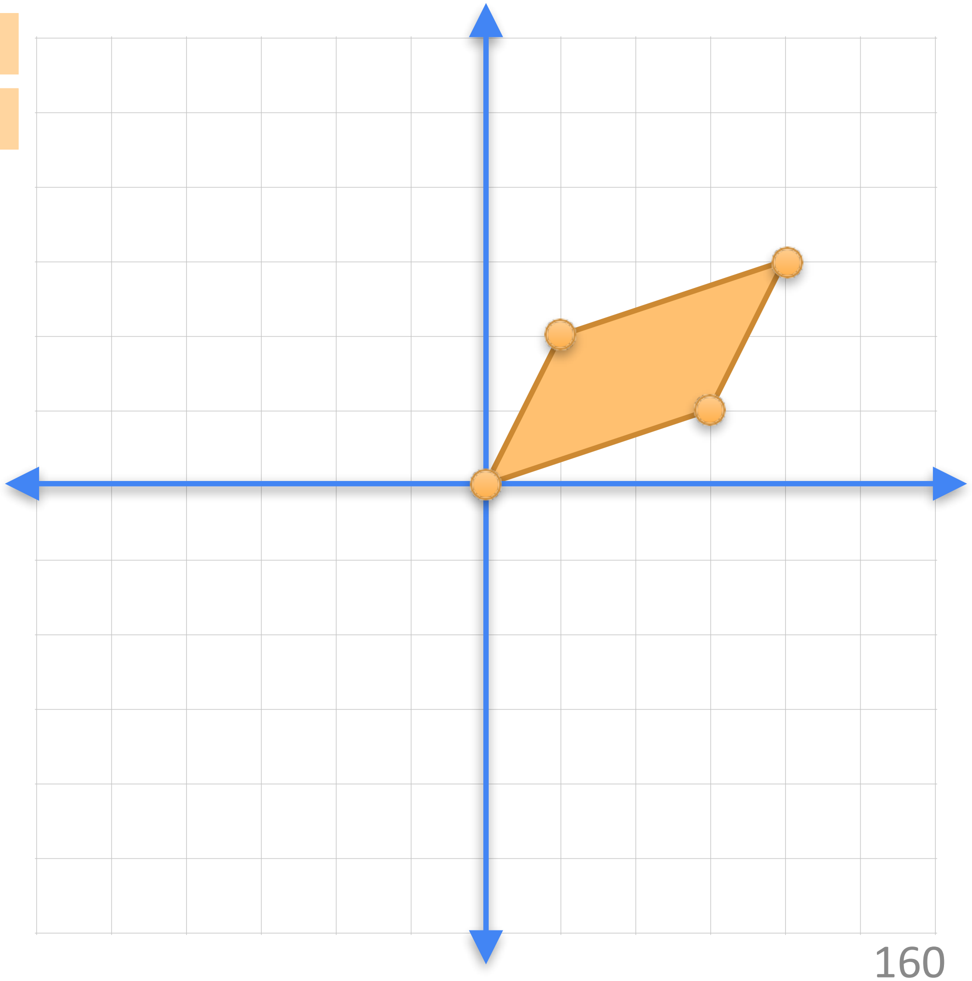
(1,1) → (4,3)



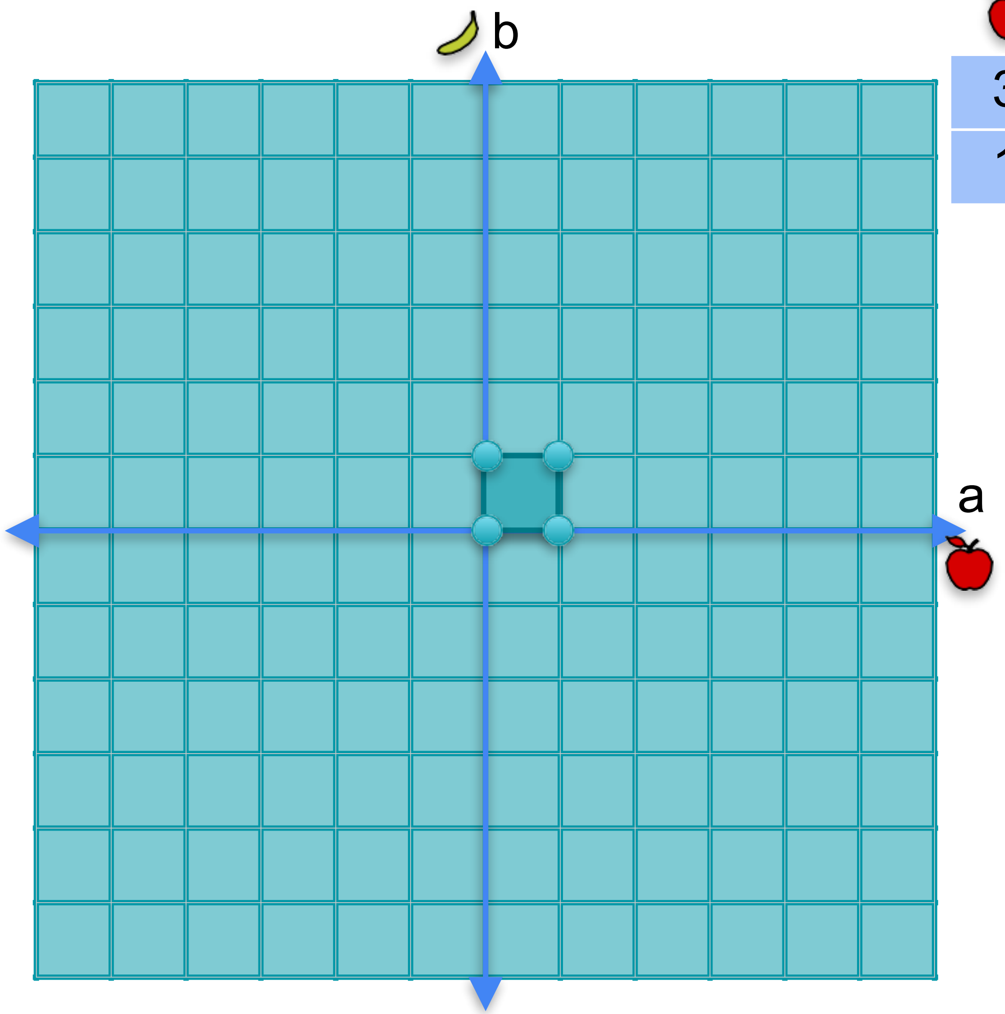
矩阵是线性变换



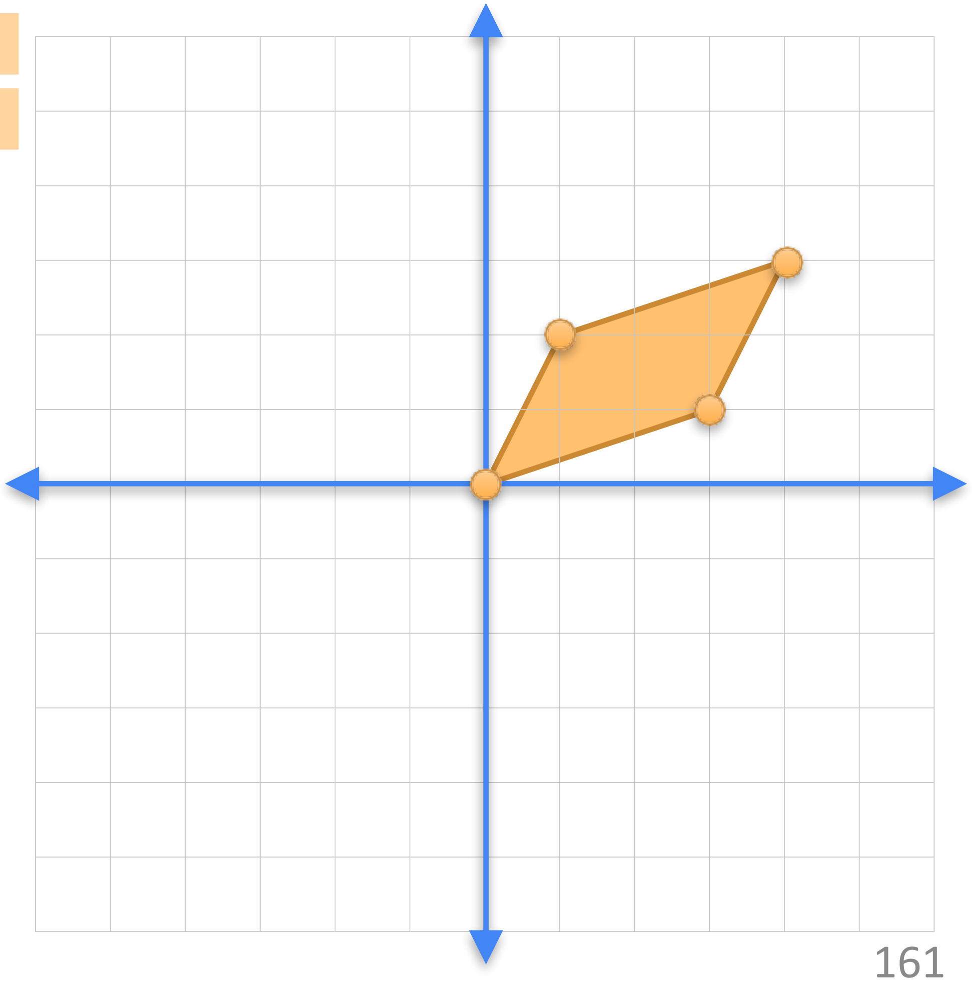
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



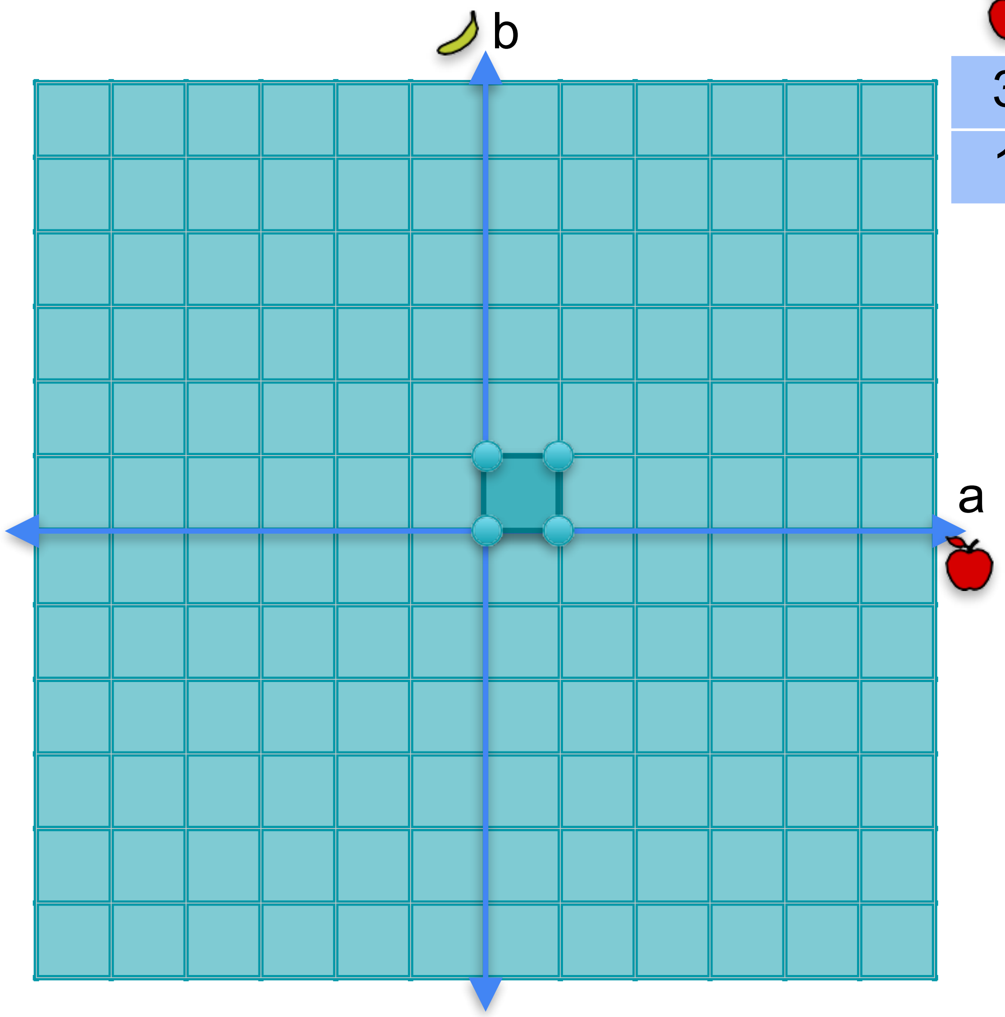
矩阵是线性变换



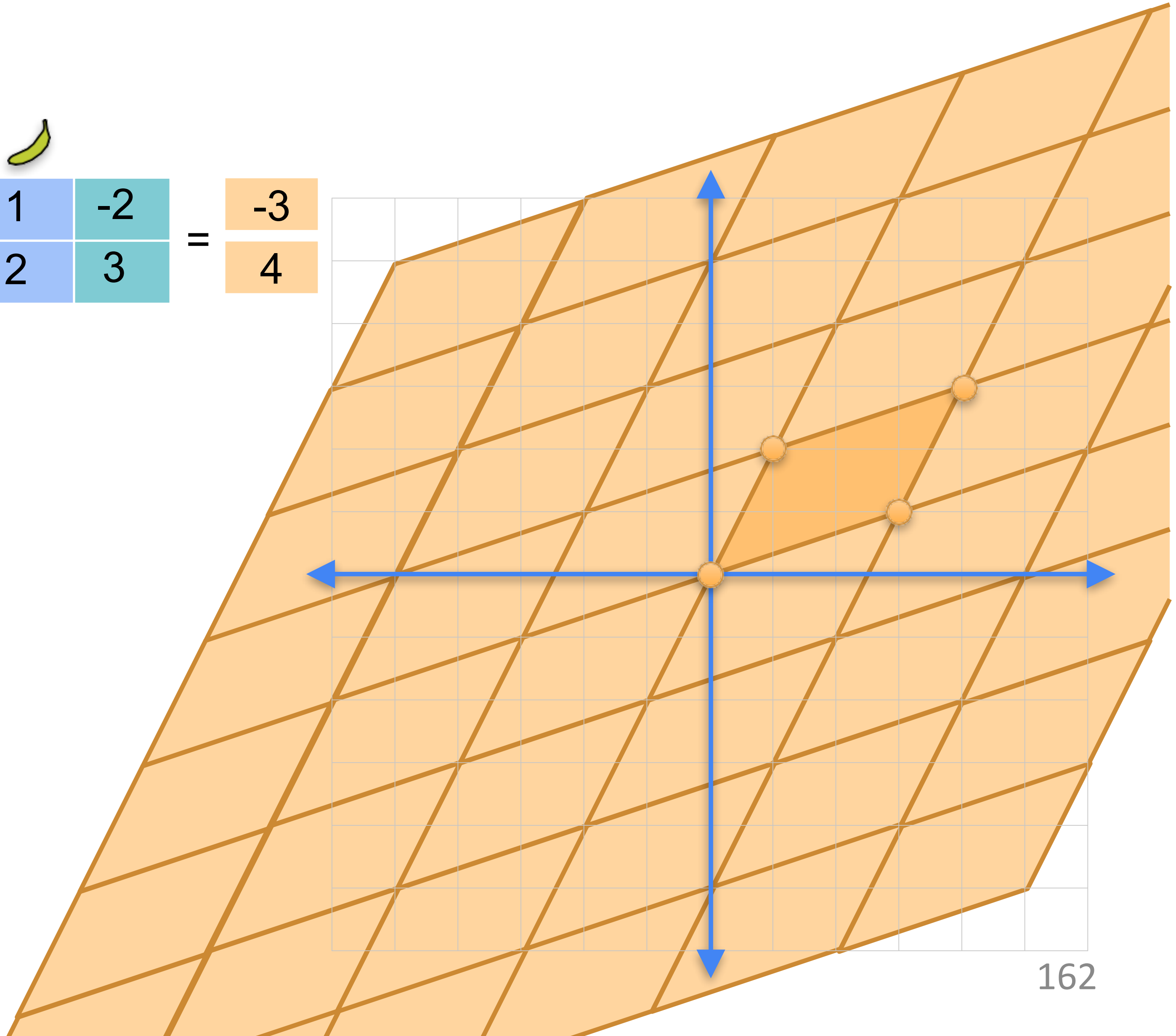
$$\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}-2\\ 3\end{bmatrix}=\begin{bmatrix}-3\\ 4\end{bmatrix}$$



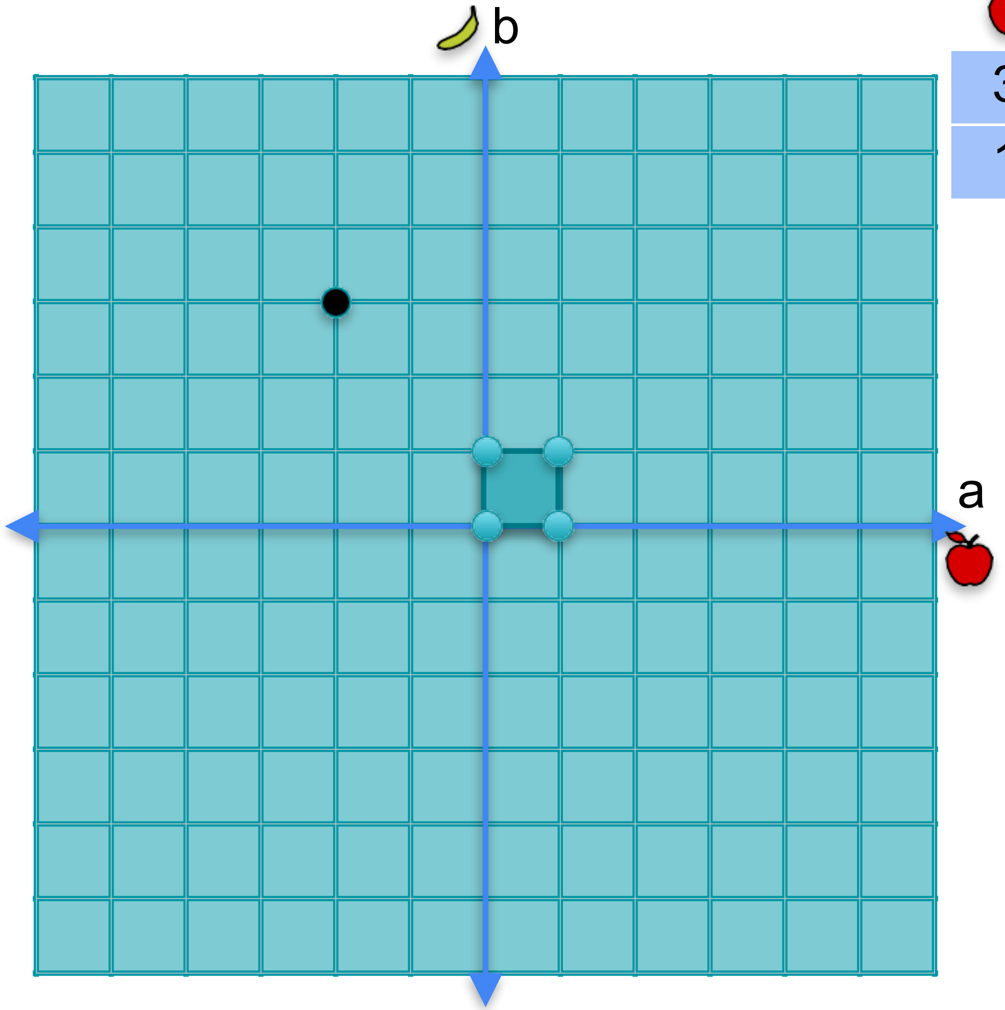
矩阵是线性变换



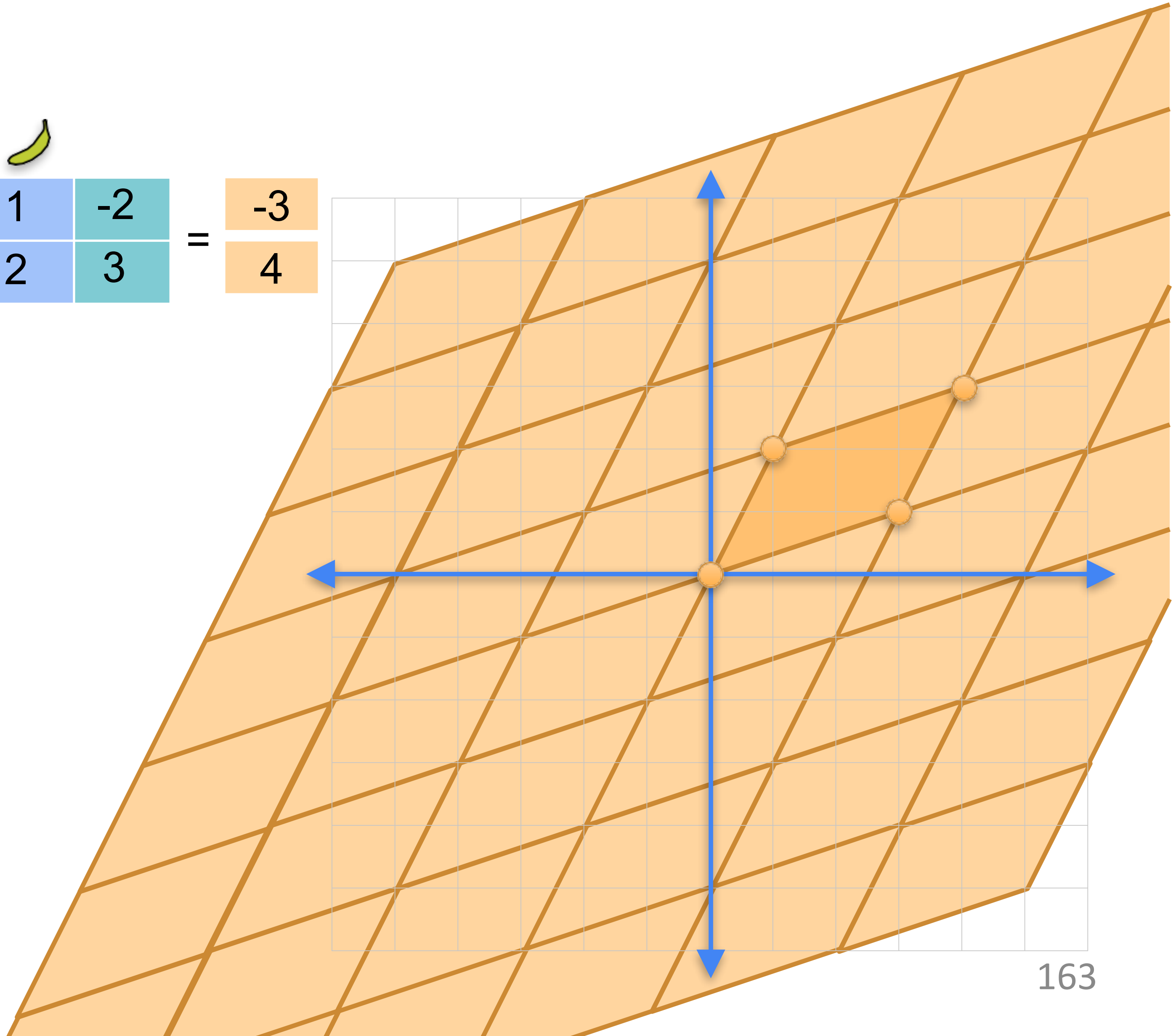
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



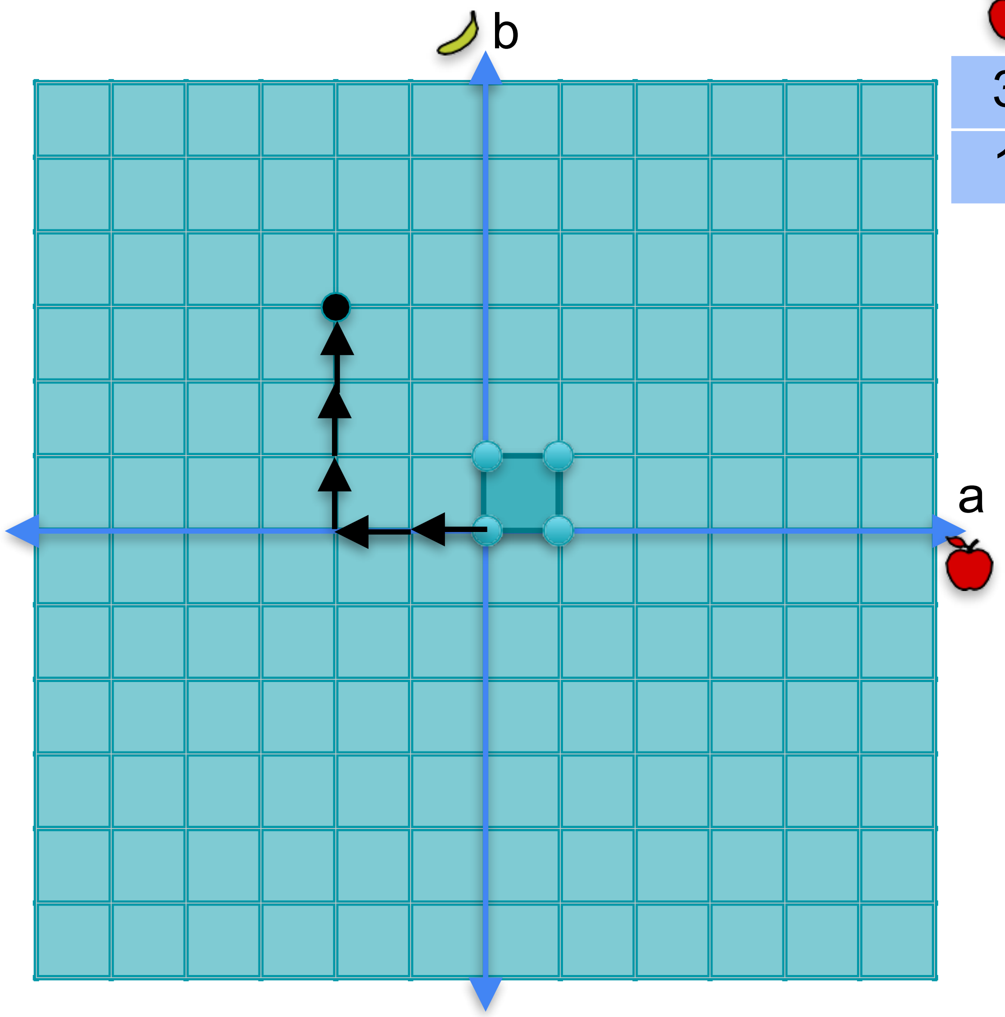
矩阵是线性变换



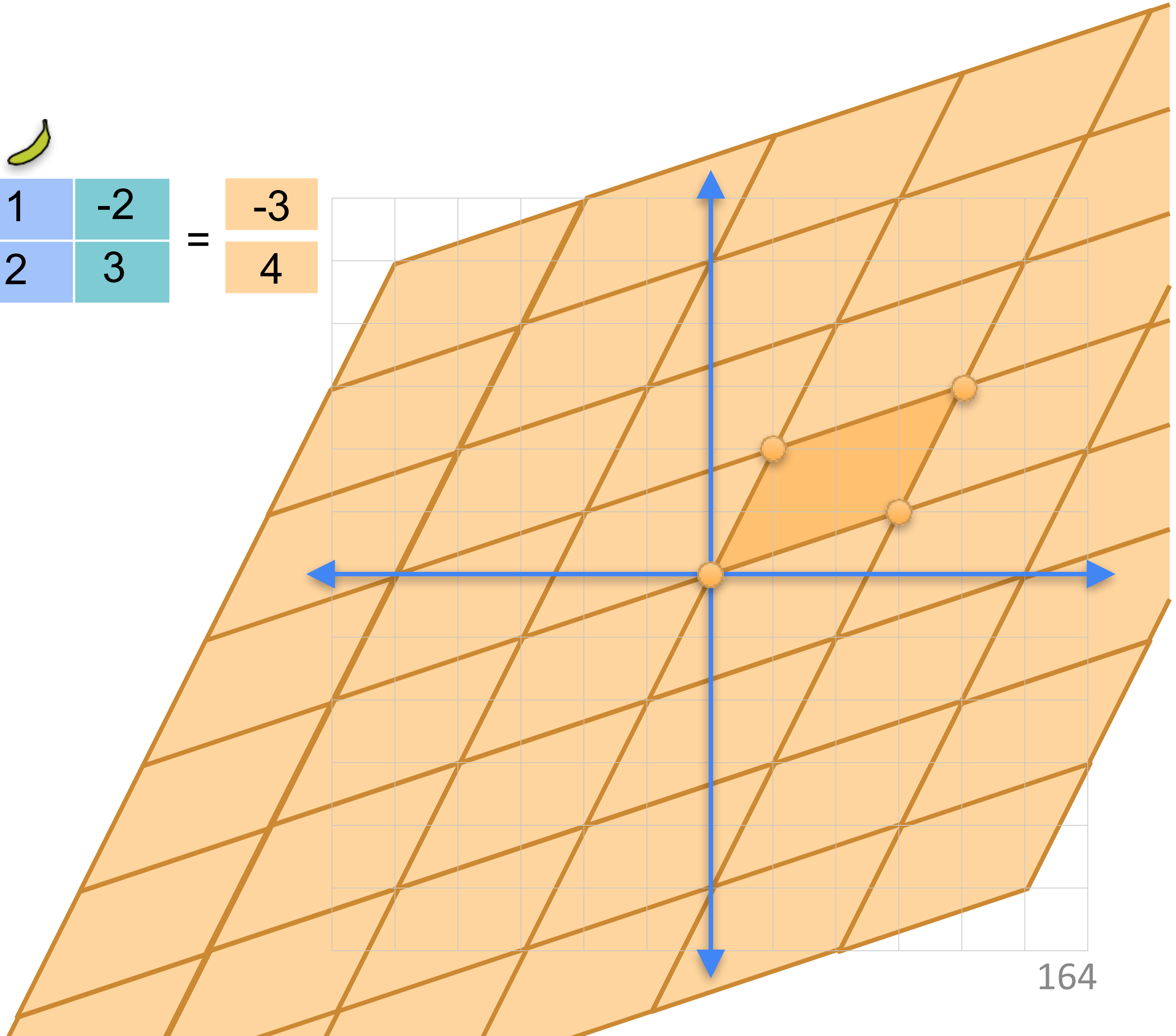
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



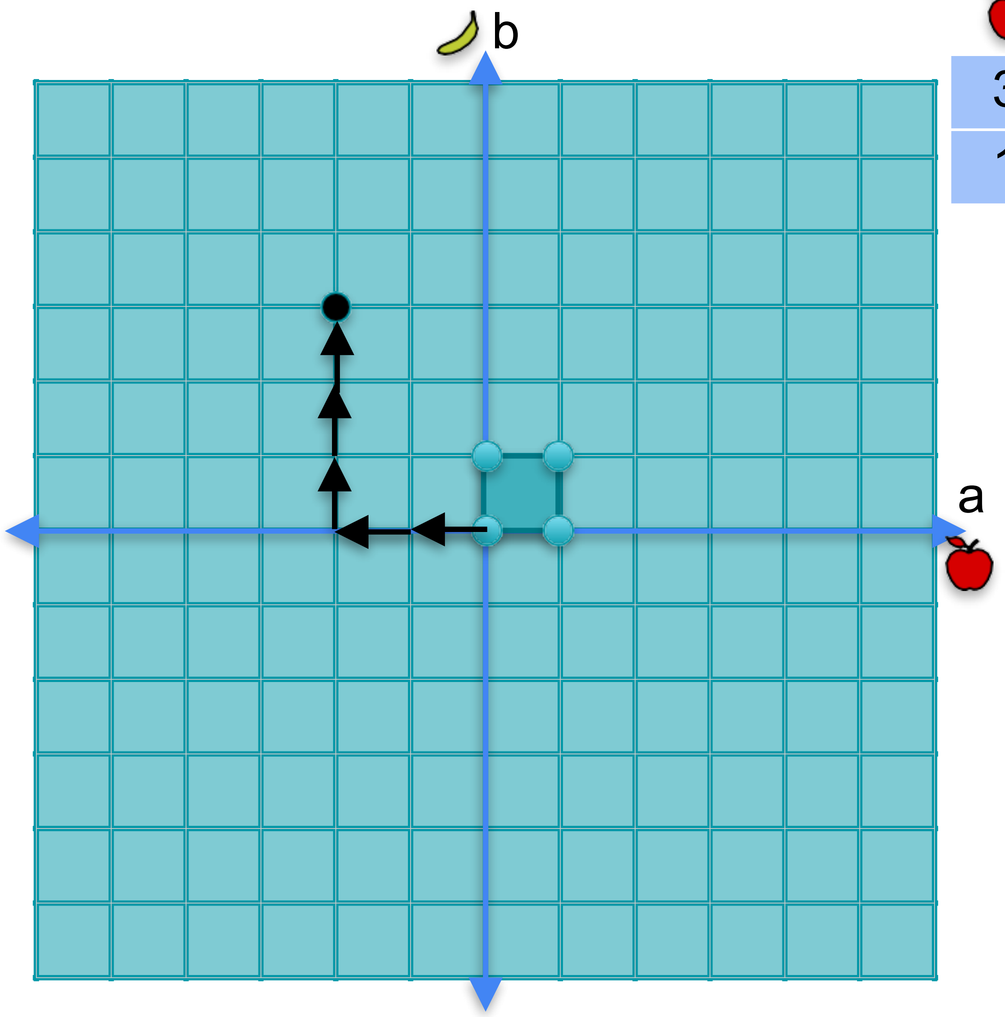
矩阵是线性变换



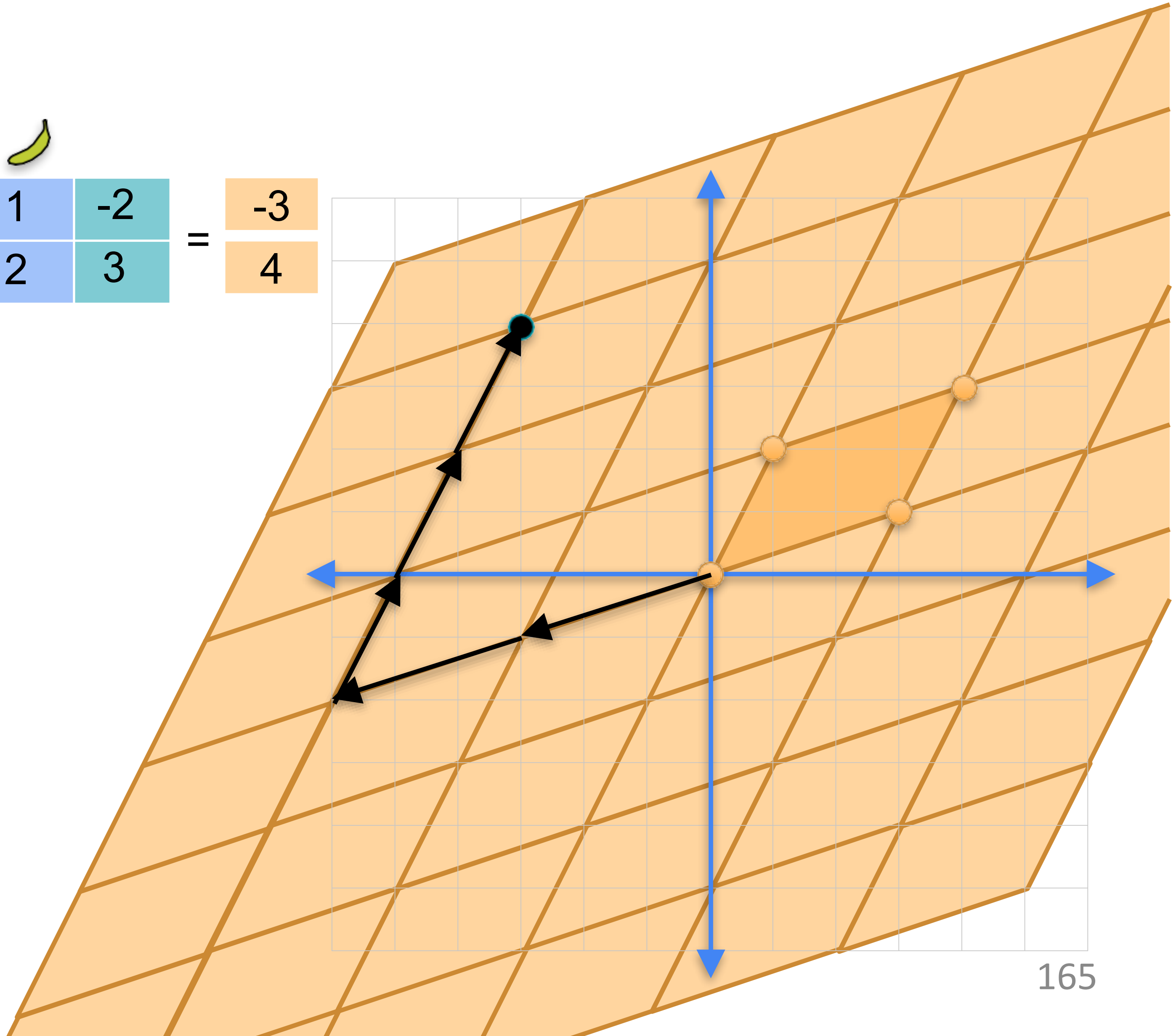
3	1	-2	=	-3
1	2	3		4



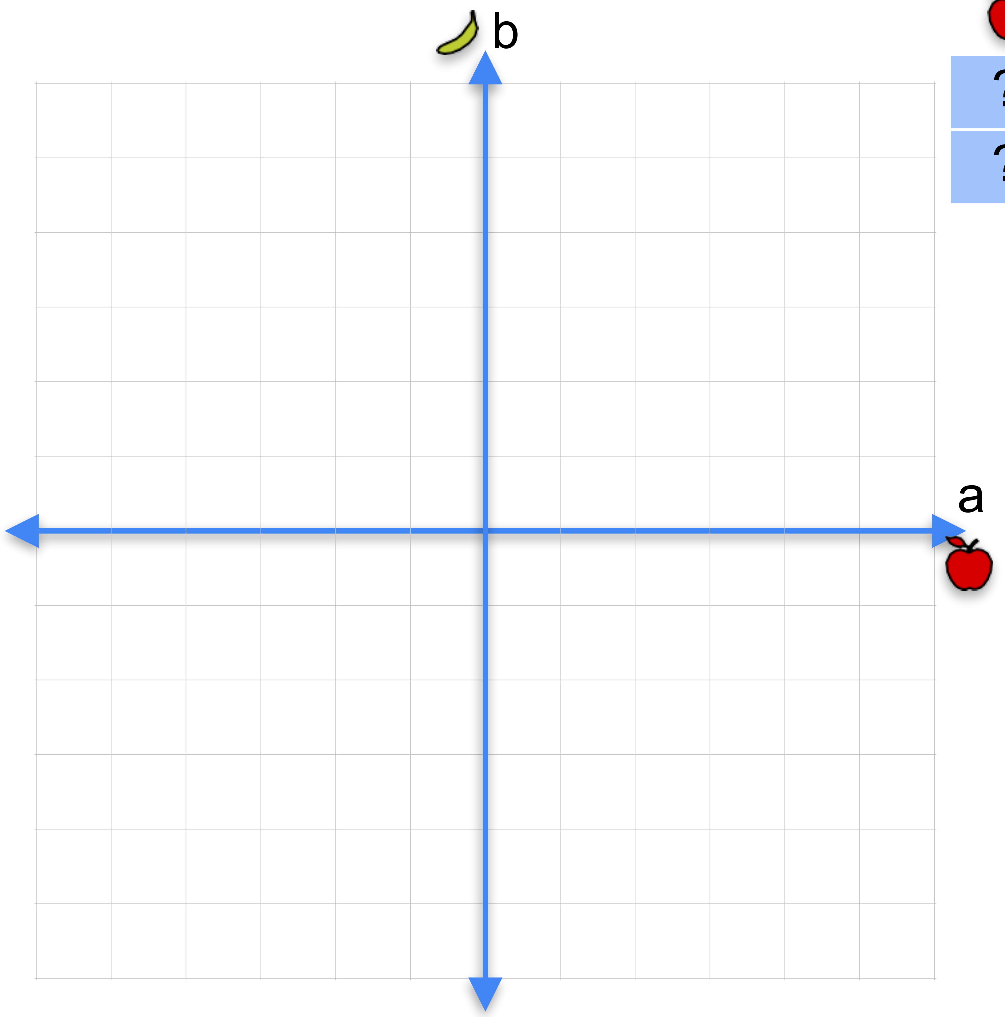
矩阵是线性变换



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



线性变换是矩阵



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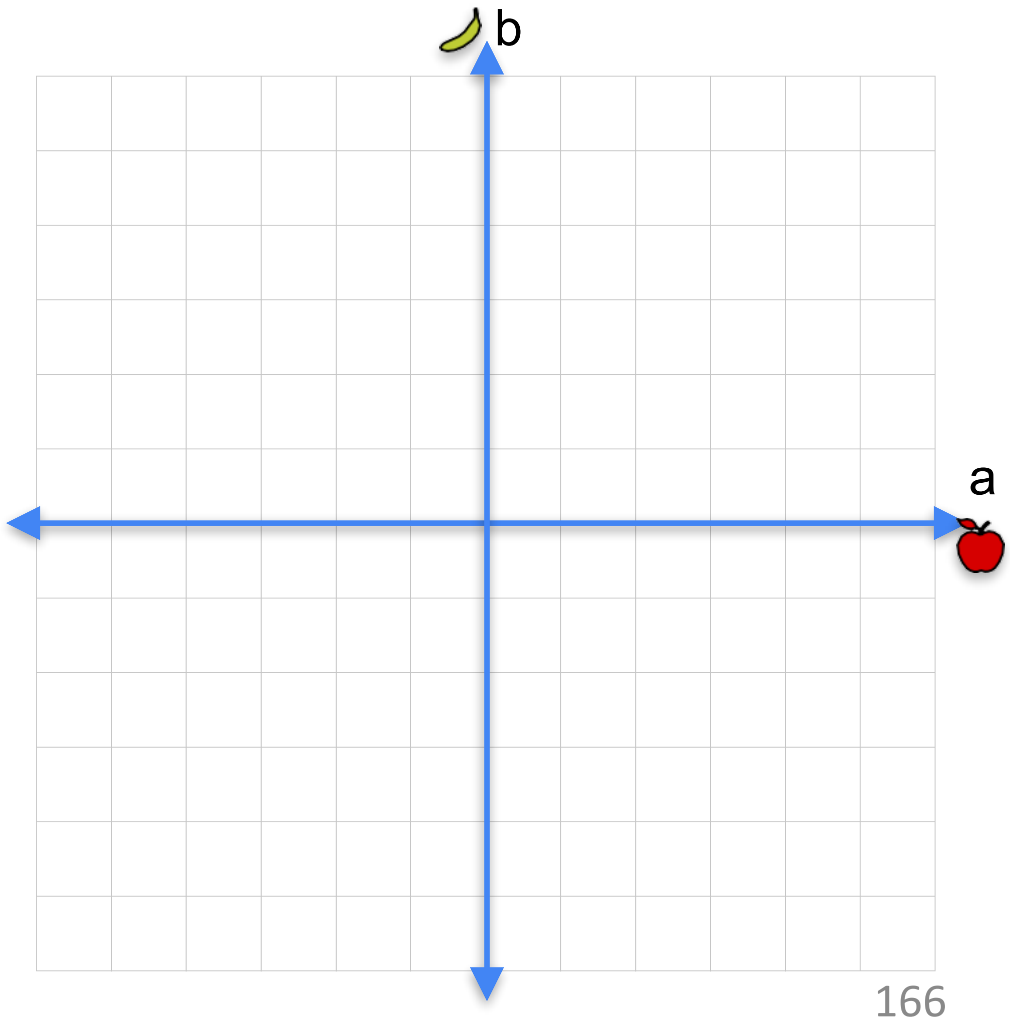
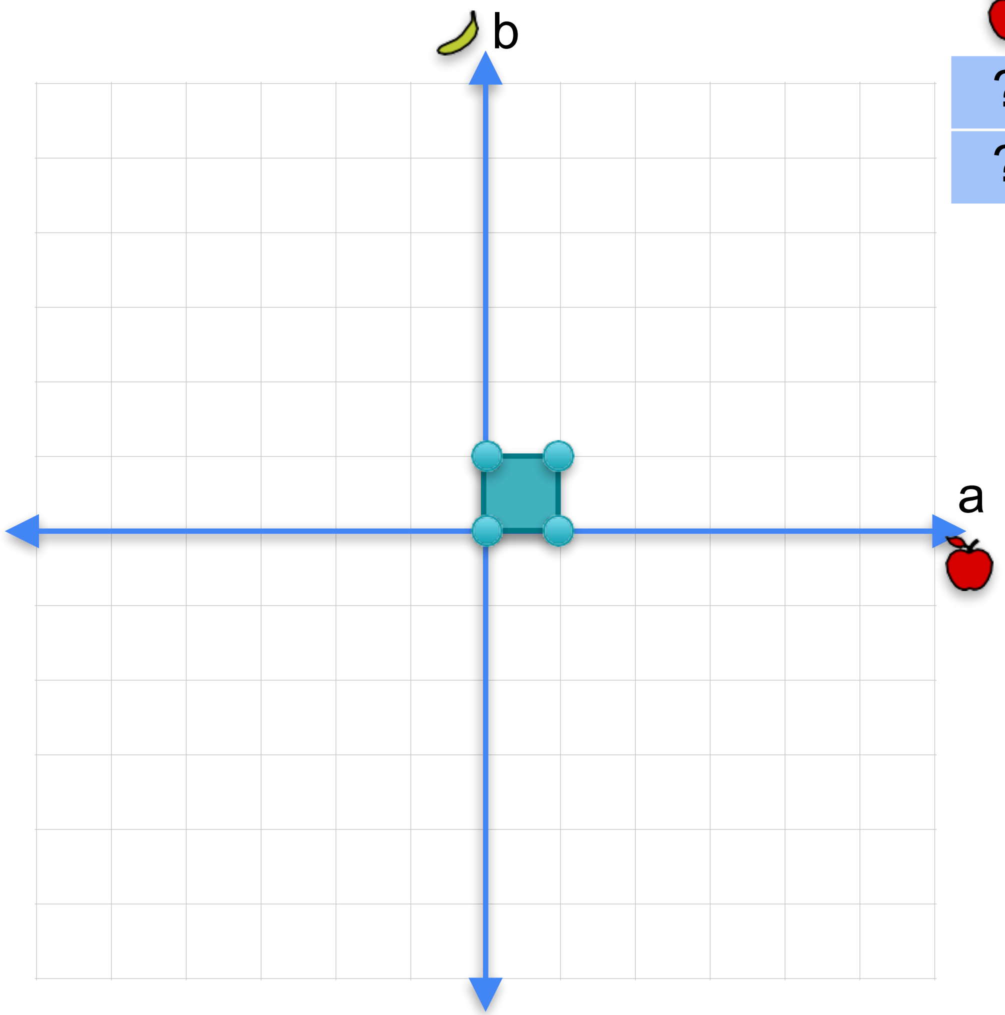


线性变换是矩阵



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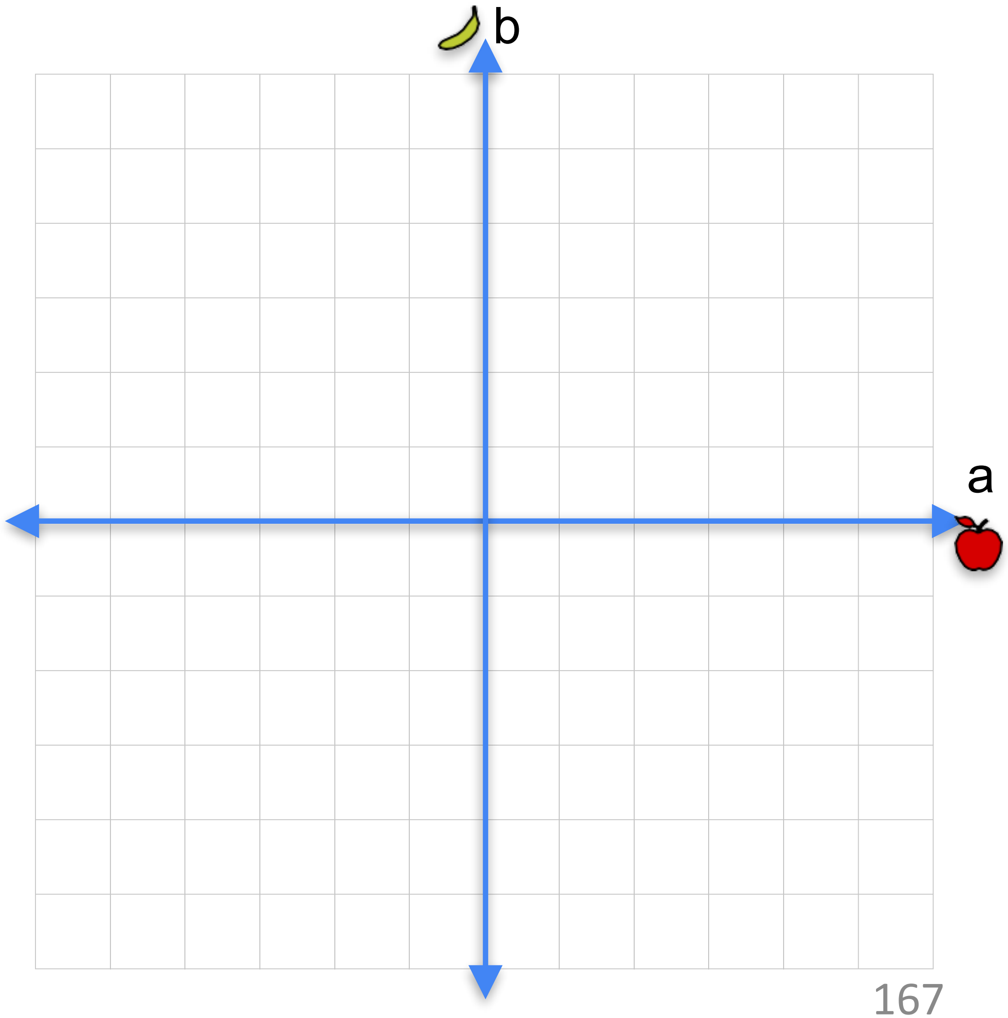
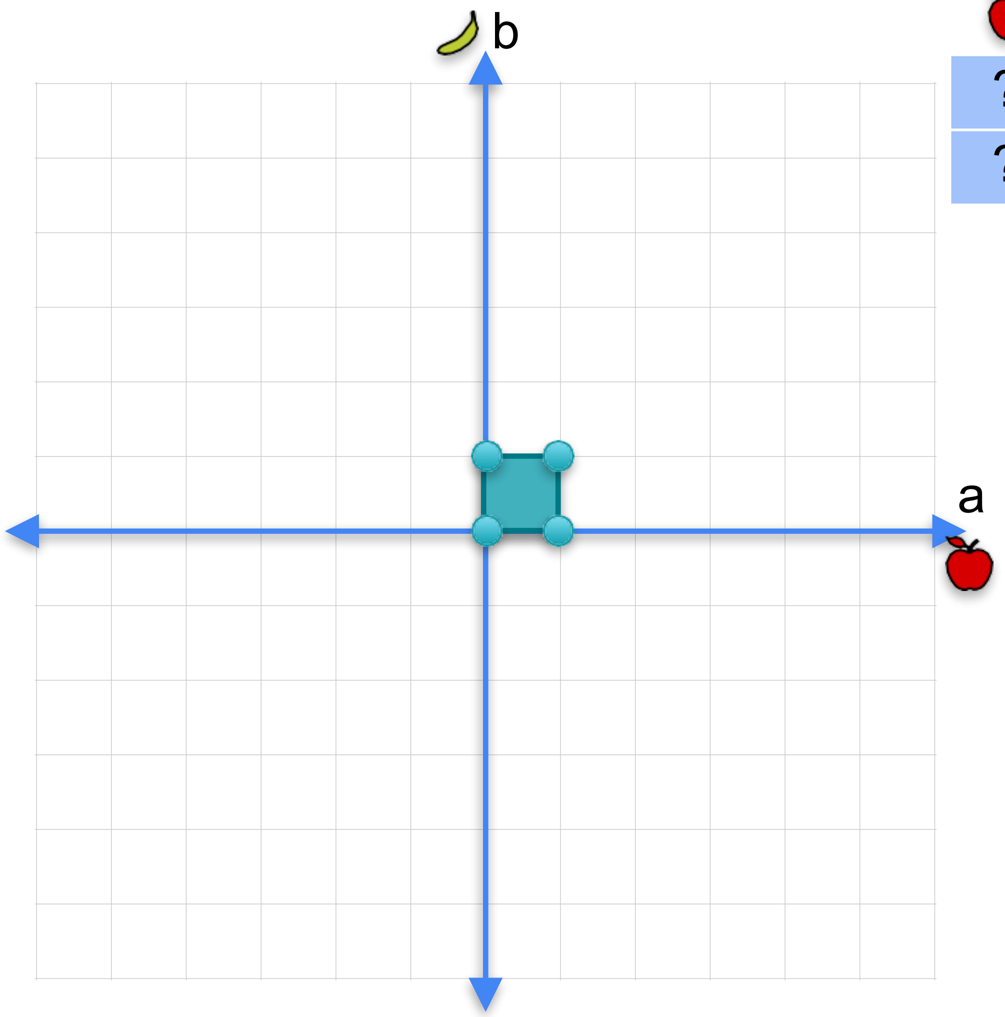


线性变换是矩阵



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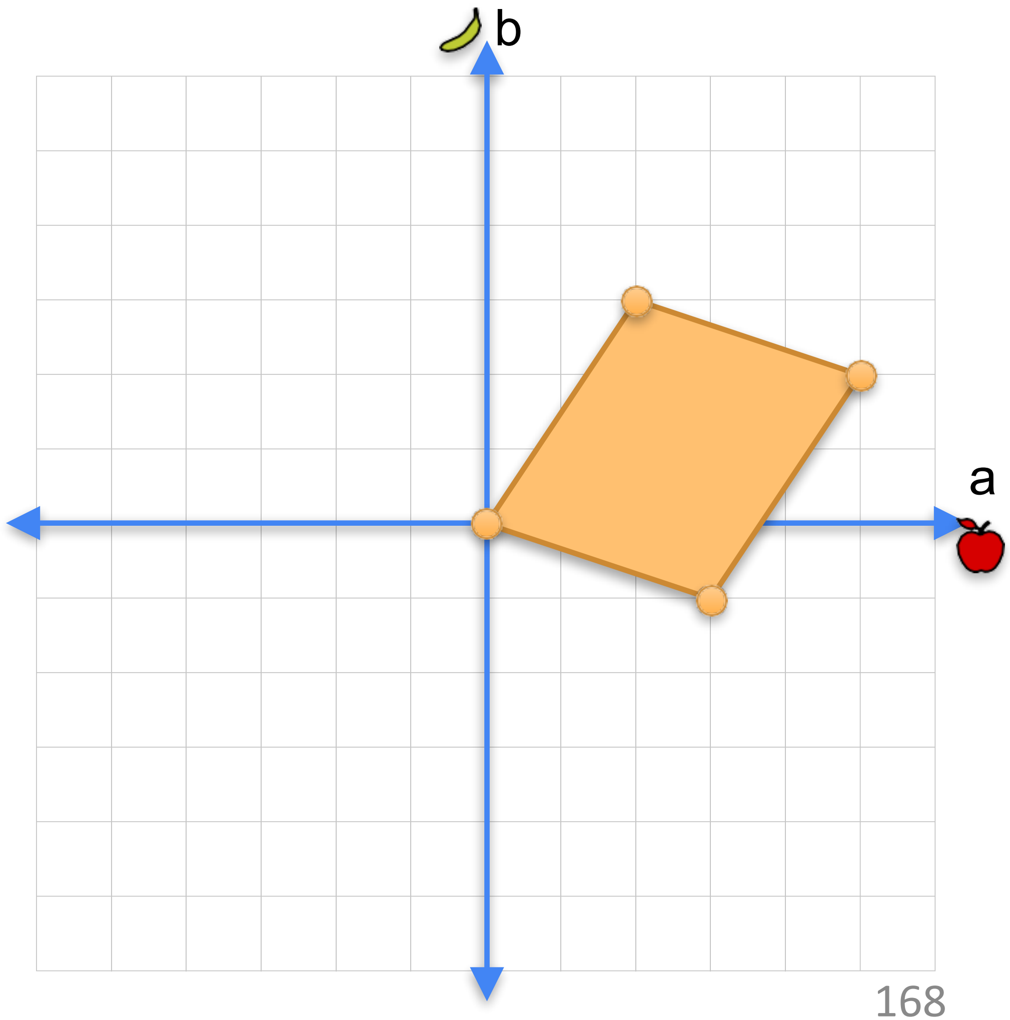
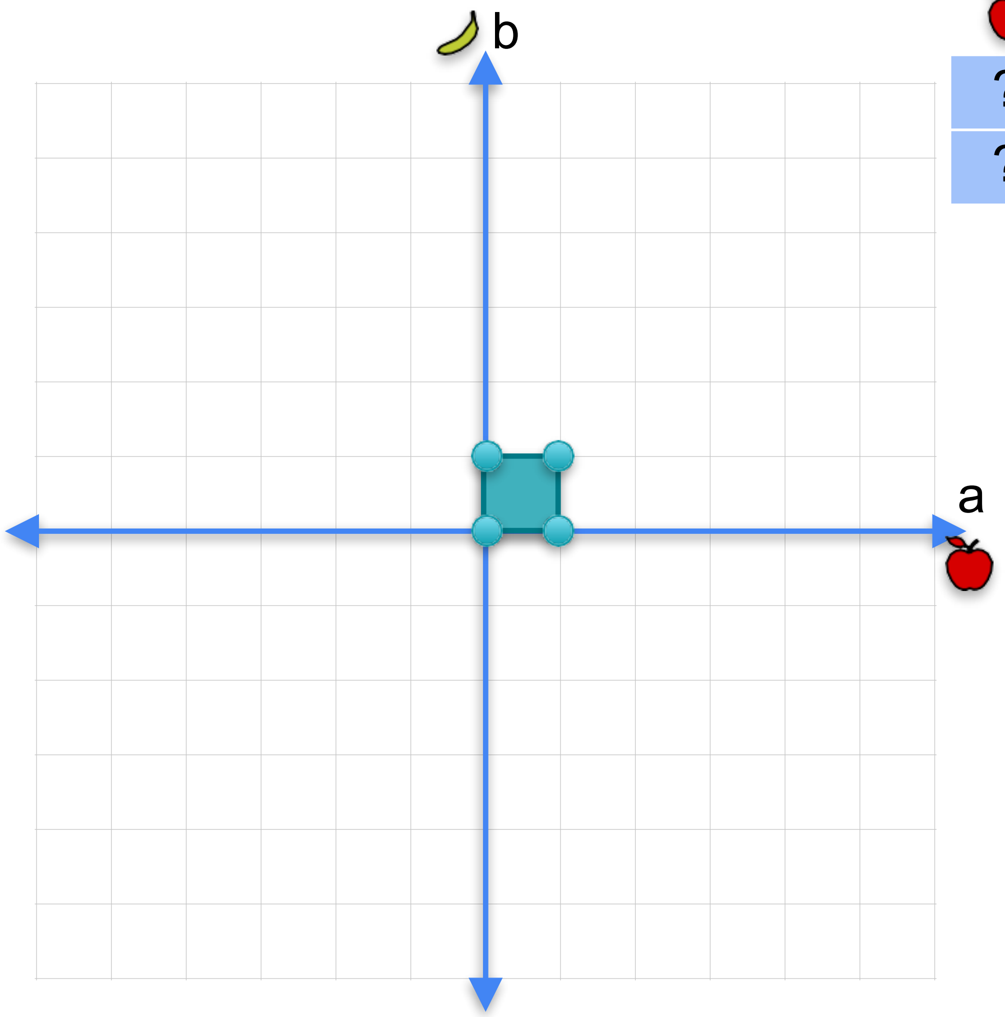


线性变换是矩阵



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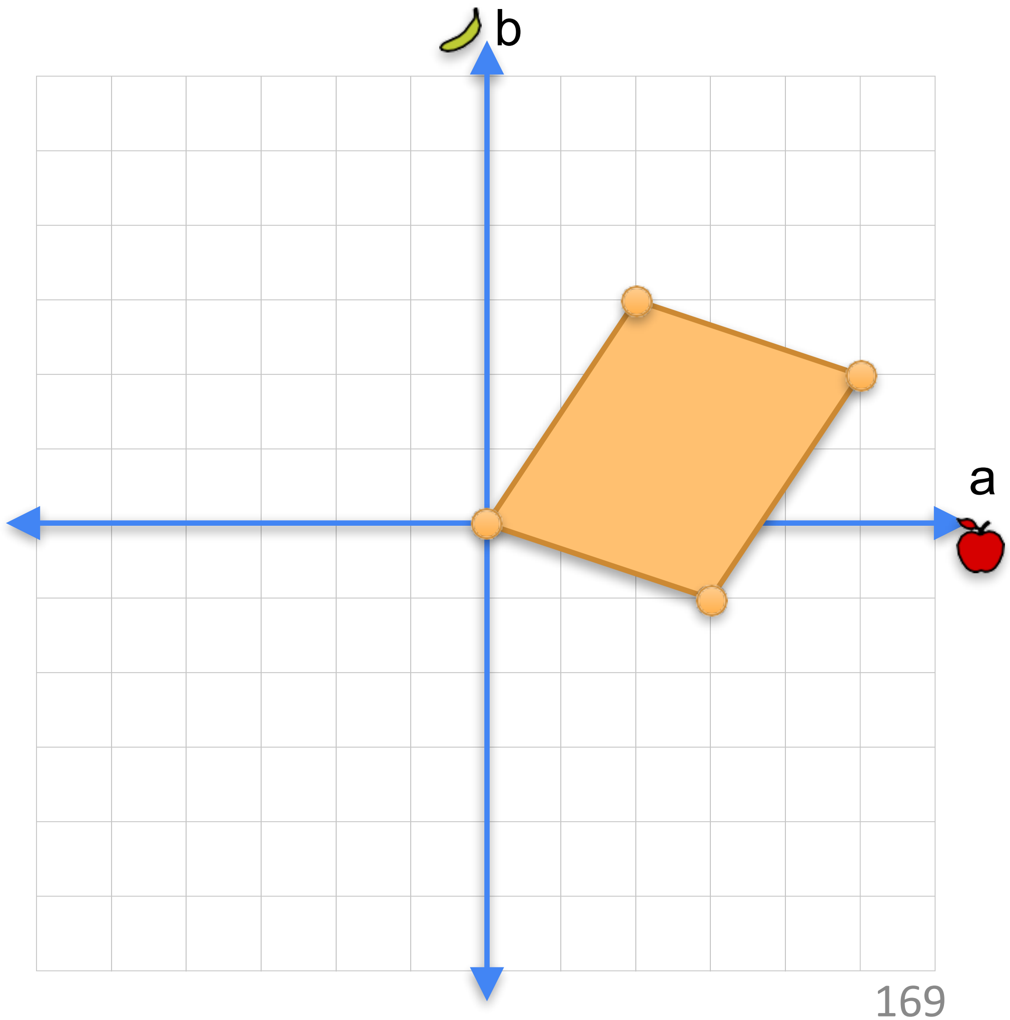
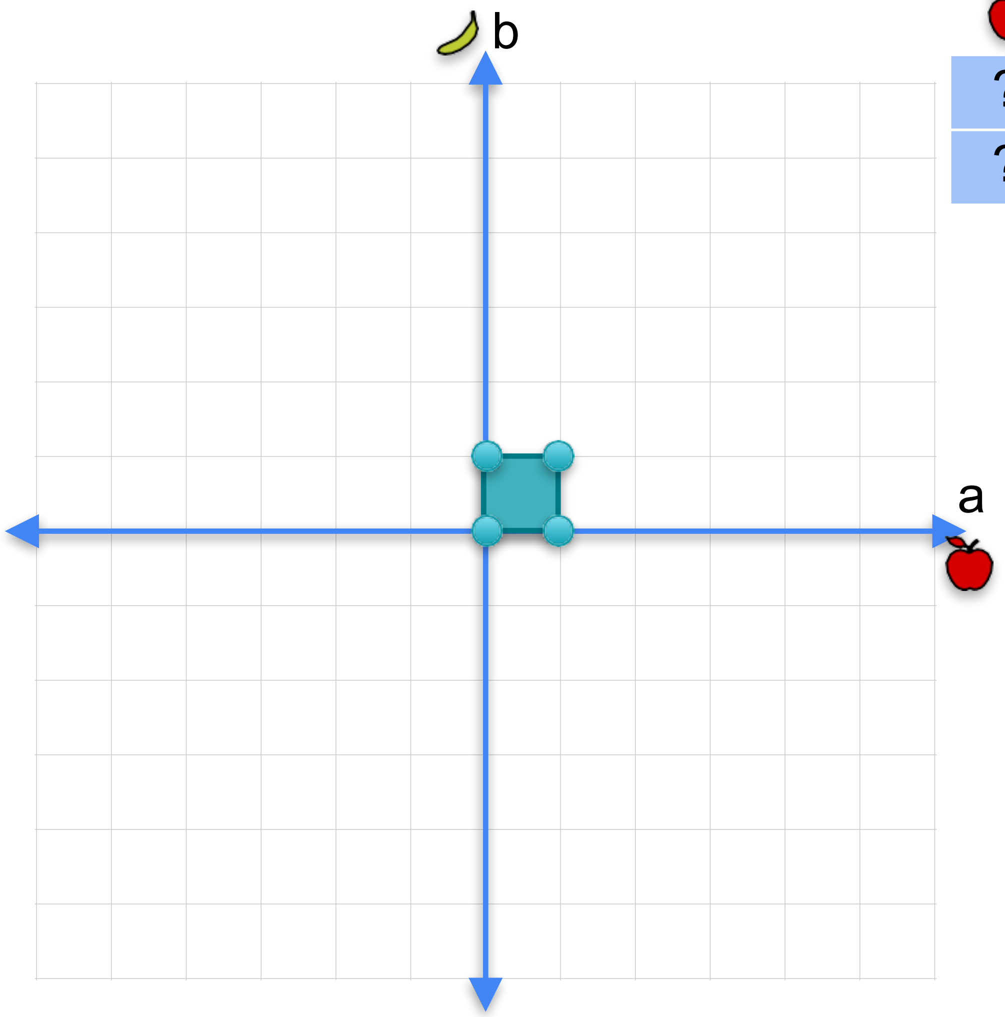
(0,0) → (0,0)



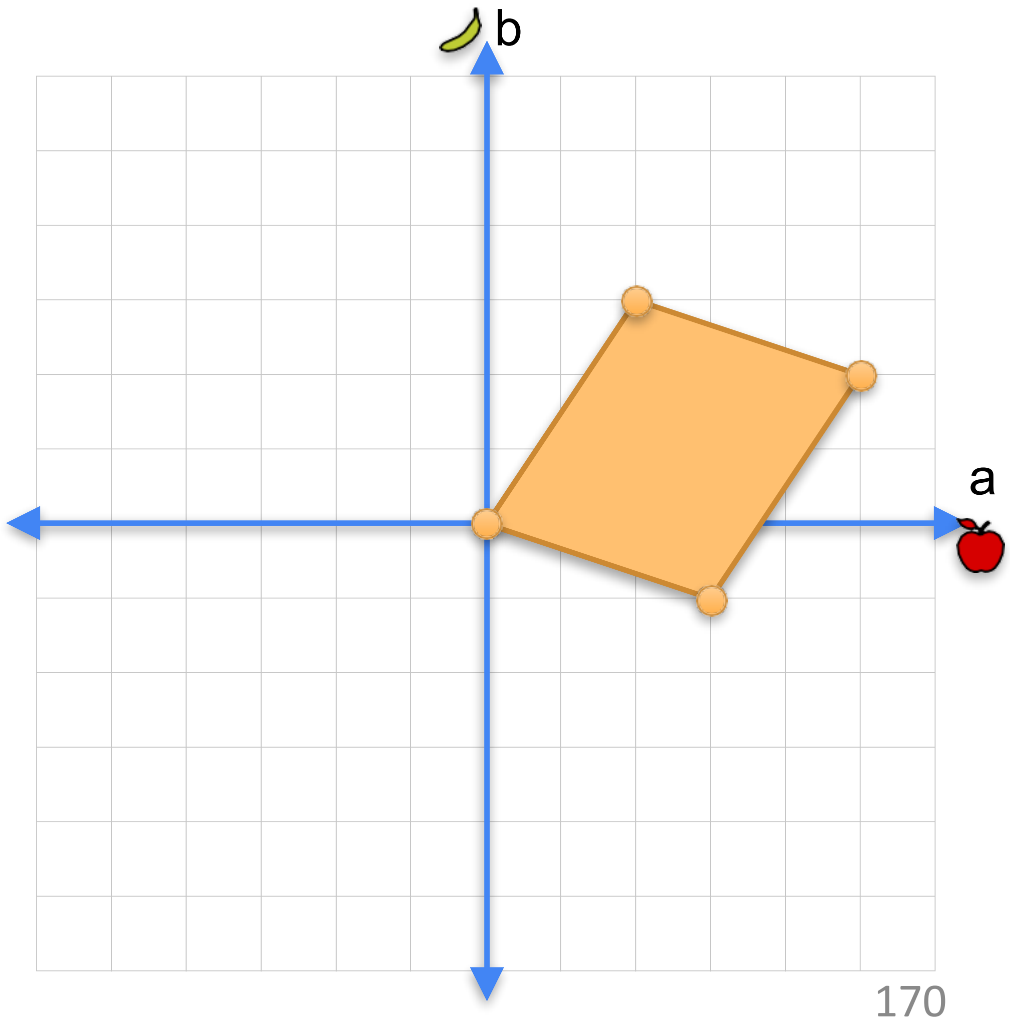
线性变换是矩阵



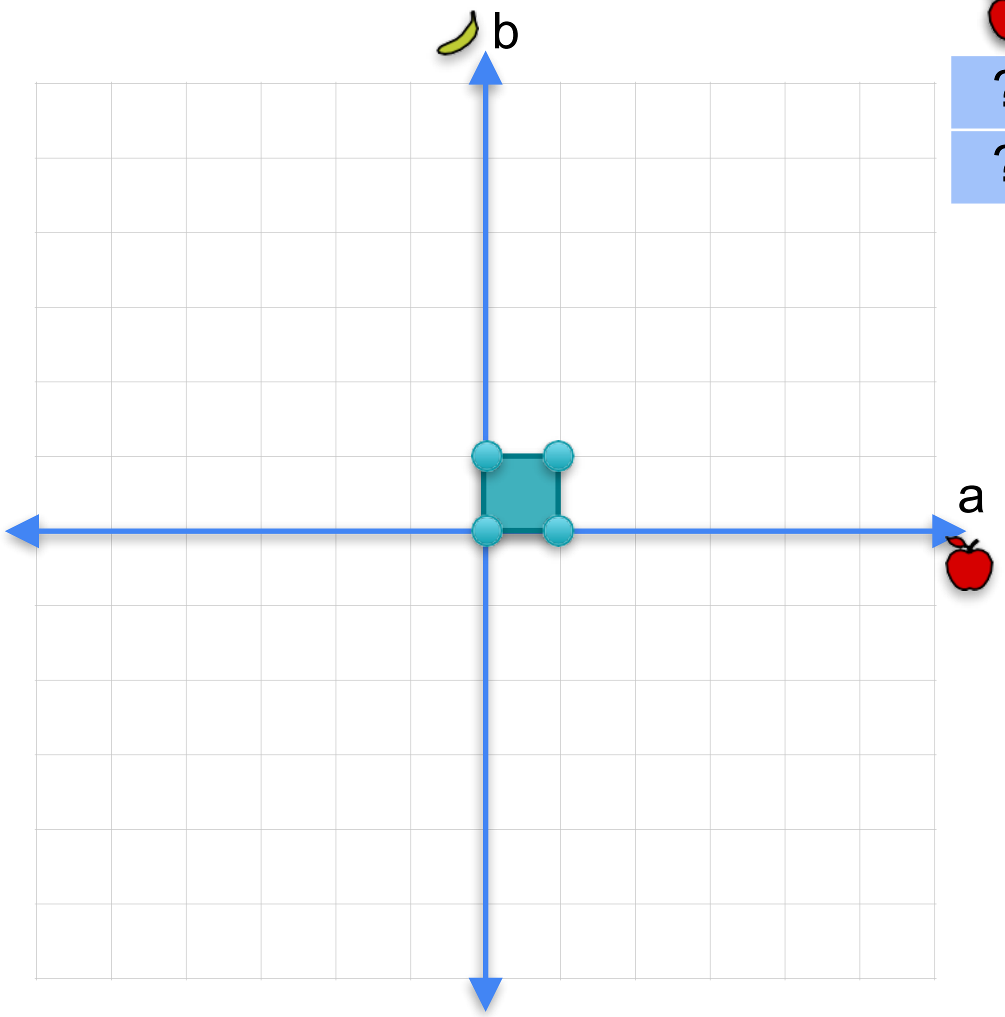
?	?
?	?

(0,0) → (0,0)

(1,0) → (3,-1)



线性变换是矩阵

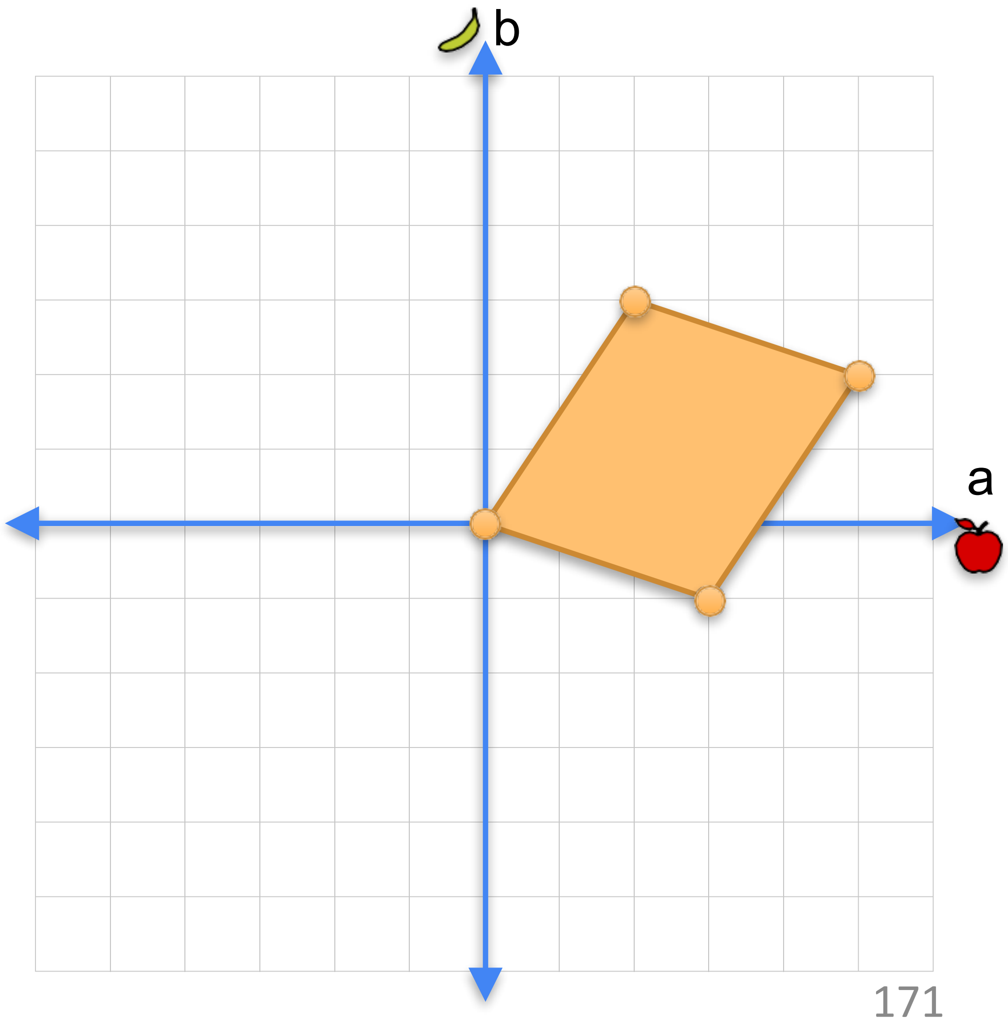


?	?
?	?

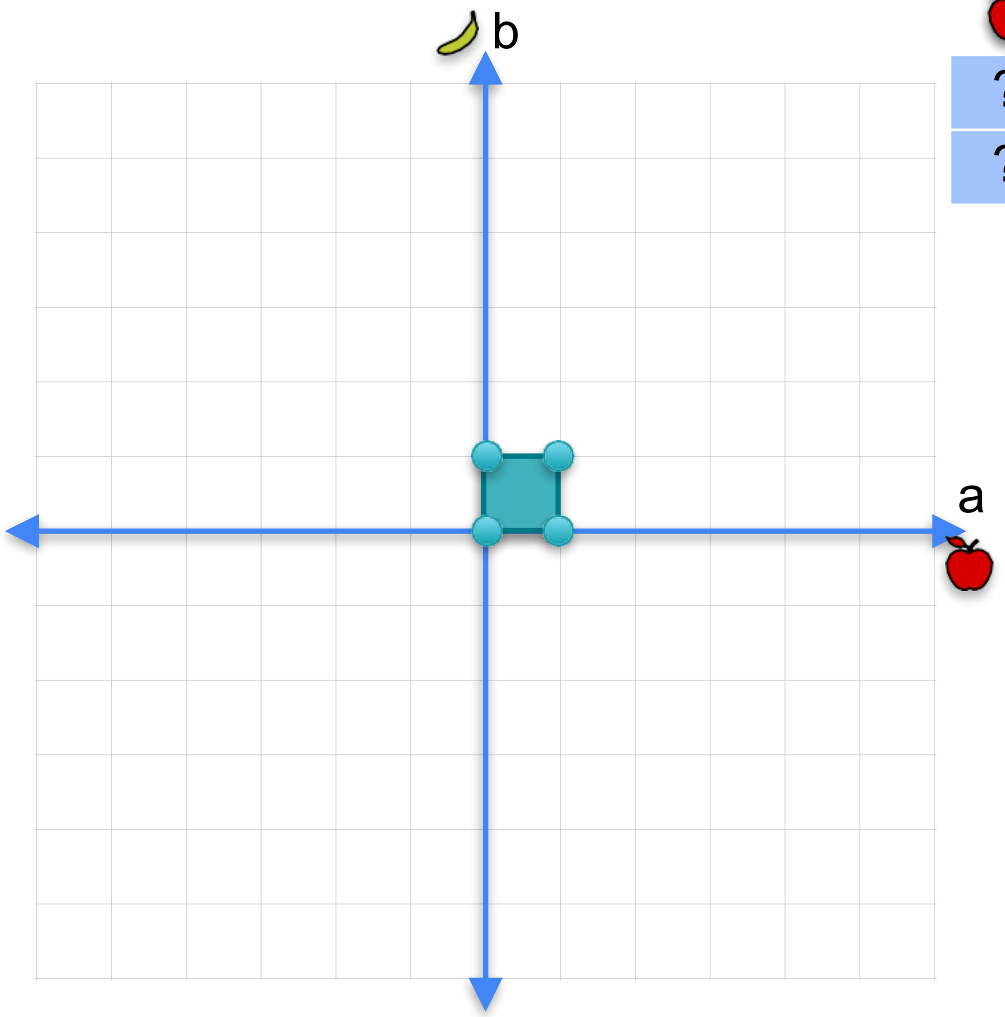
(0,0) → (0,0)

(1,0) → (3,-1)

(0,1) → (2,3)



线性变换是矩阵



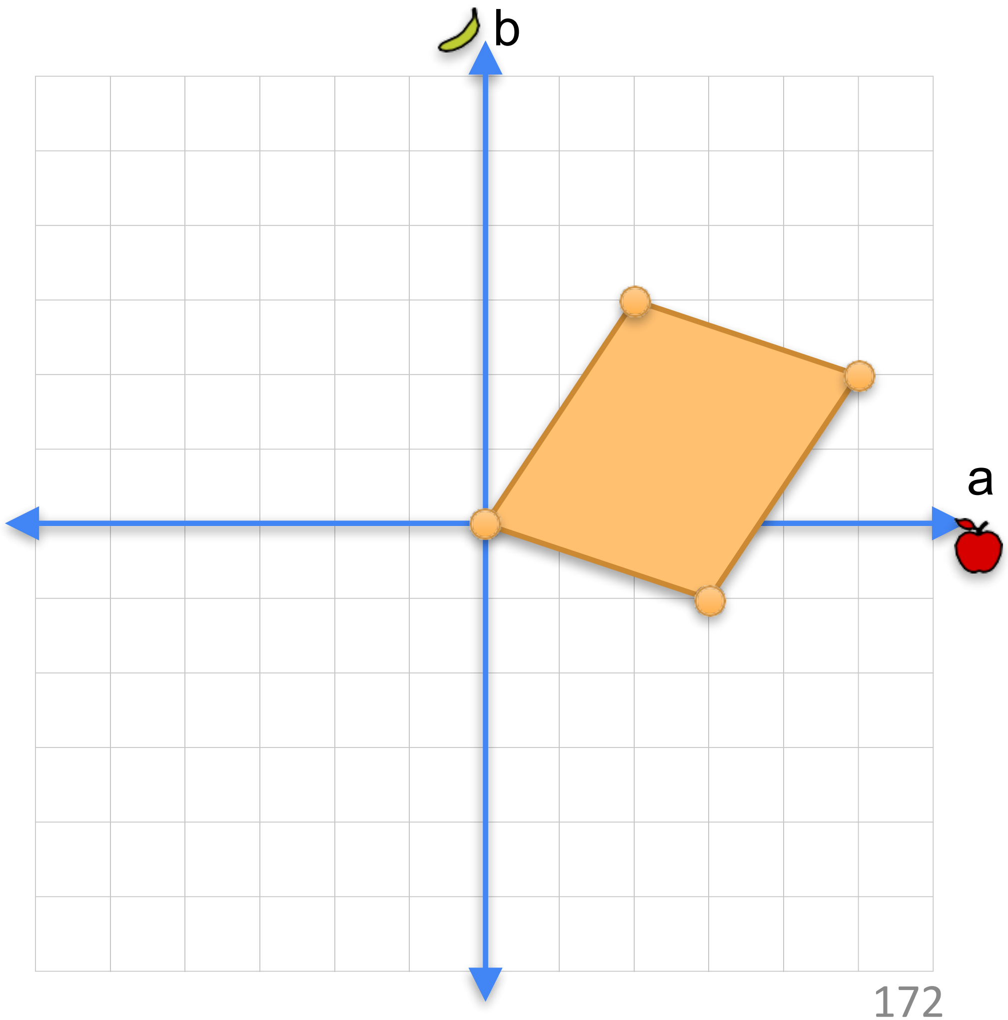
?	?
?	?

(0,0) → (0,0)

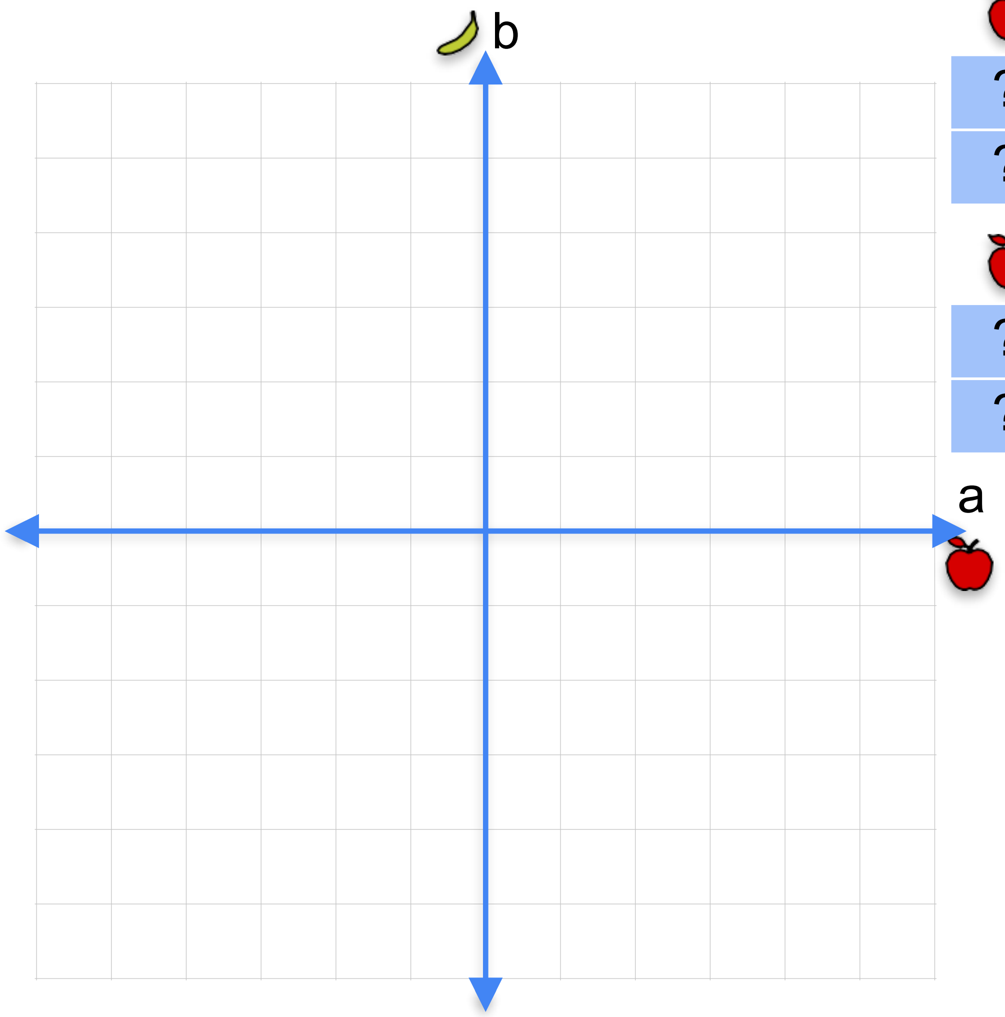
(1,0) → (3,-1)

(0,1) → (2,3)

(1,1) → (5,2)



线性变换是矩阵

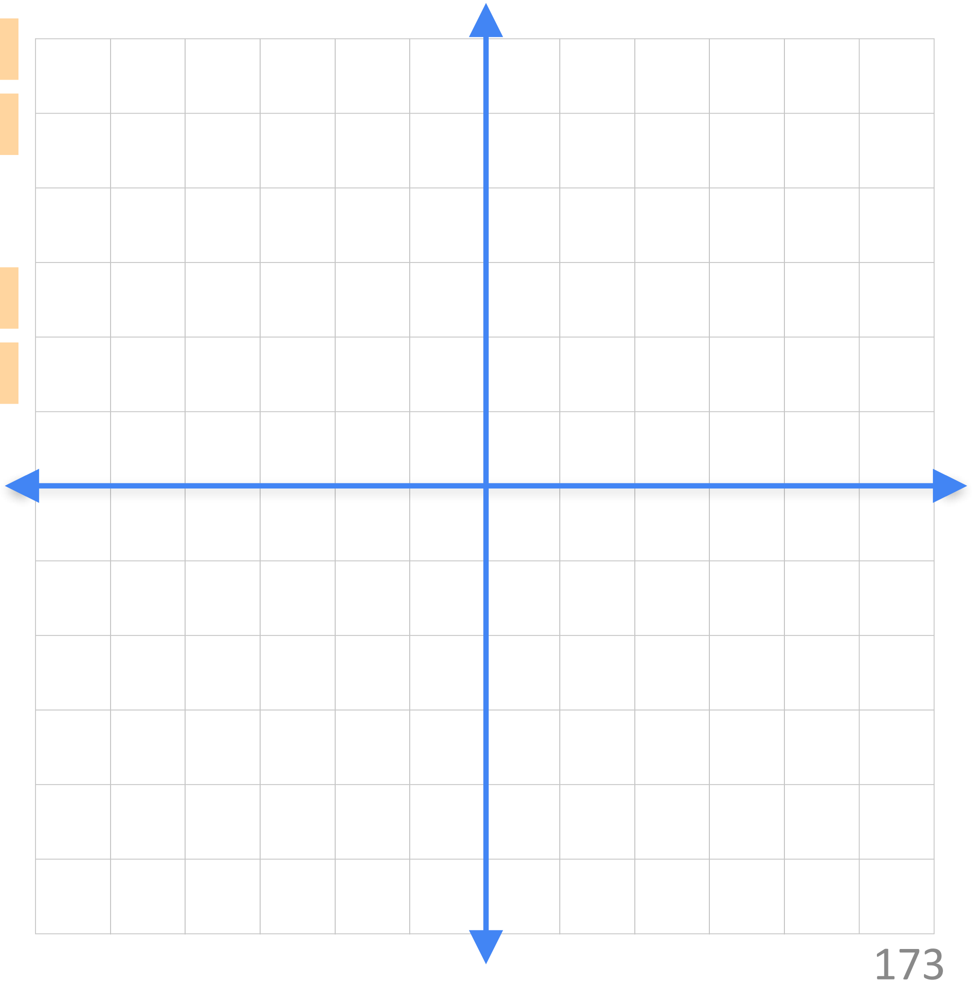


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

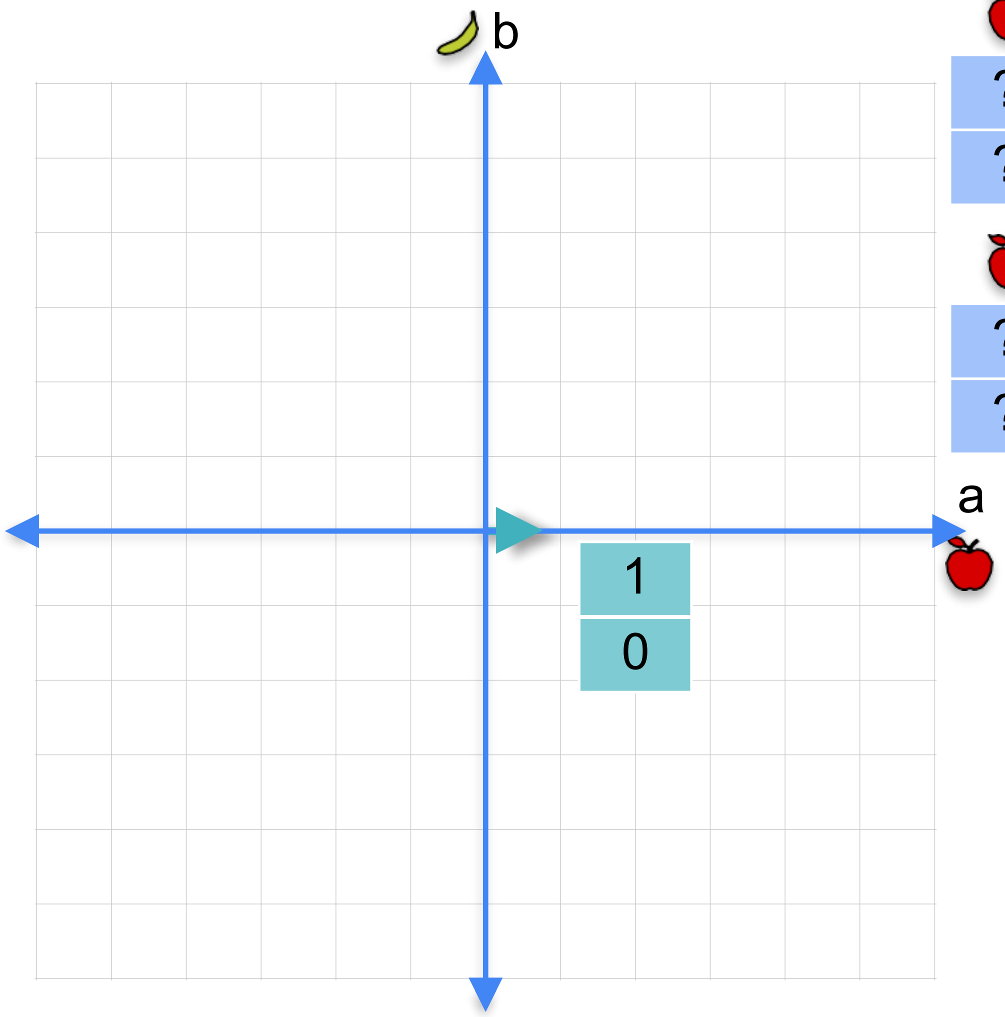
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

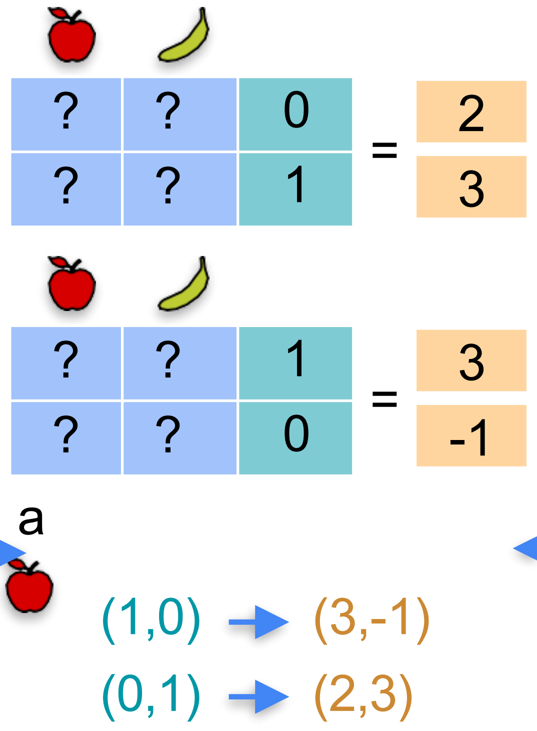
(1,0) → (3,-1)

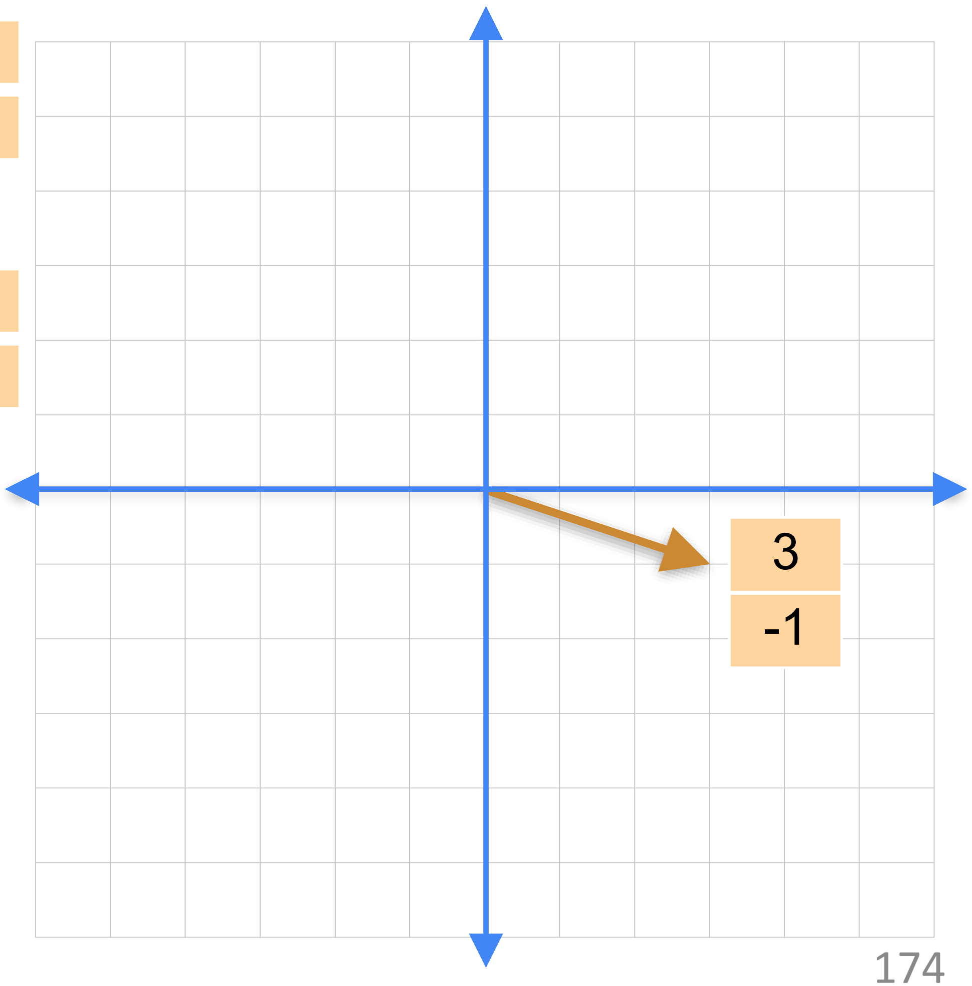
(0,1) → (2,3)



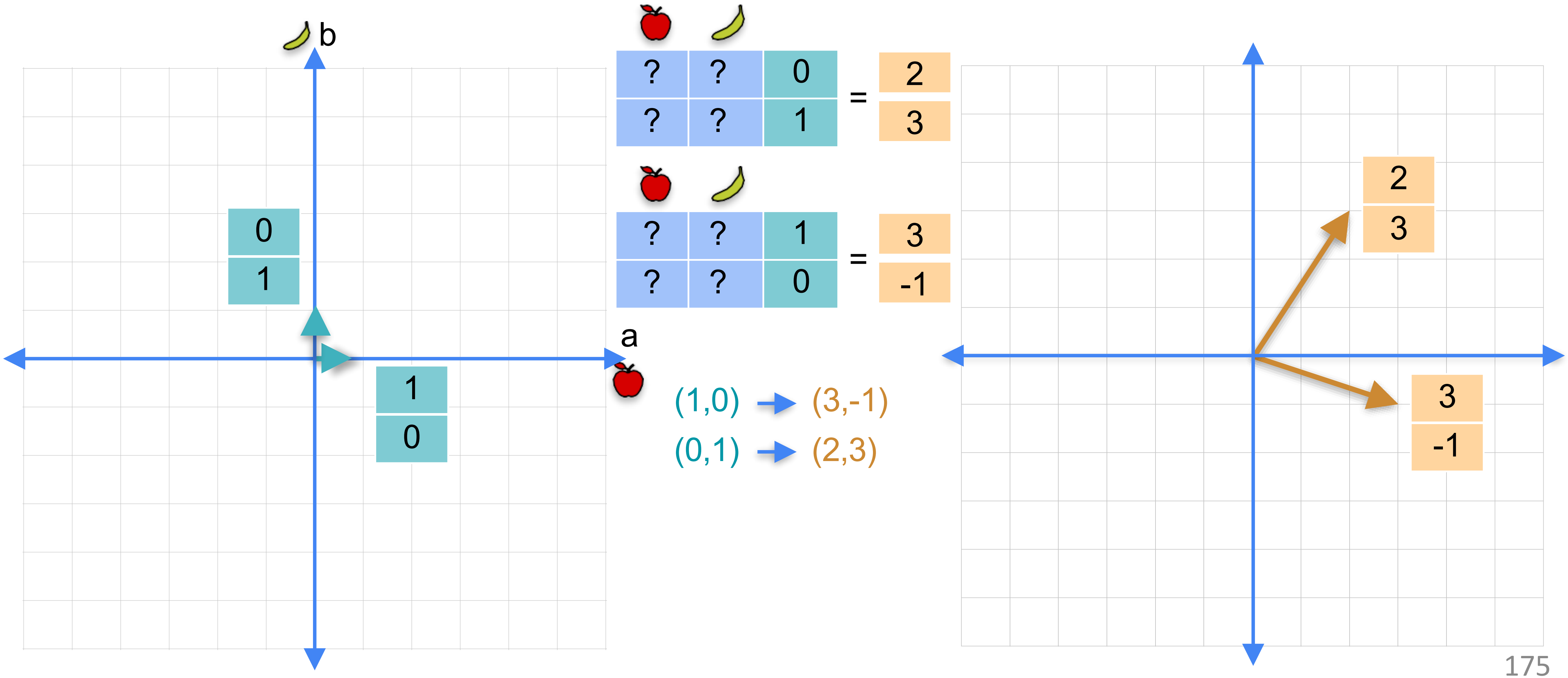
线性变换是矩阵



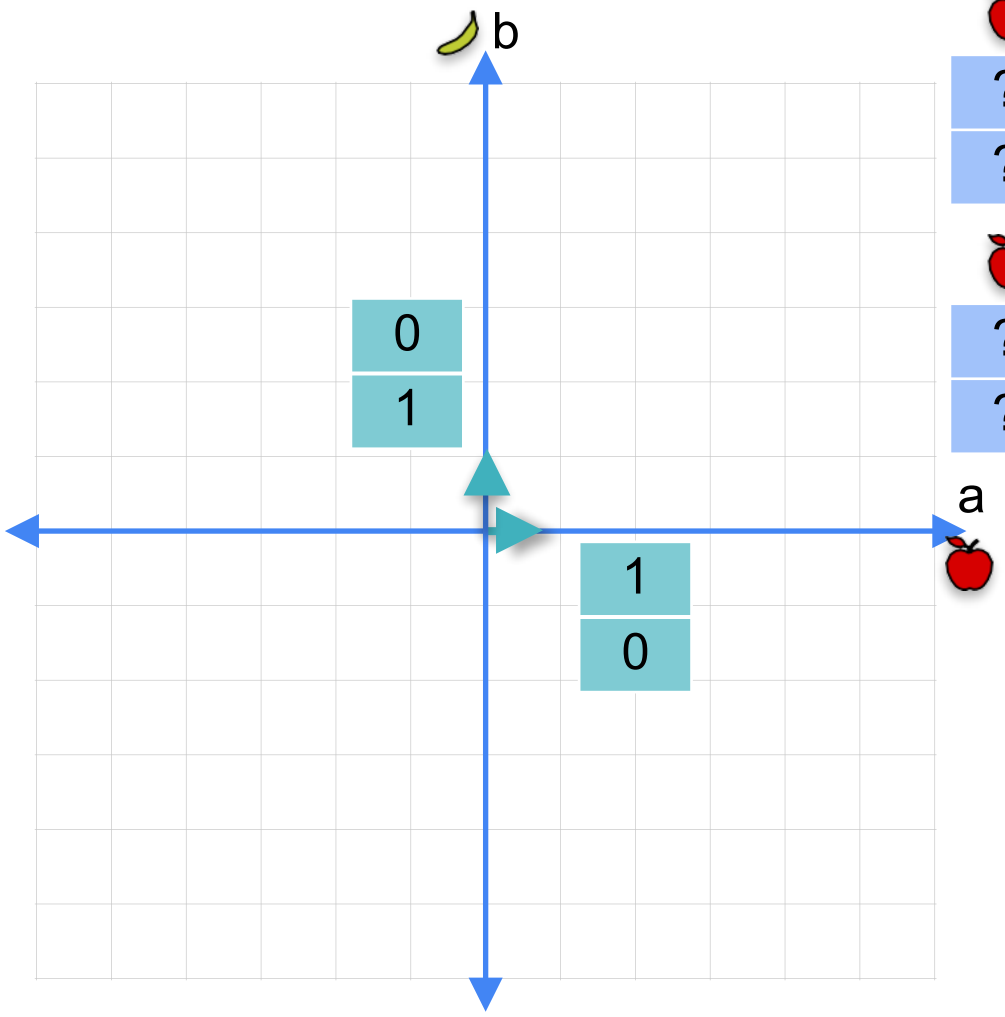




线性变换是矩阵



线性变换是矩阵

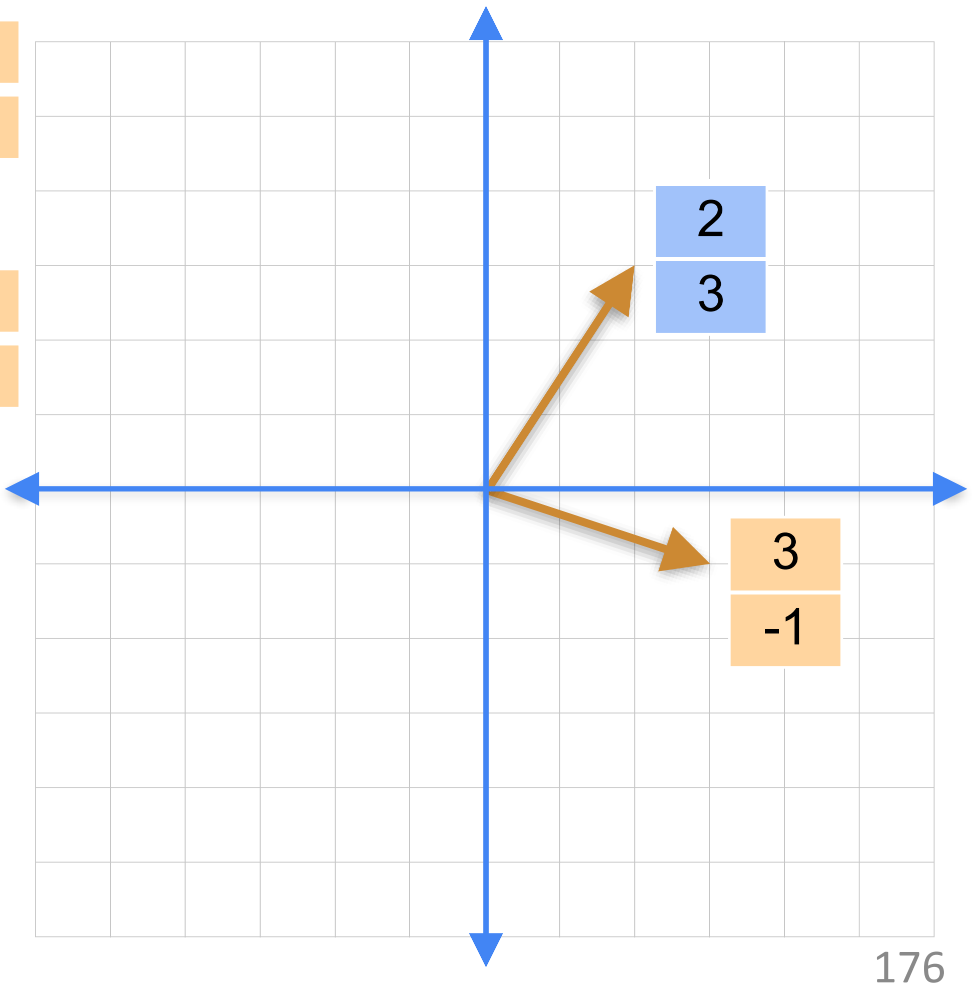


$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

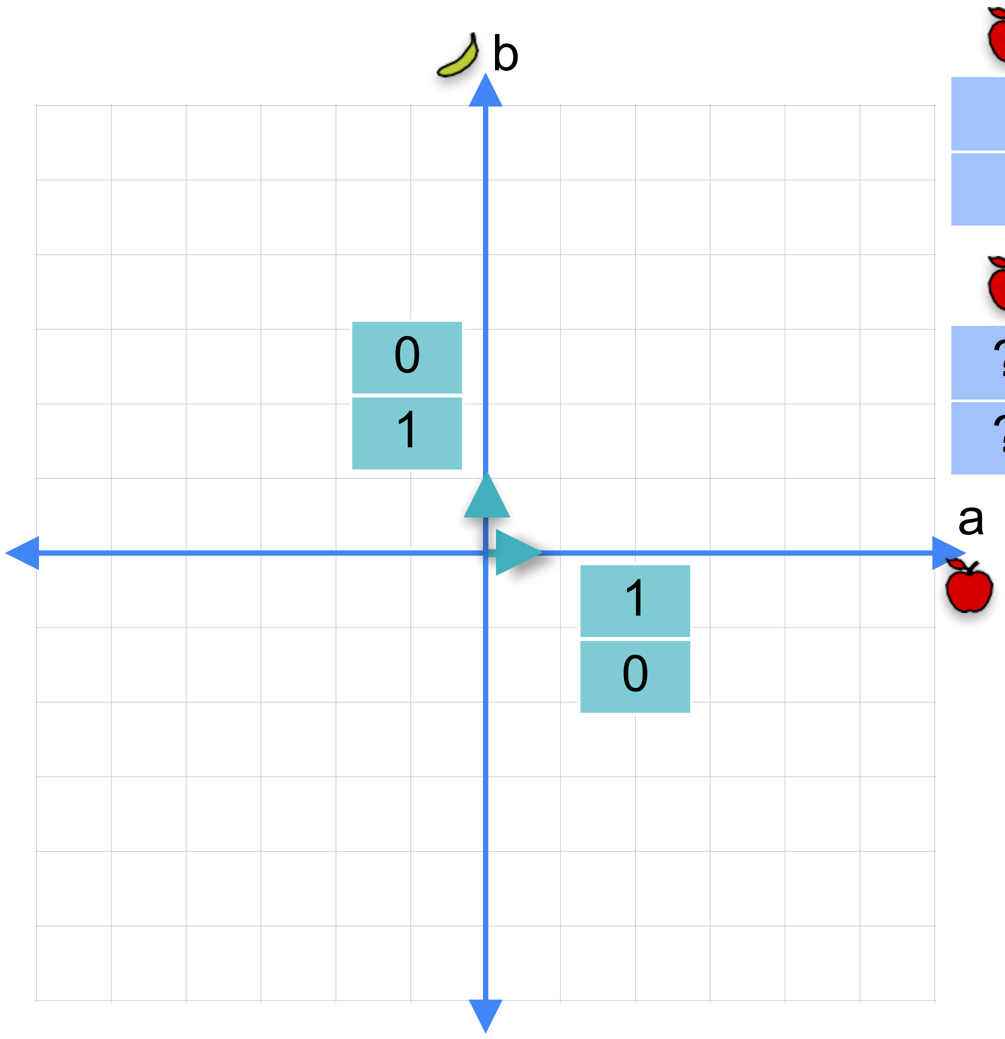
$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

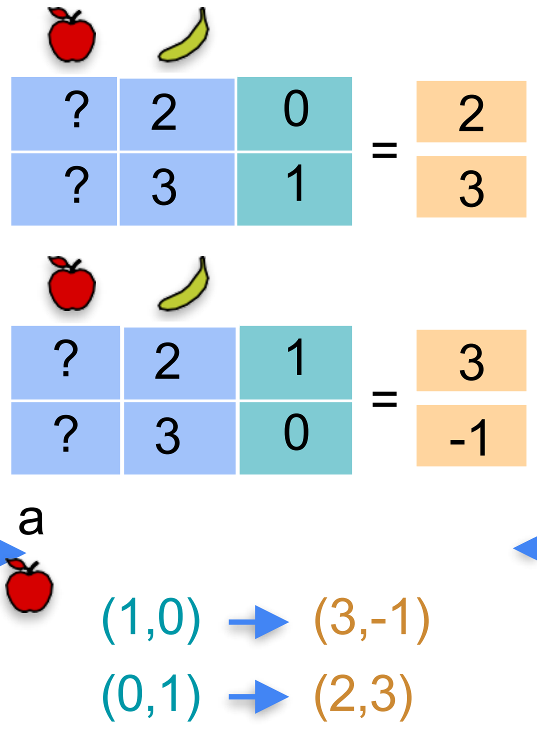
(1,0) → (3,-1)

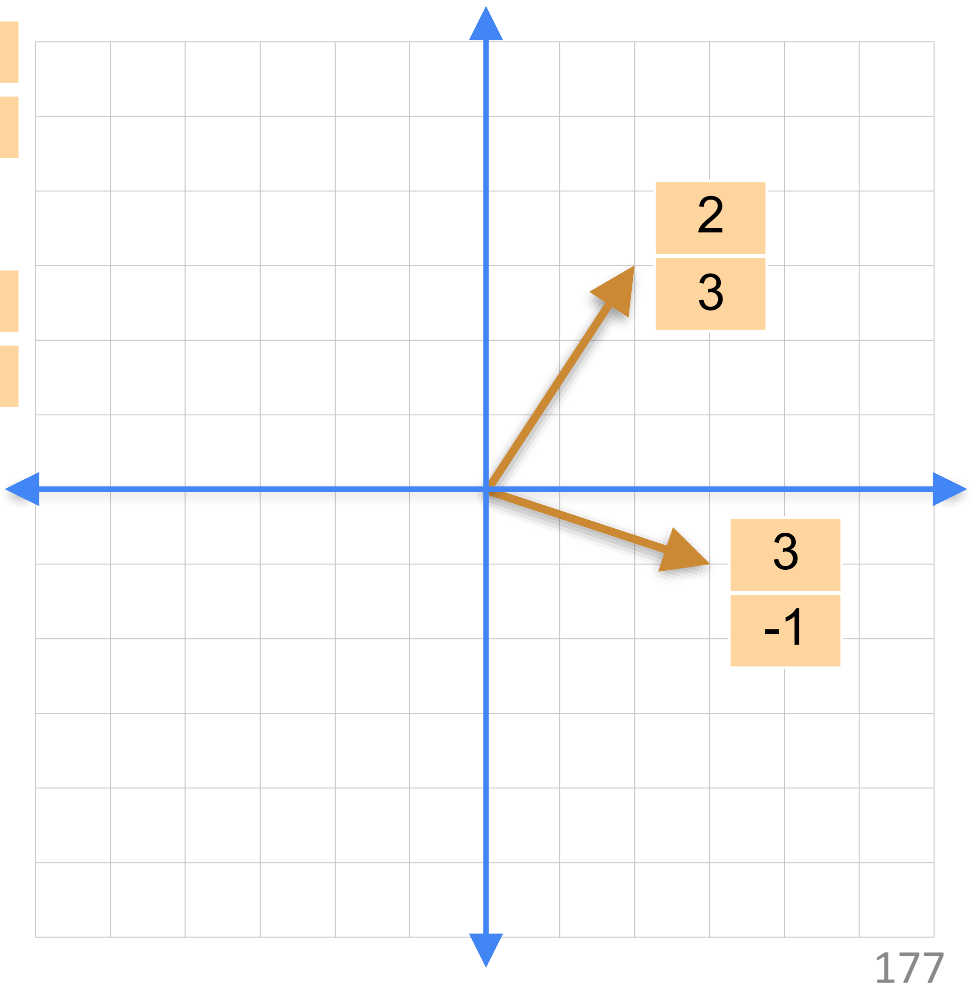
(0,1) → (2,3)



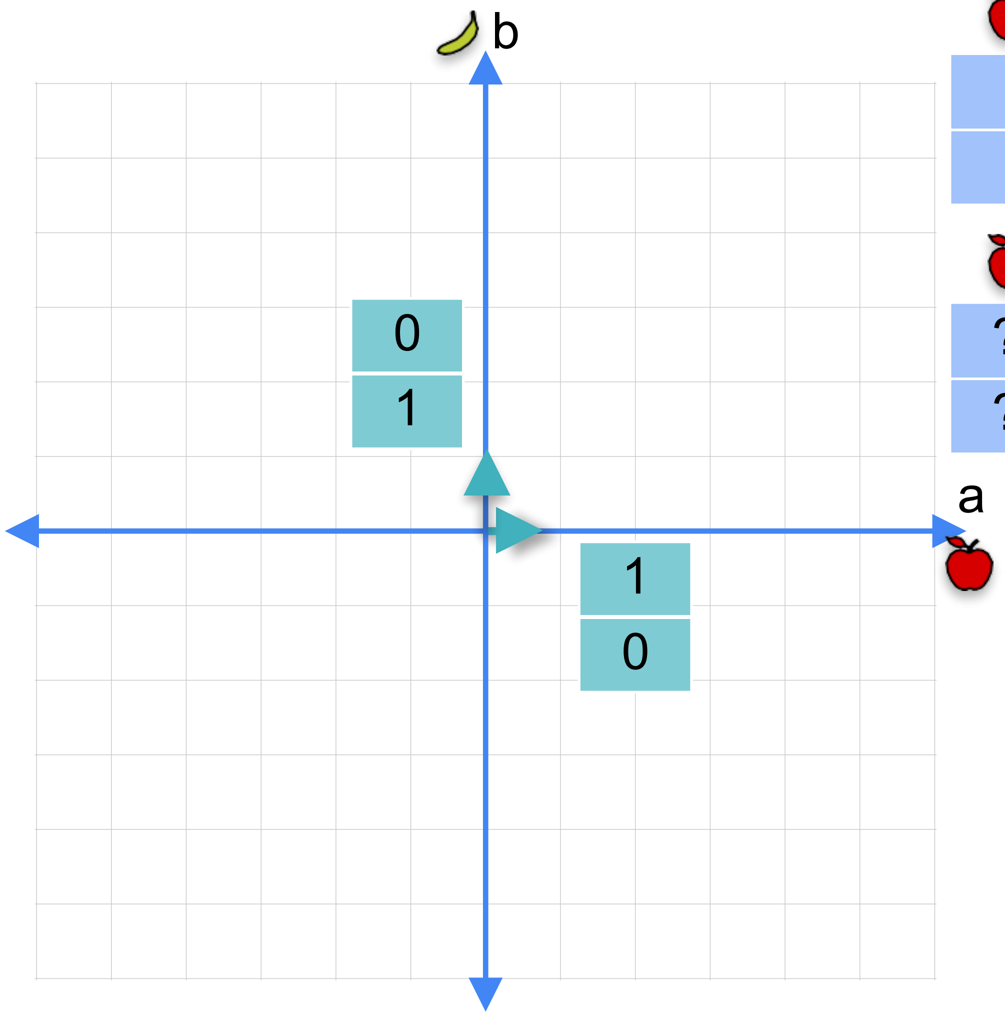
线性变换是矩阵







线性变换是矩阵

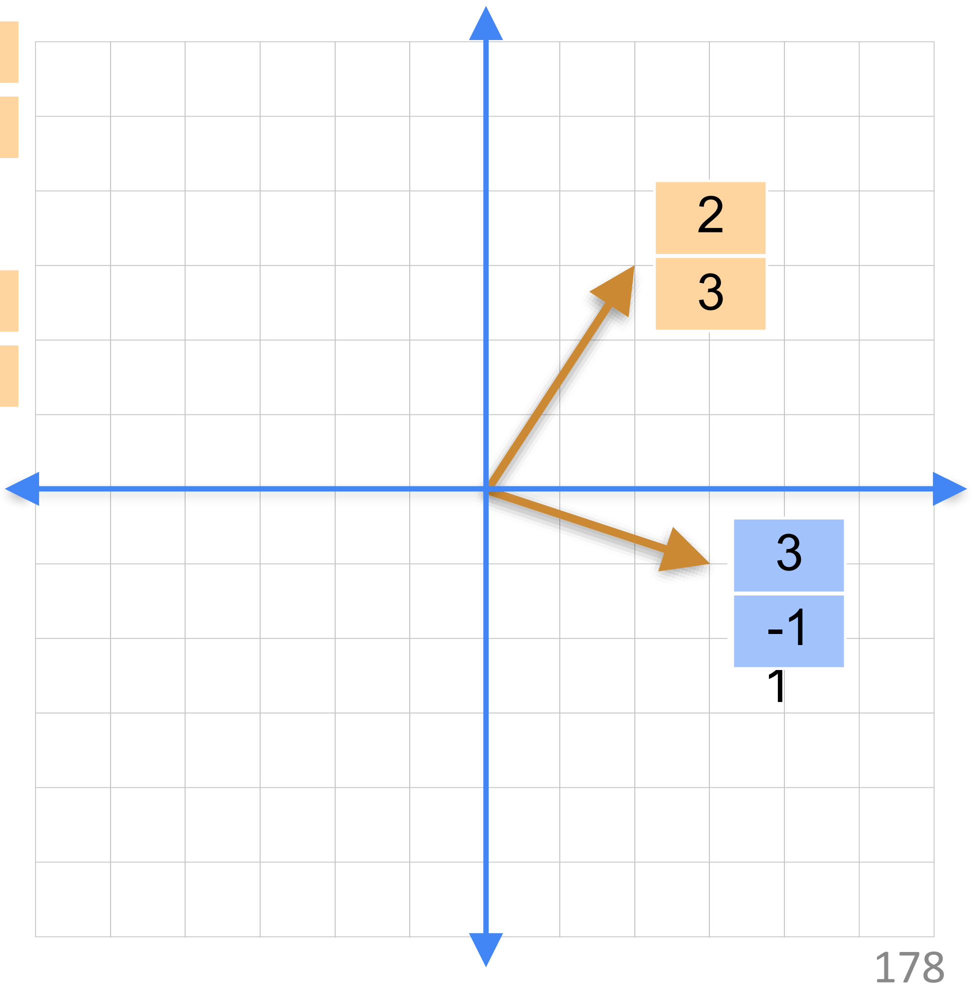


$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

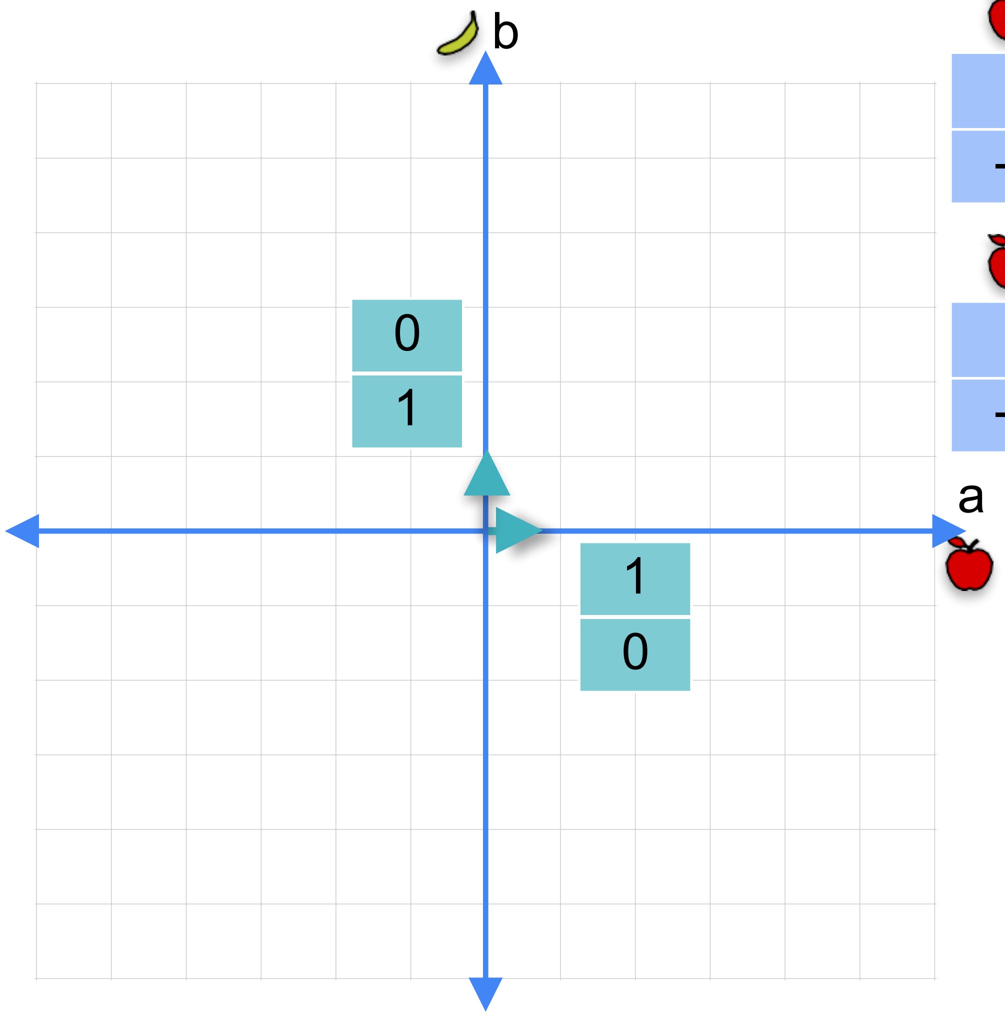
$$\begin{bmatrix} ? & 2 \\ ? & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

(1,0) → (3,-1)

(0,1) → (2,3)



线性变换是矩阵

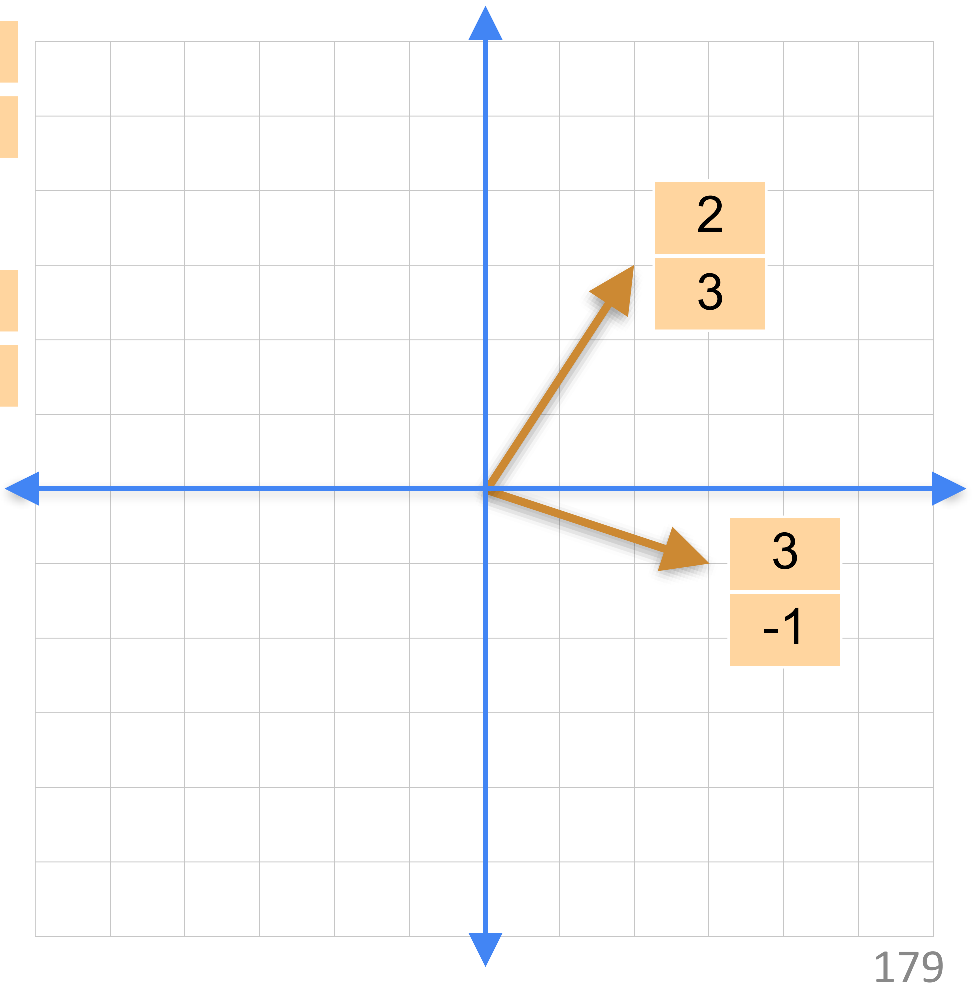


$$\begin{bmatrix}3&2\\-1&3\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}$$

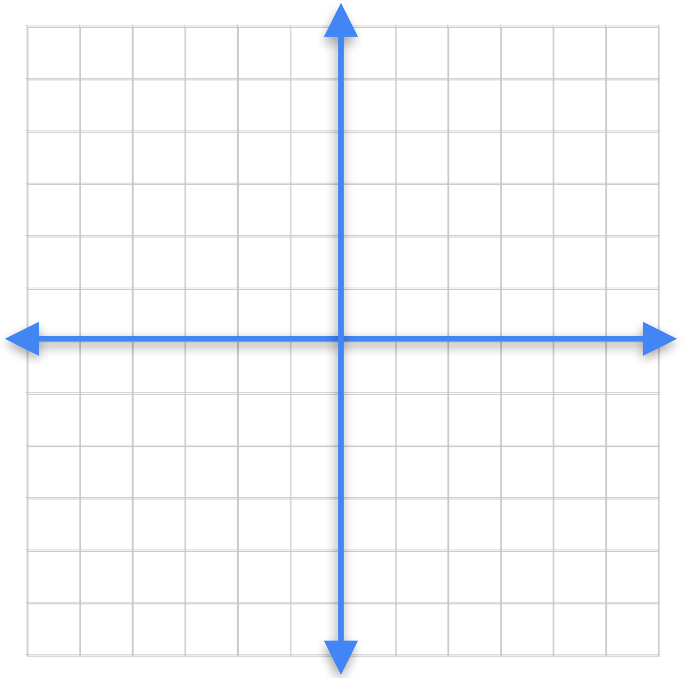
$$\begin{bmatrix}3&2\\-1&3\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$$

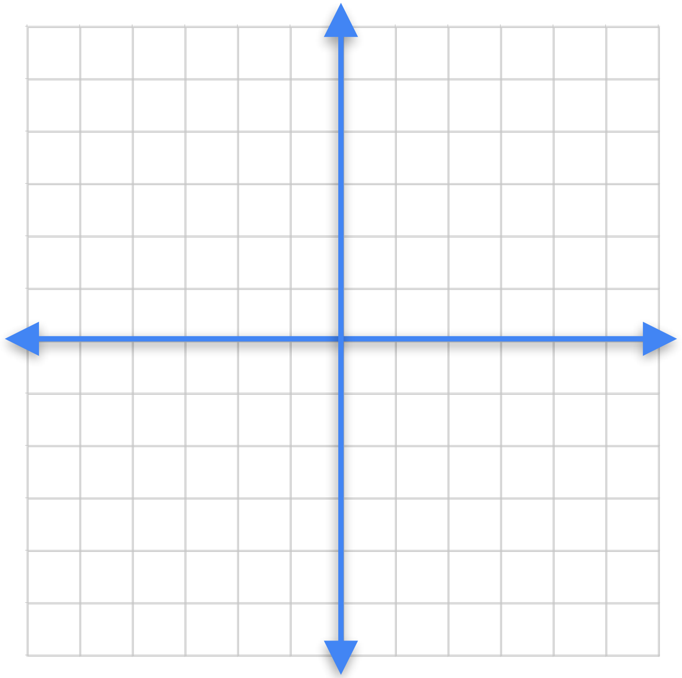
(1,0) → (3,-1)

(0,1) → (2,3)

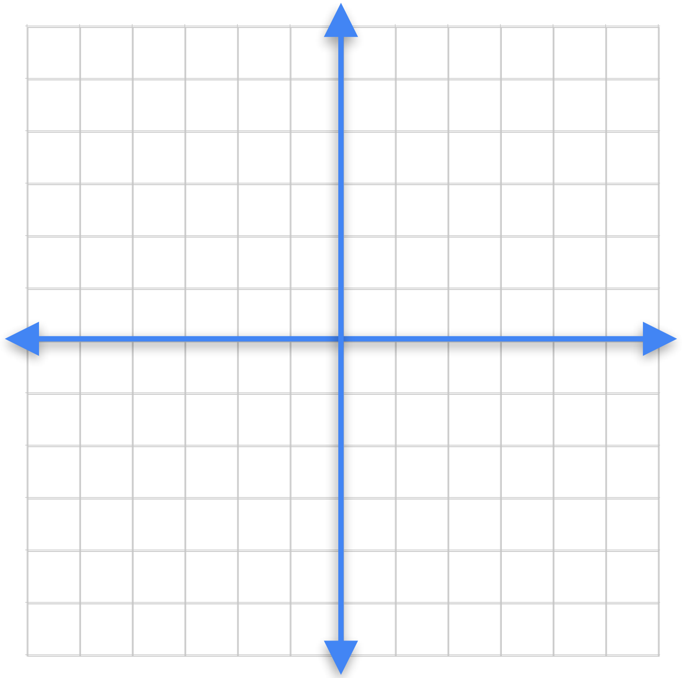


多次线性变换

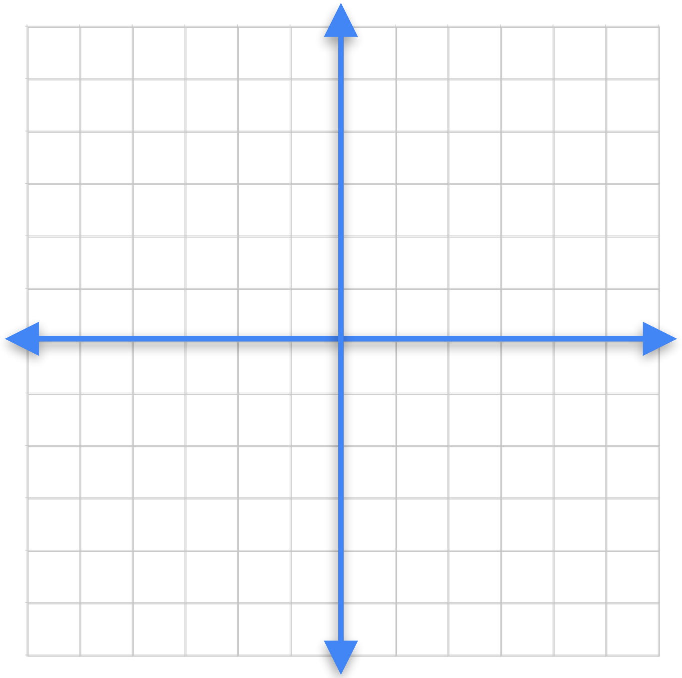




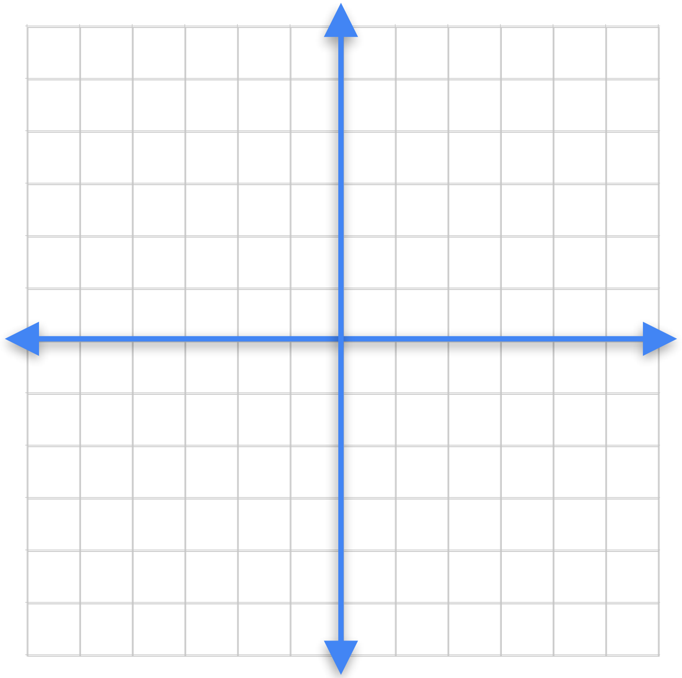
多次线性变换



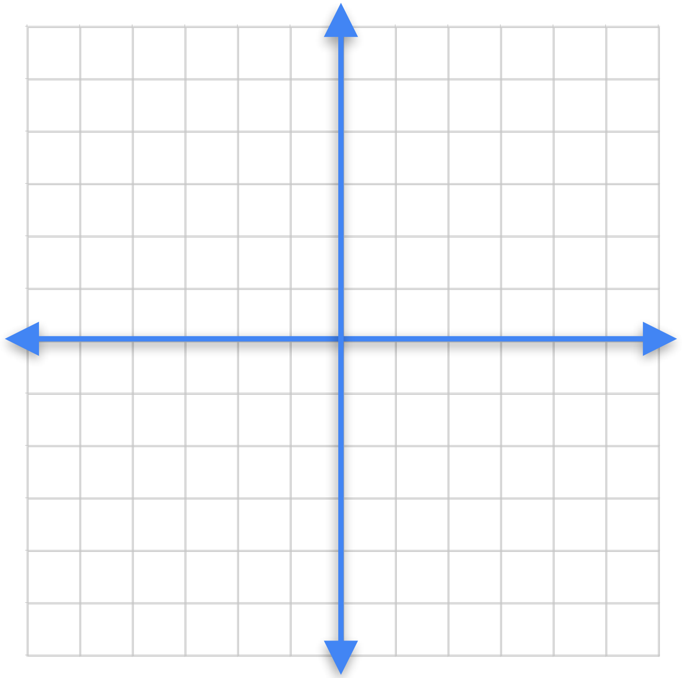
3	1
1	2



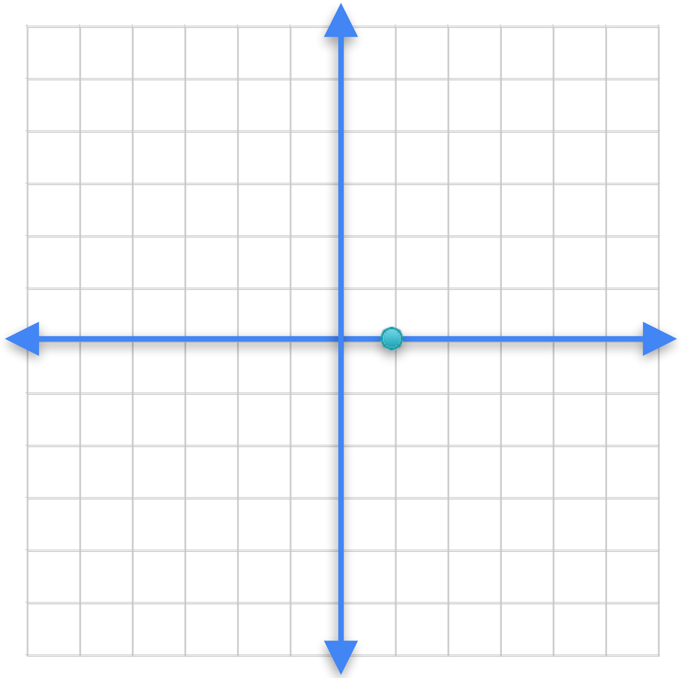
多次线性变换



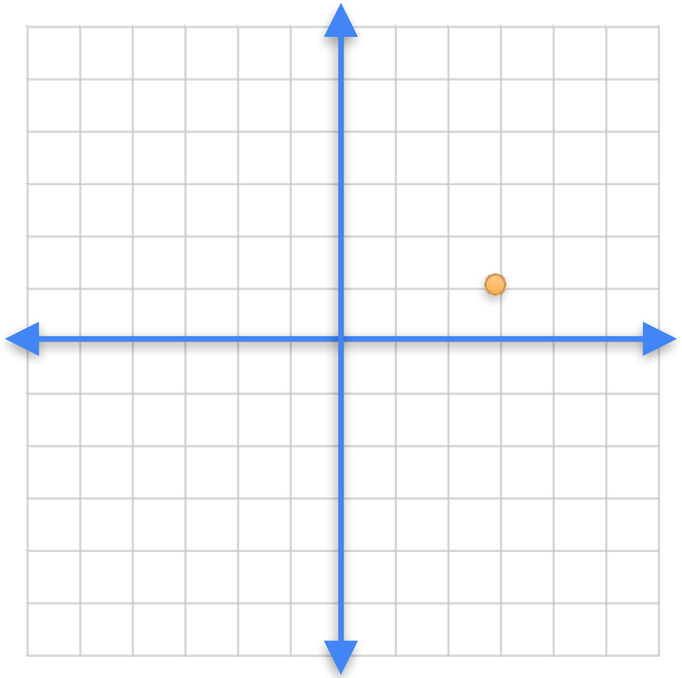
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



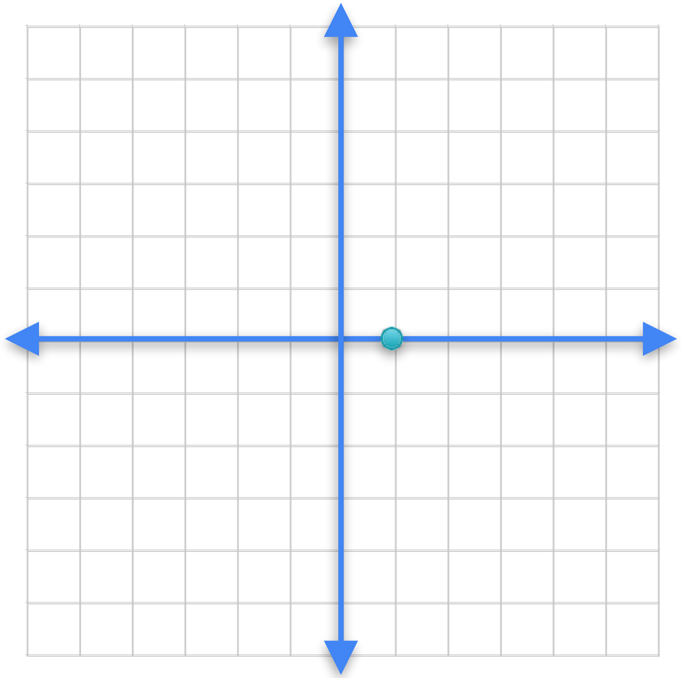
多次线性变换



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



多次线性变换



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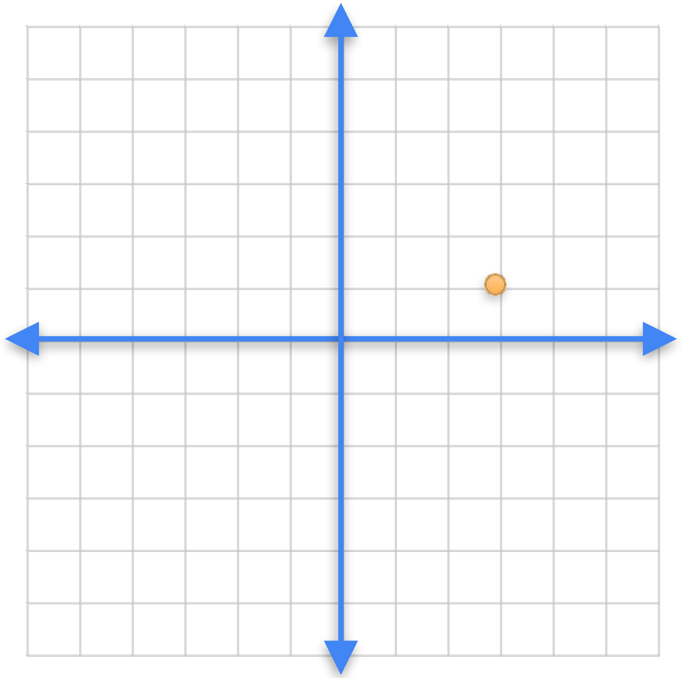

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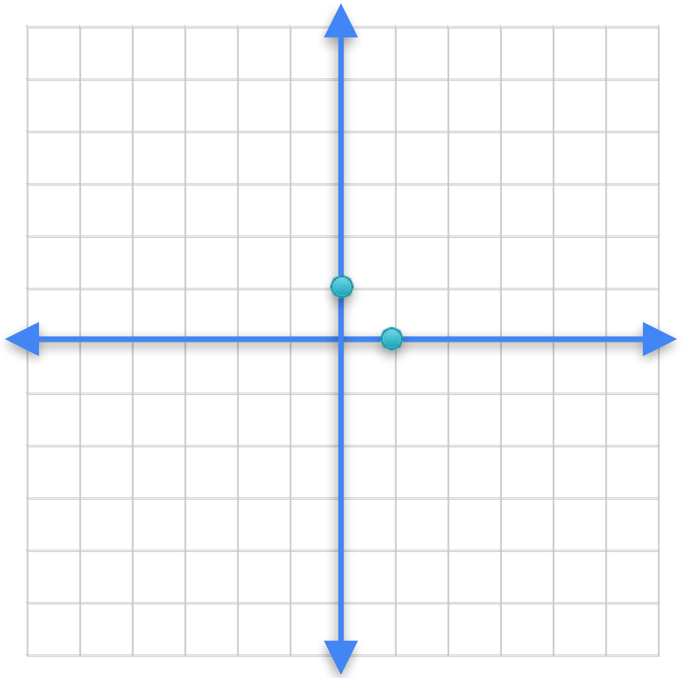

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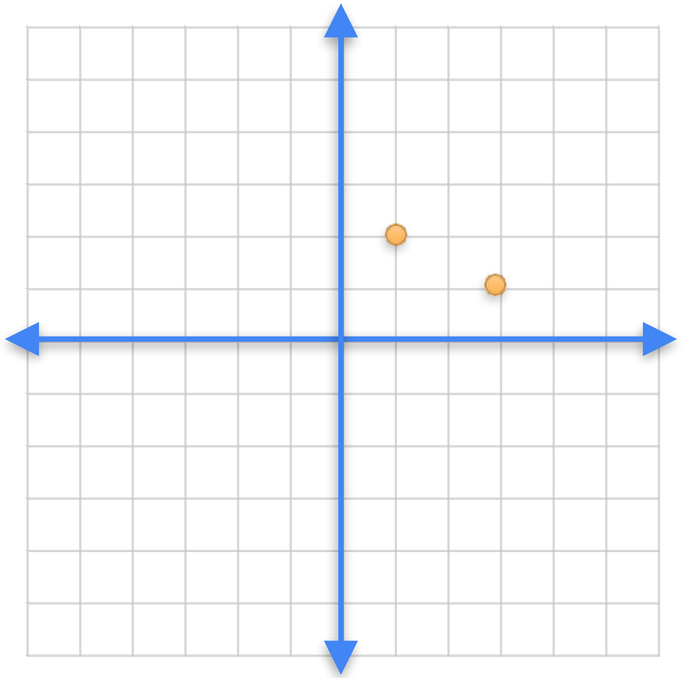
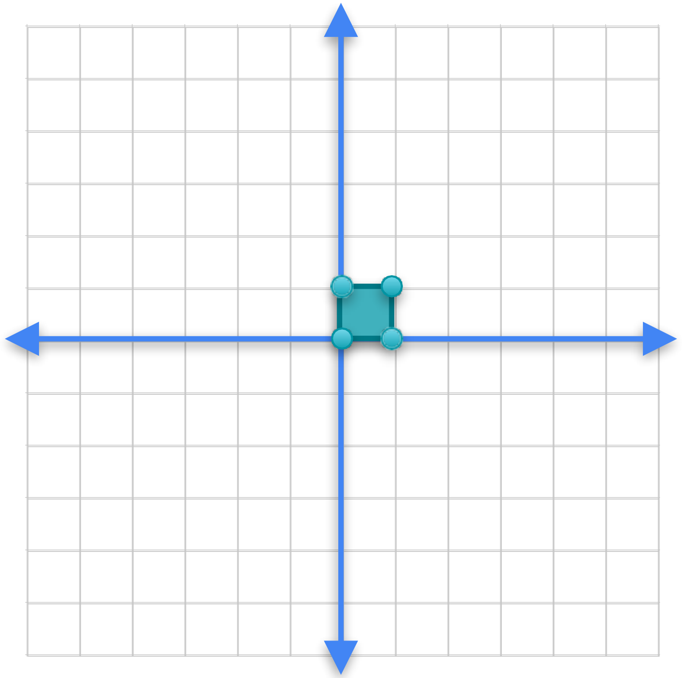


多次线性变换



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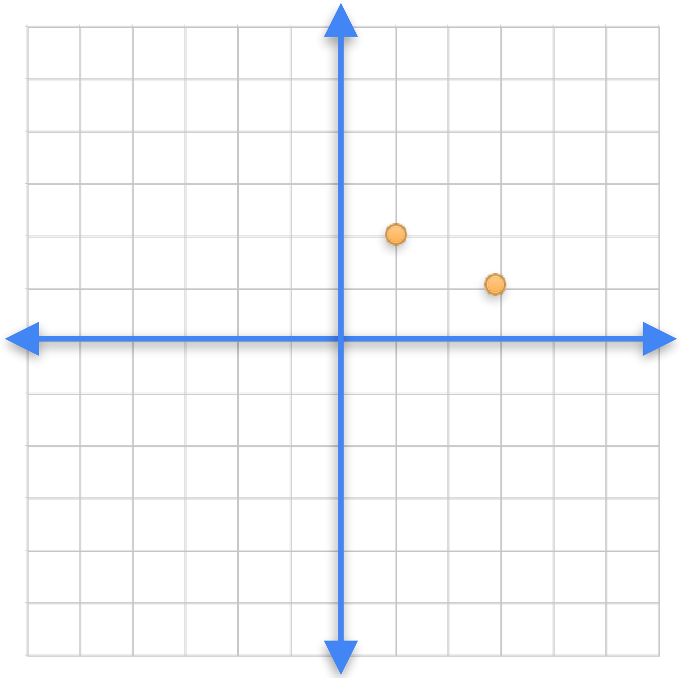
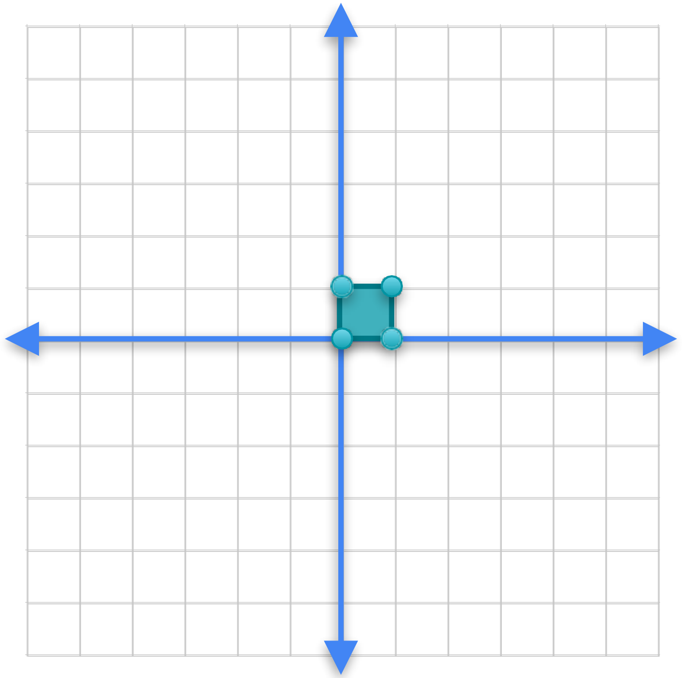
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



多次线性变换



$$\begin{bmatrix}3 & 1\\1 & 2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$$

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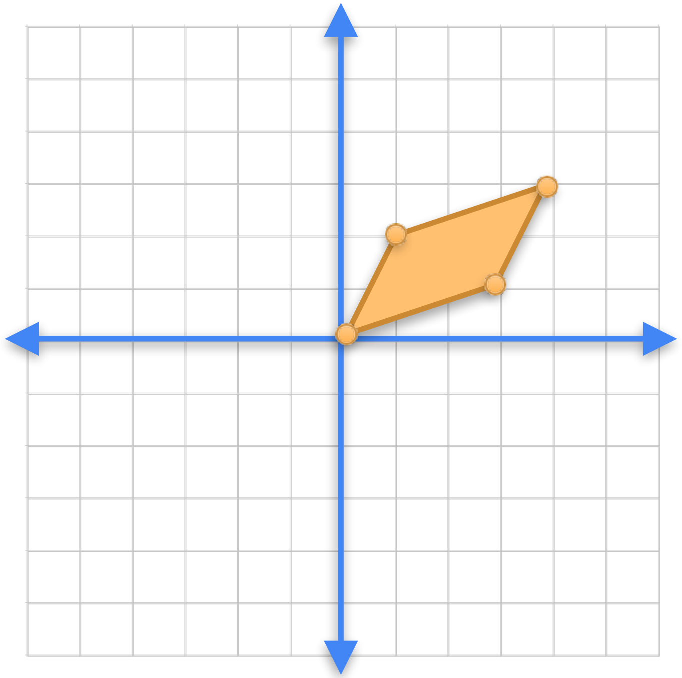
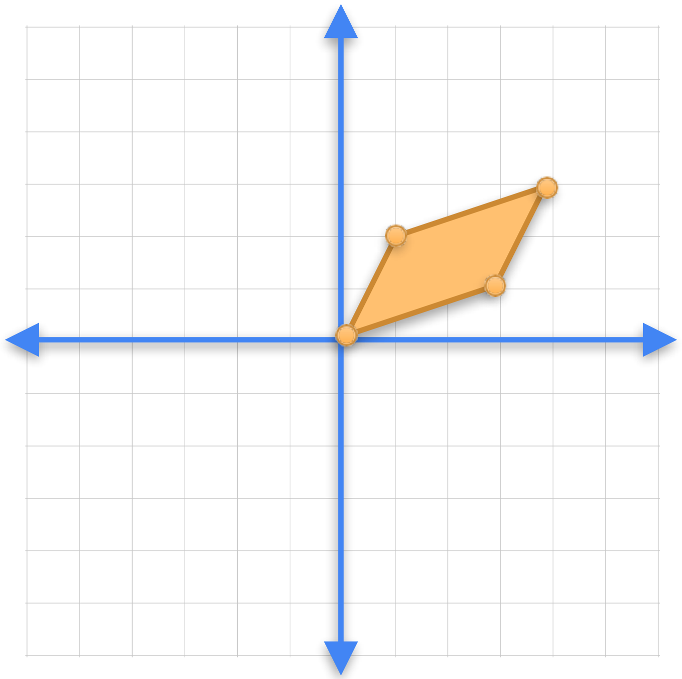
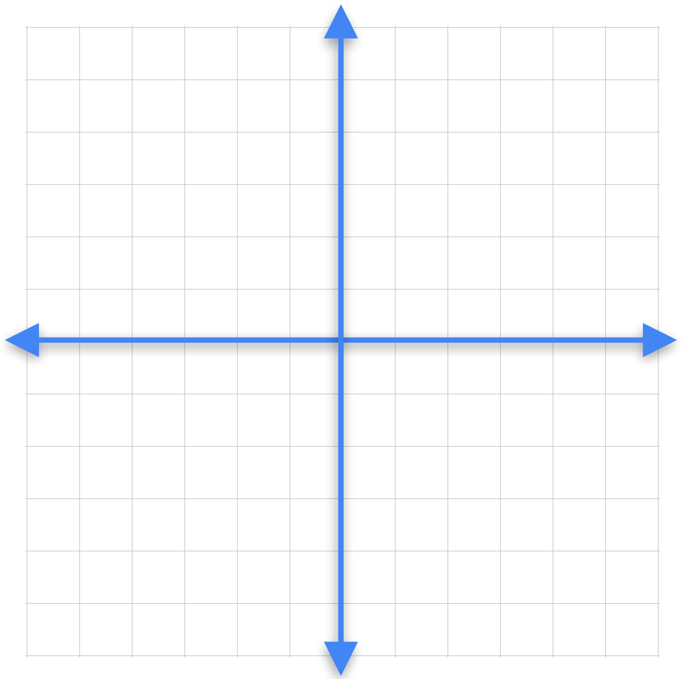


多次线性变换





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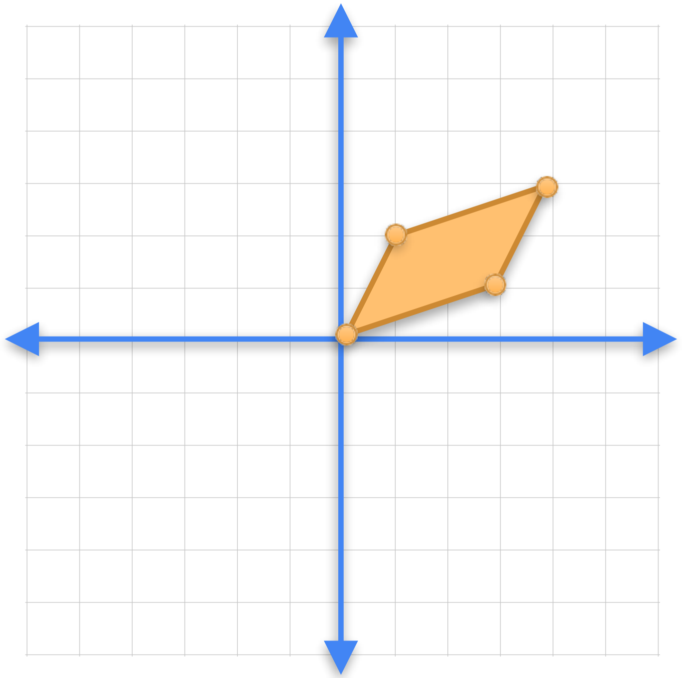
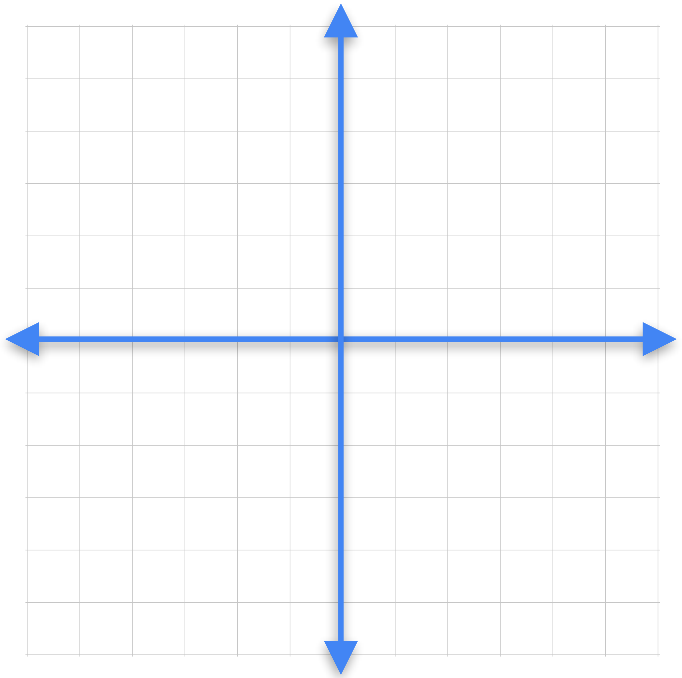
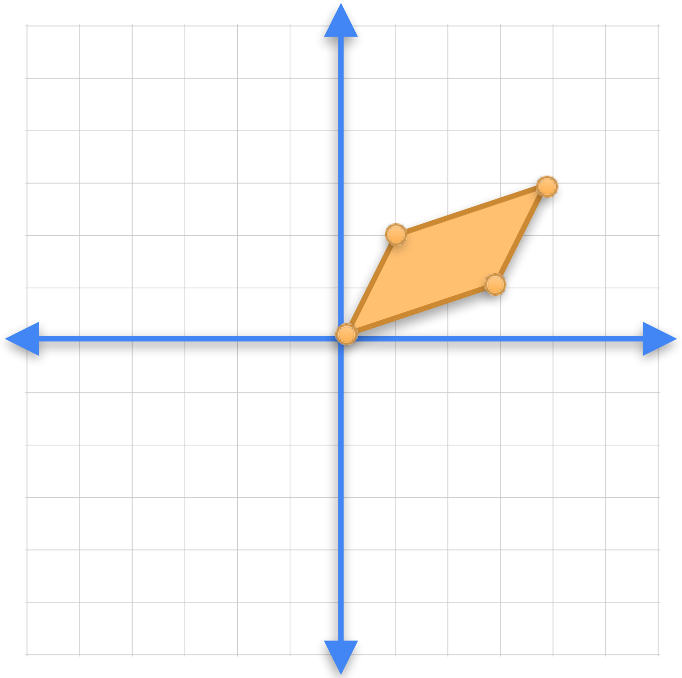


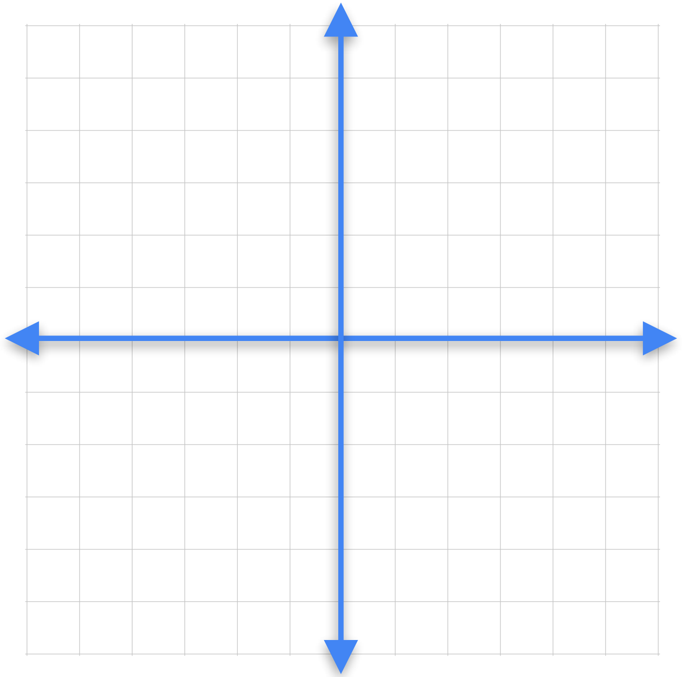
2	-1
0	2



多次线性变换



$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



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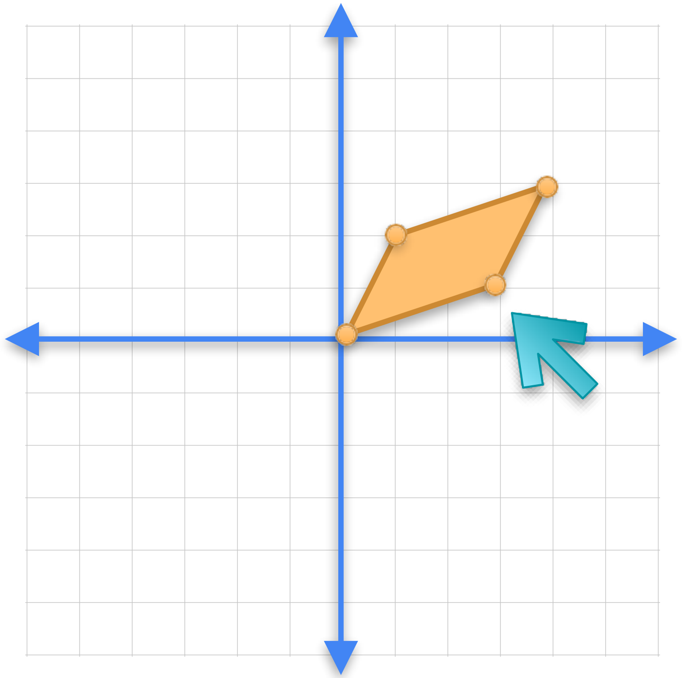

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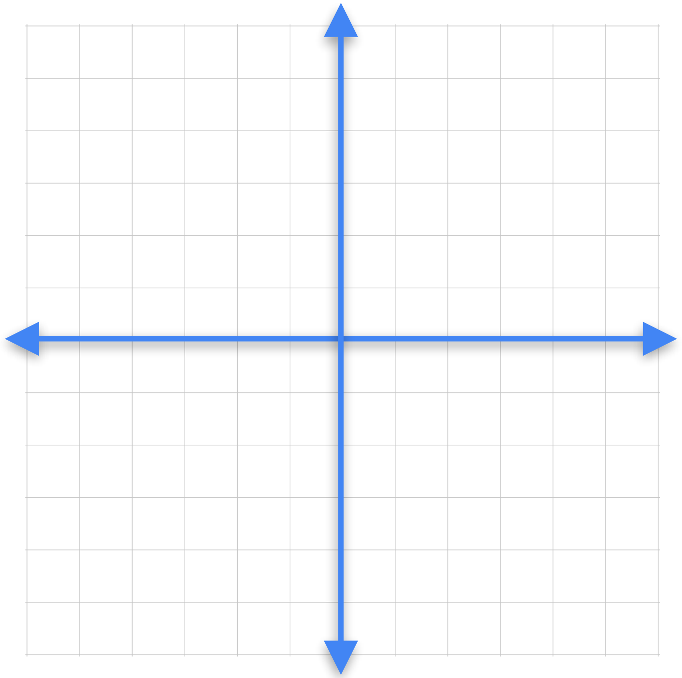
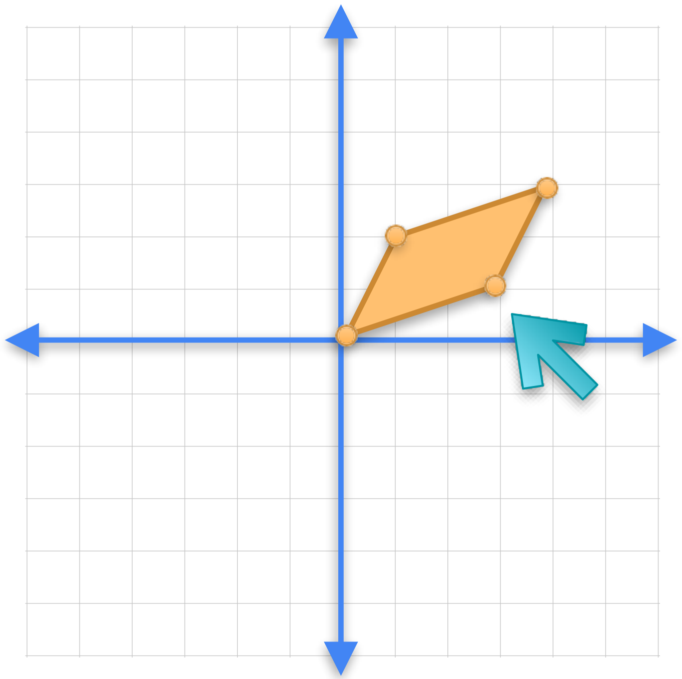


多次线性变换



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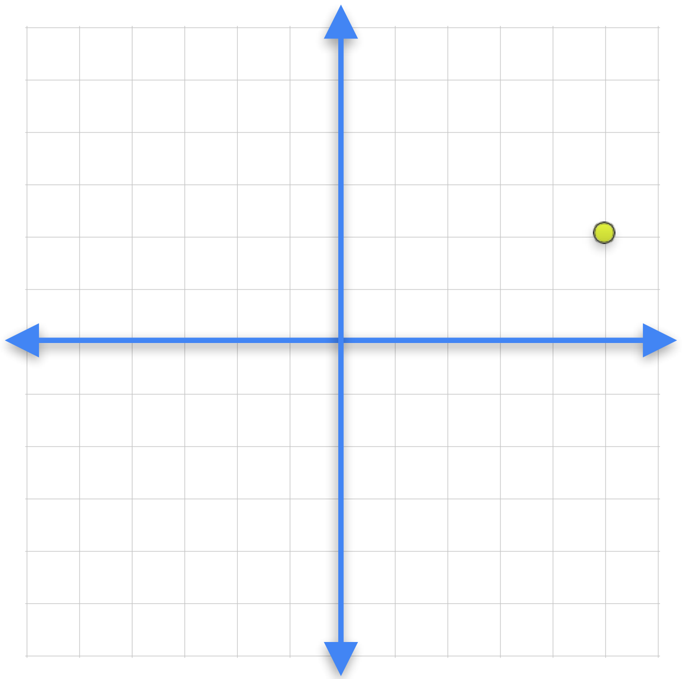
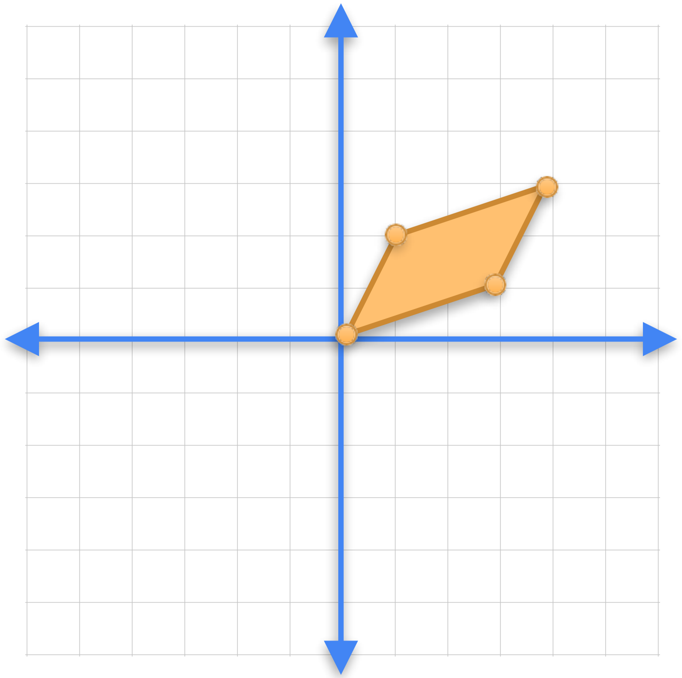
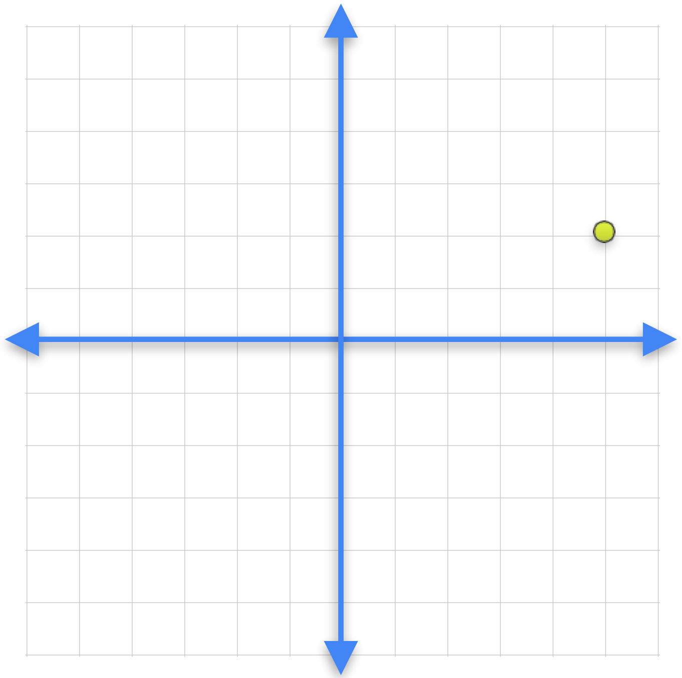


多次线性变换

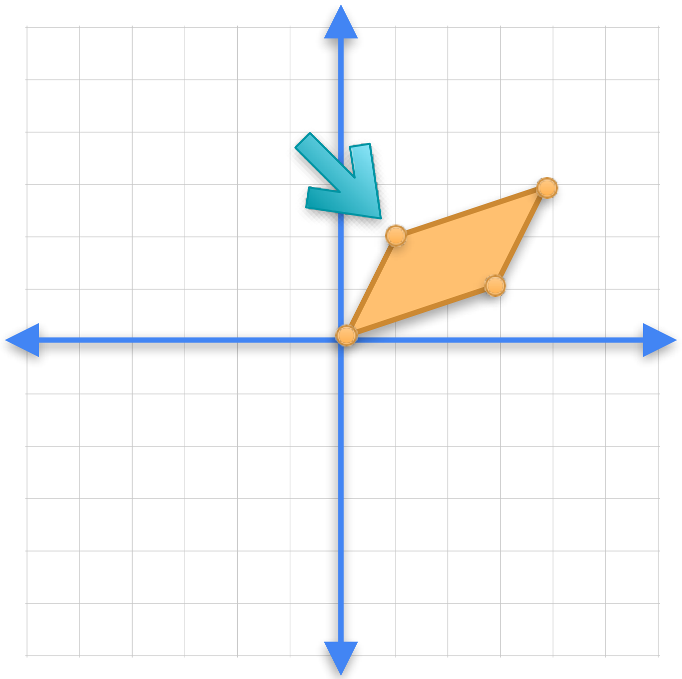


$$\begin{bmatrix}2 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}3\\1\end{bmatrix}=\begin{bmatrix}5\\2\end{bmatrix}$$

$$\begin{bmatrix}2 & -1\\0 & 2\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0\\4\end{bmatrix}$$



多次线性变换



$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

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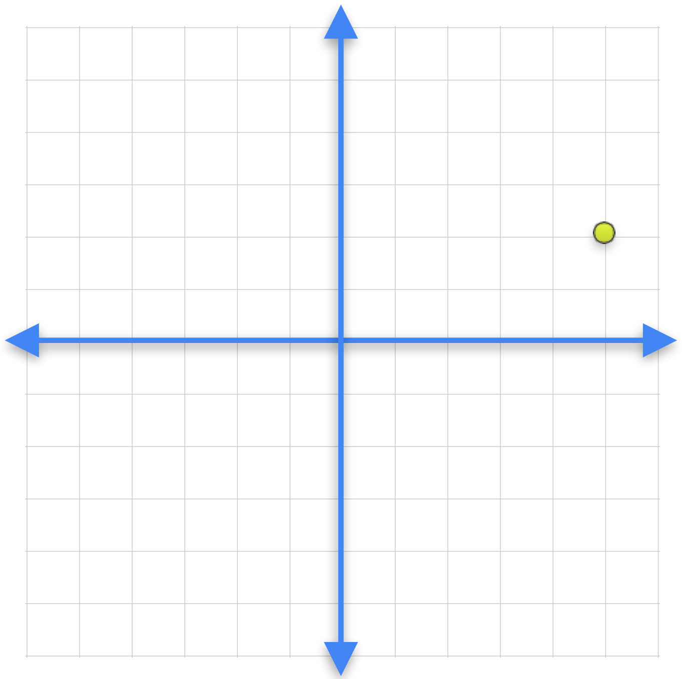
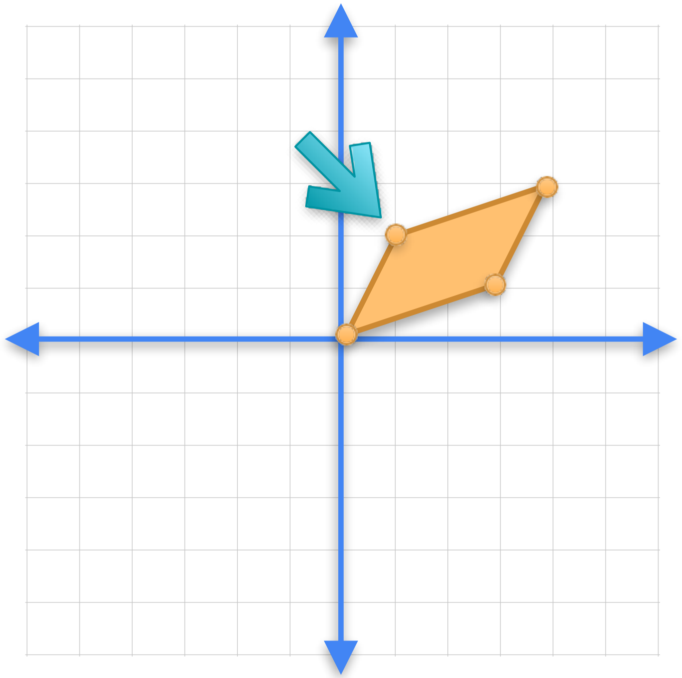


多次线性变换



$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

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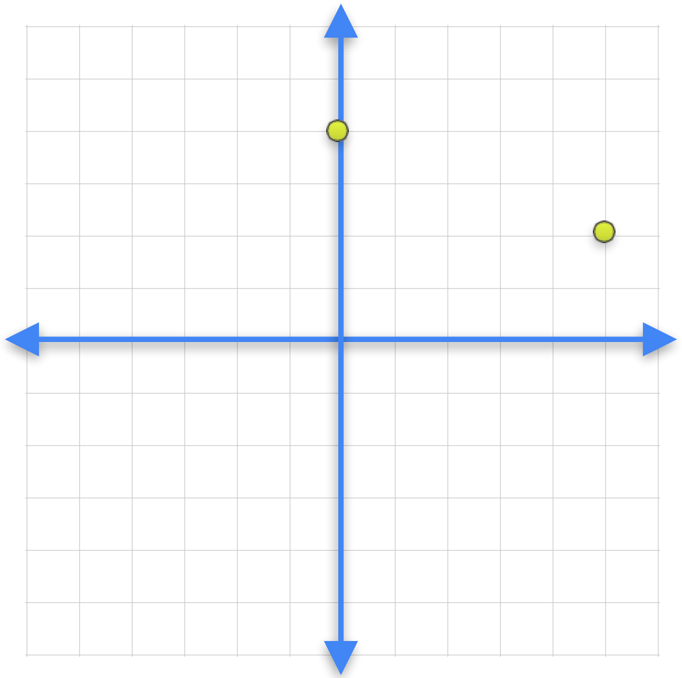
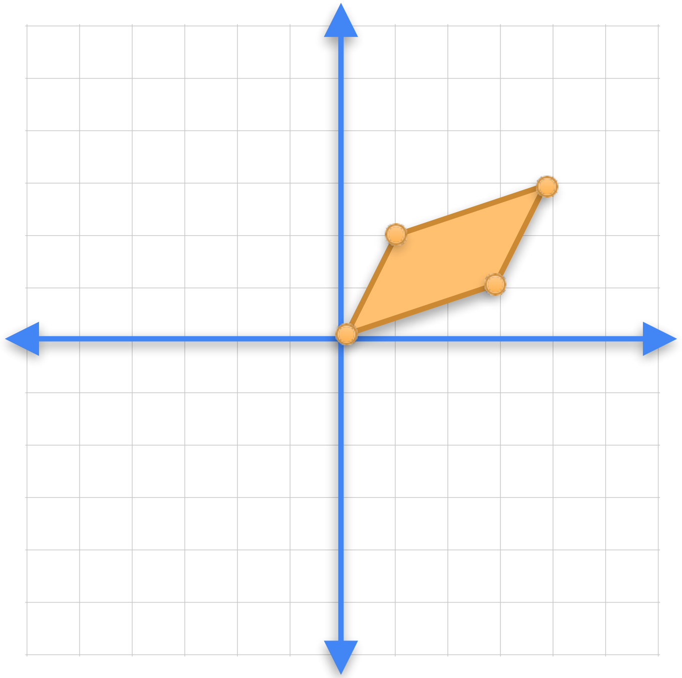


多次线性变换



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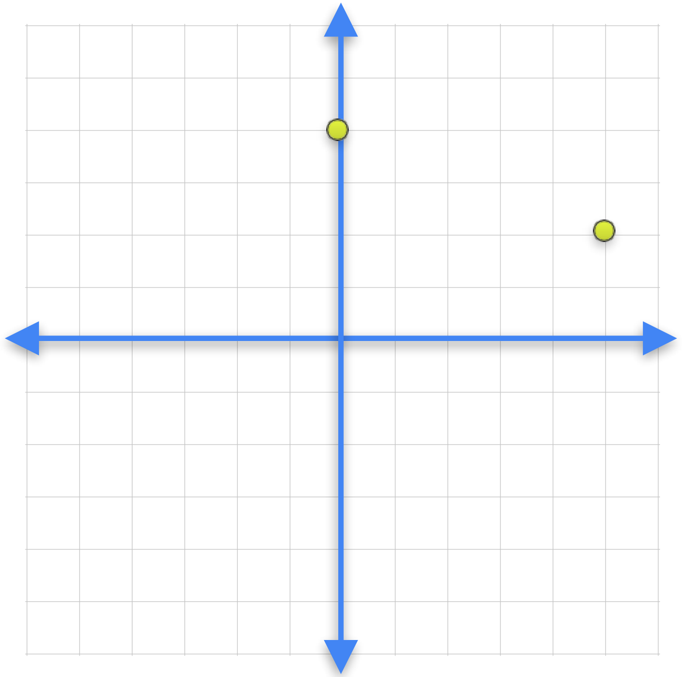

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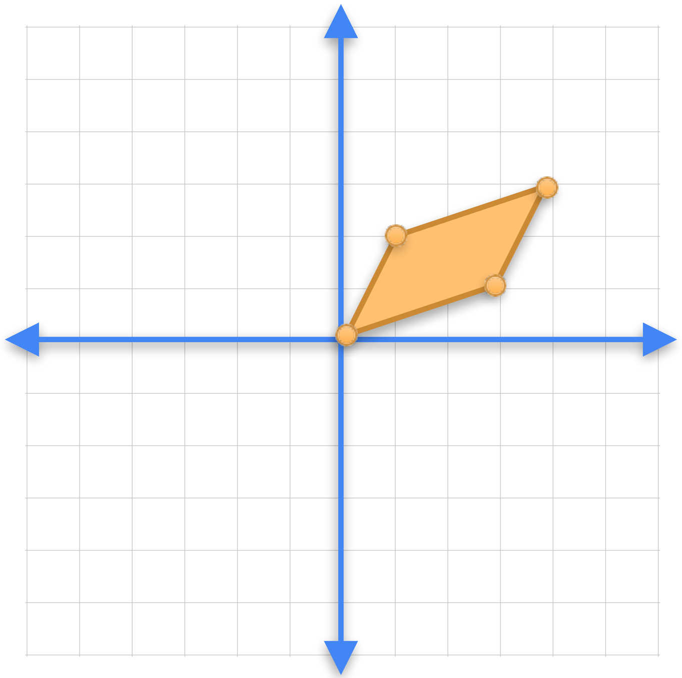

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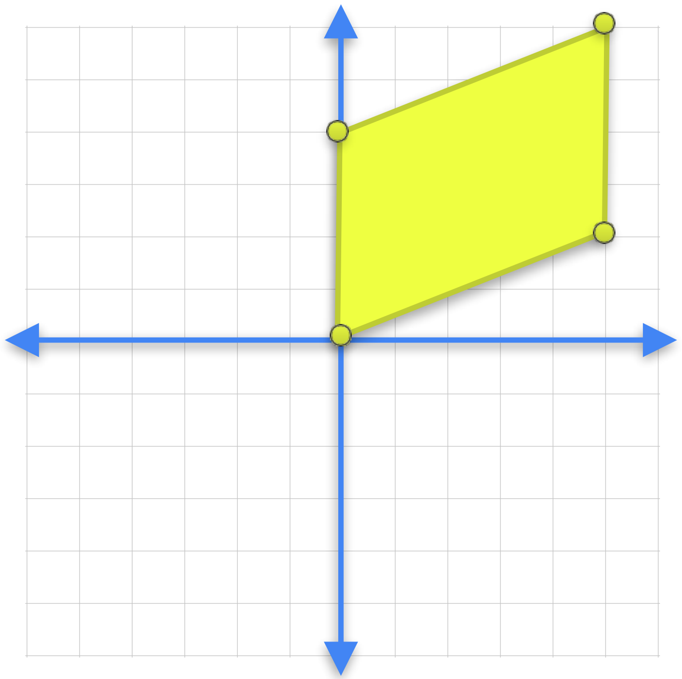

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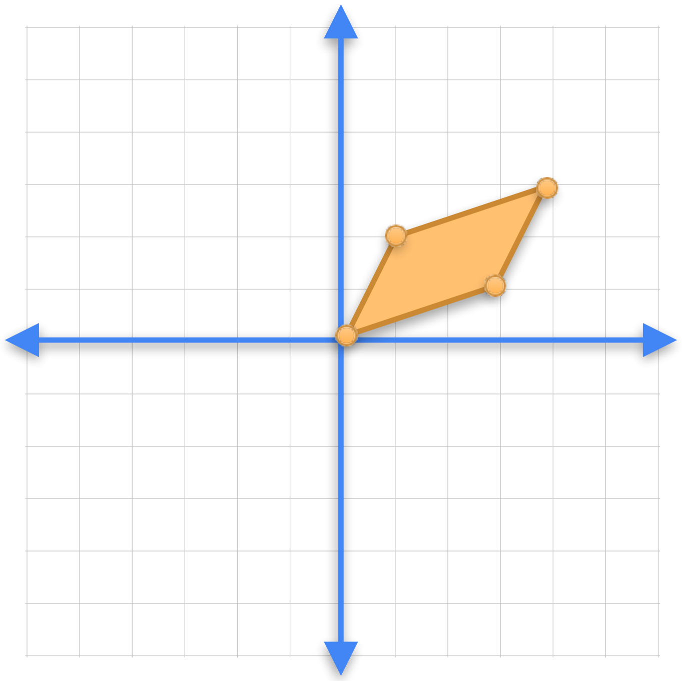

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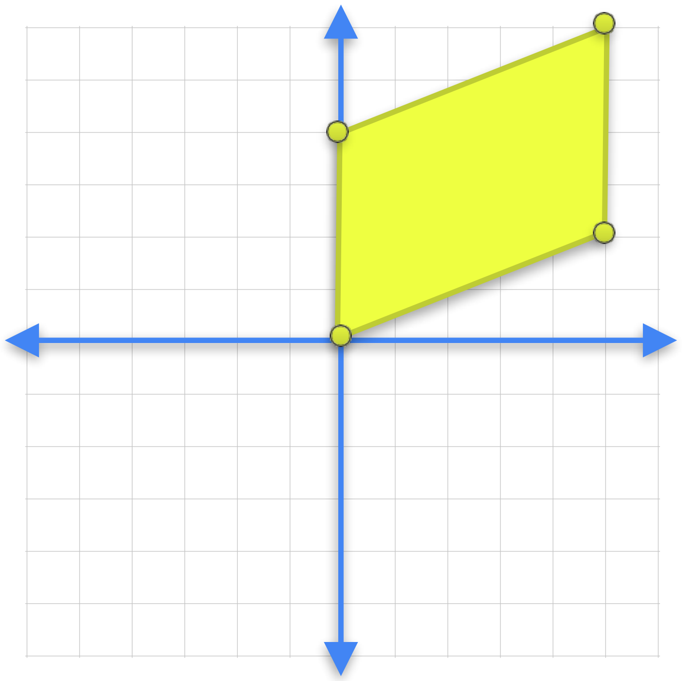

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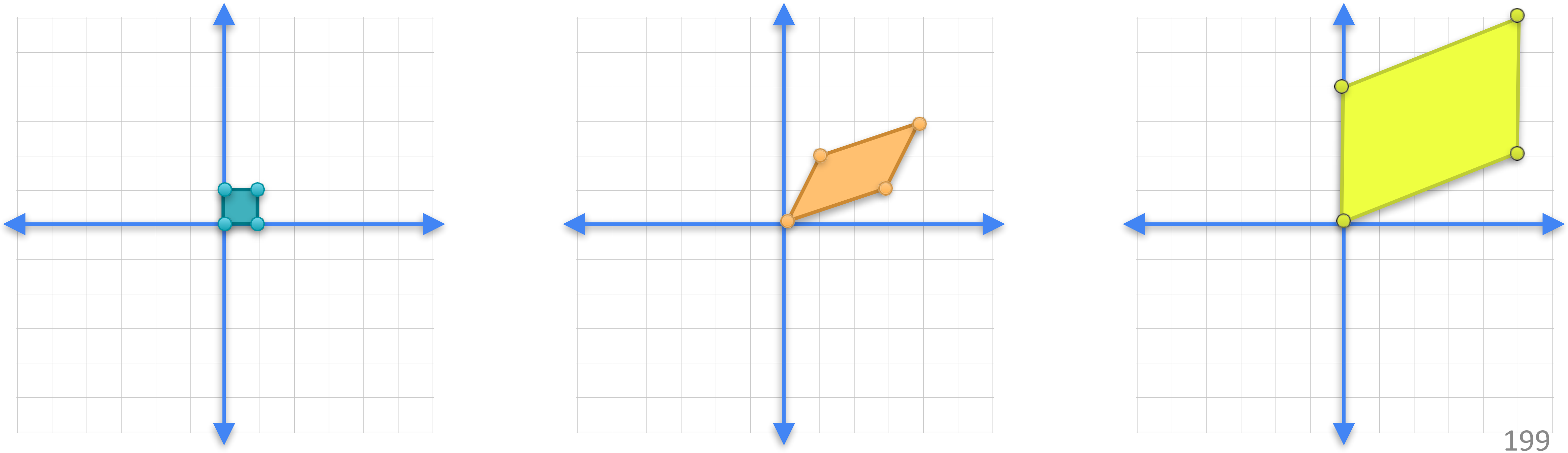

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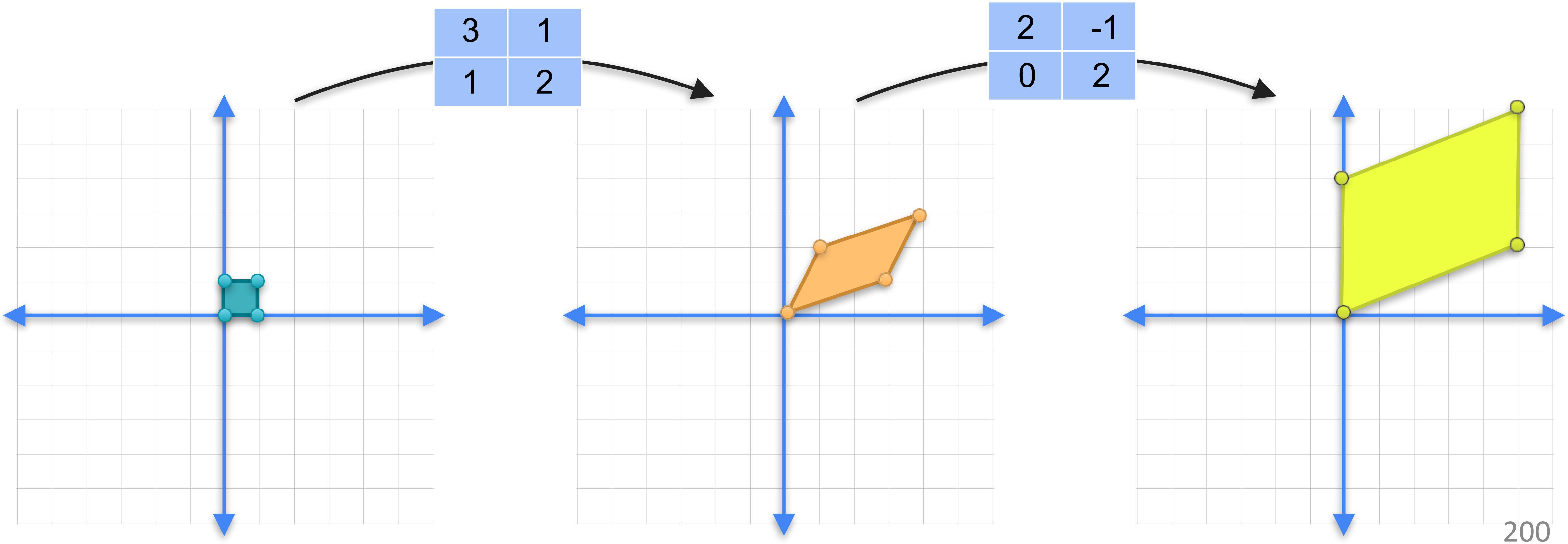

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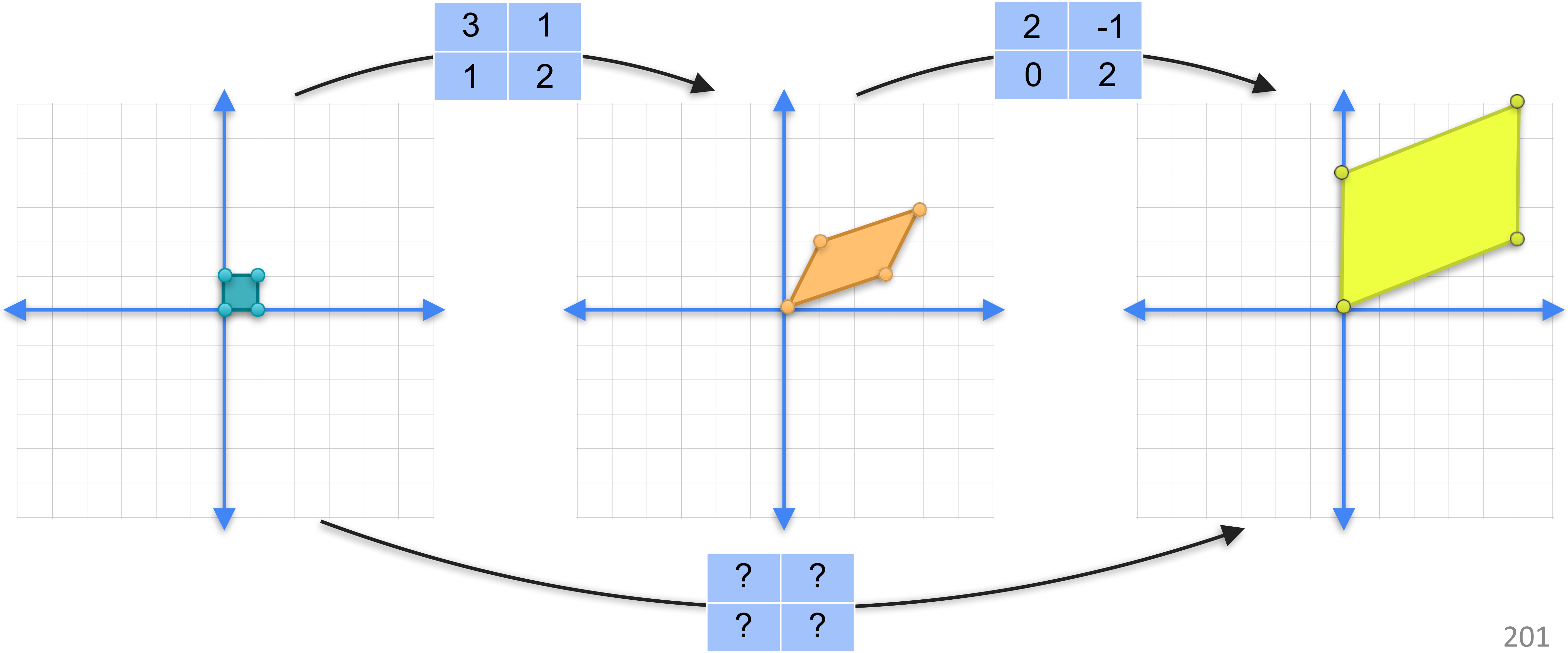

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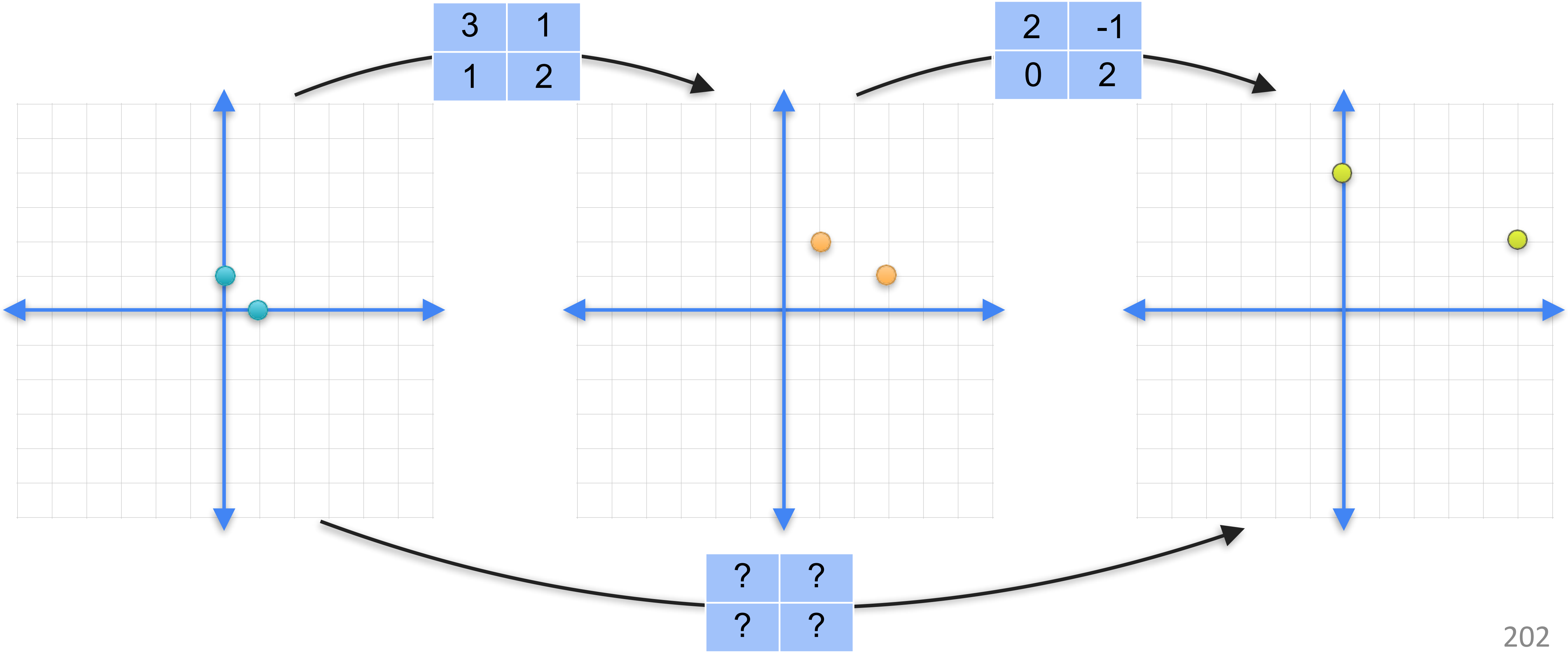

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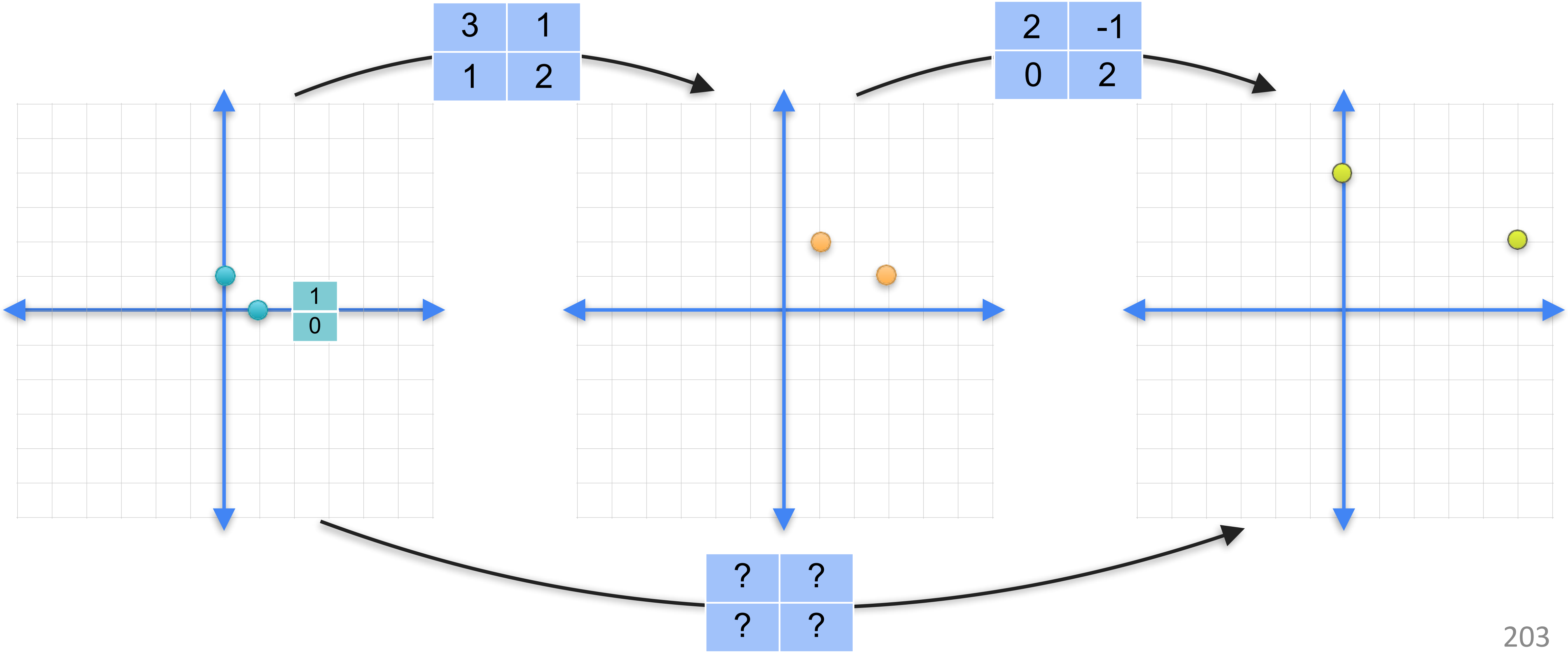

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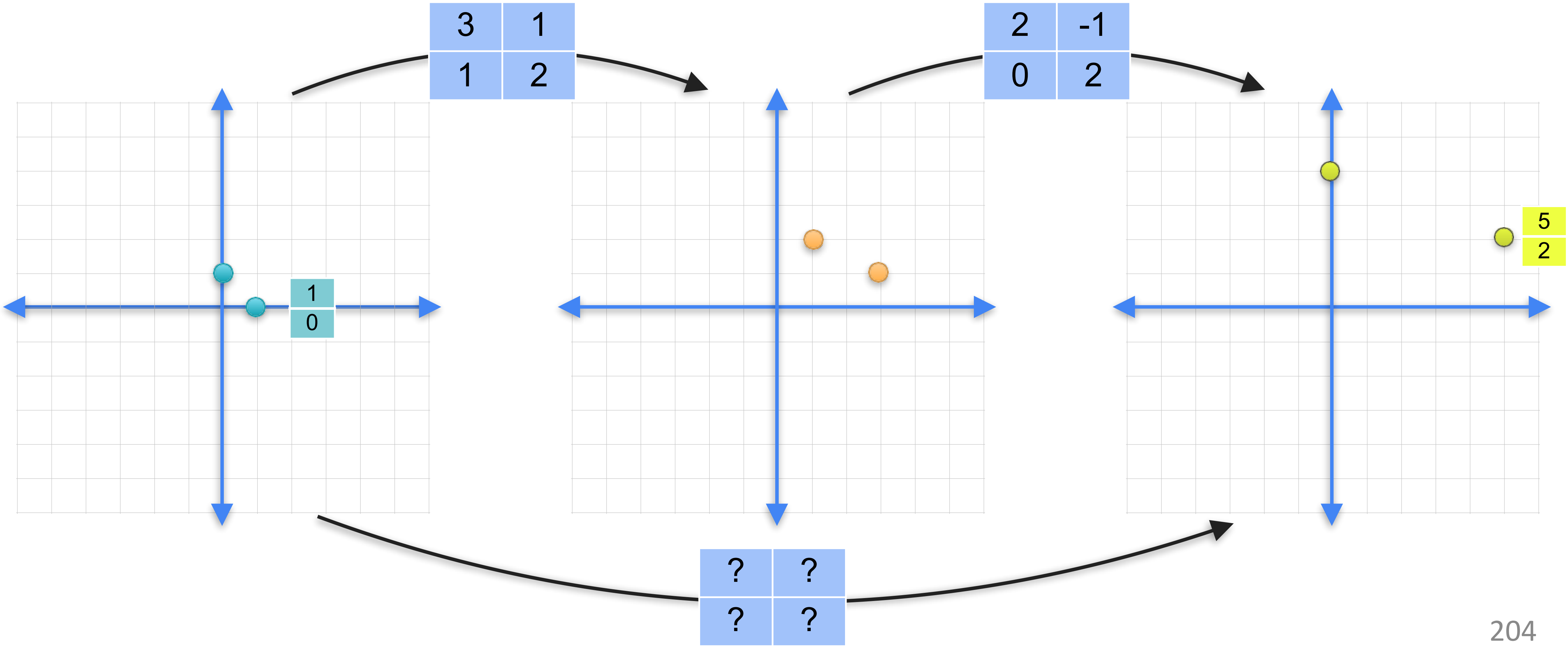

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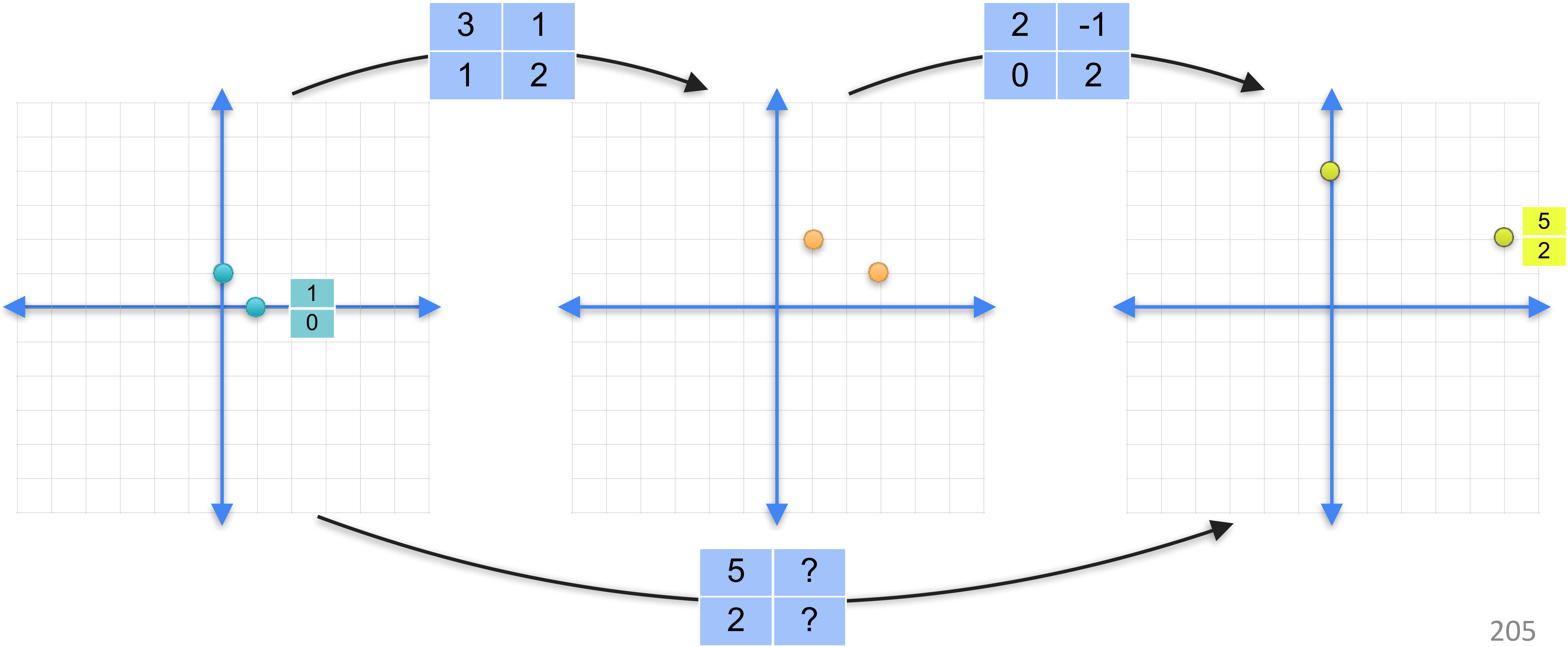

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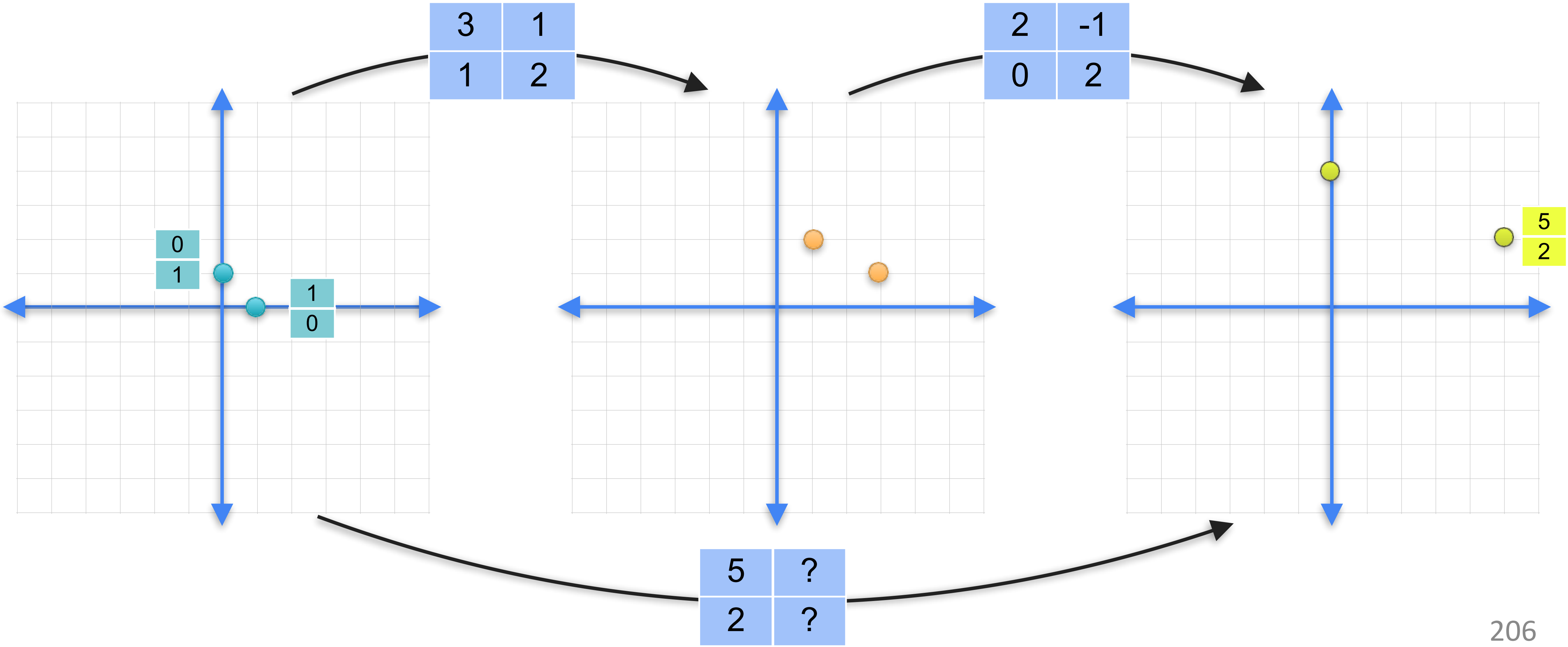

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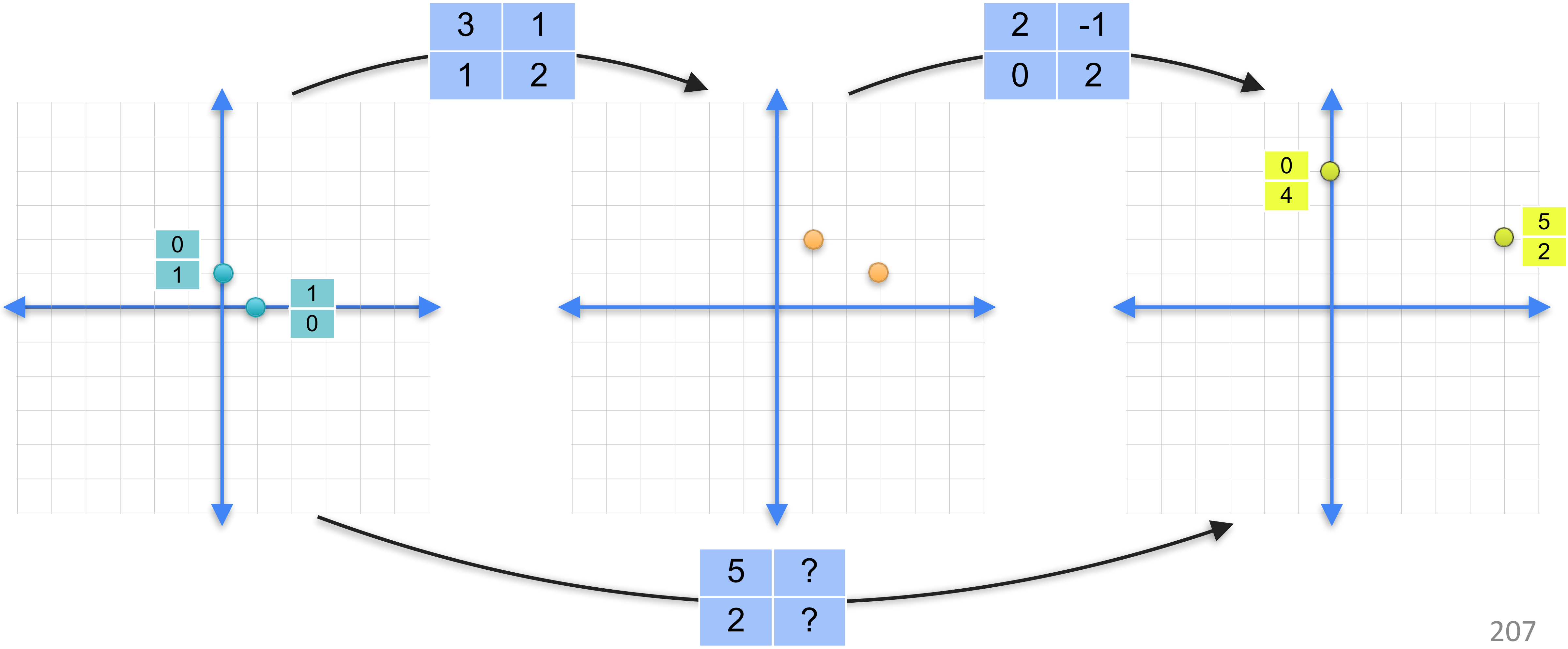

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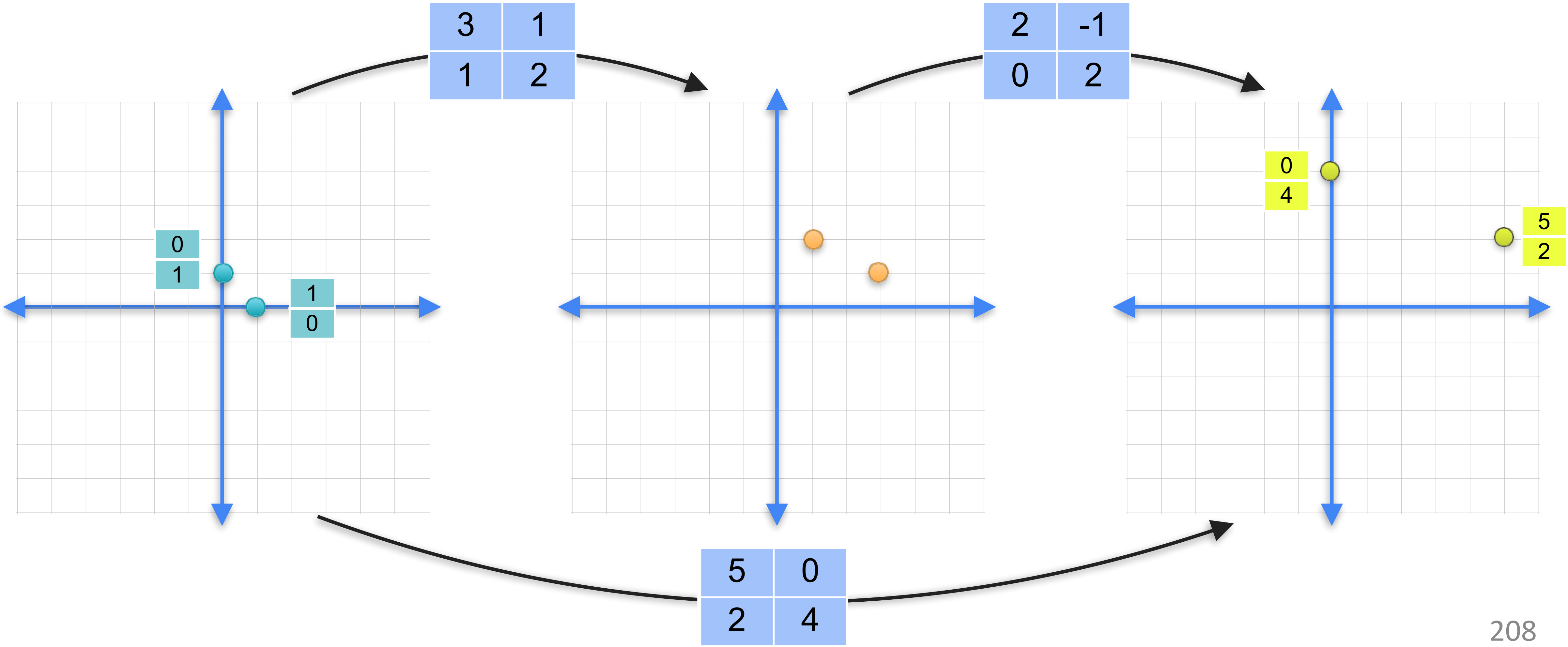
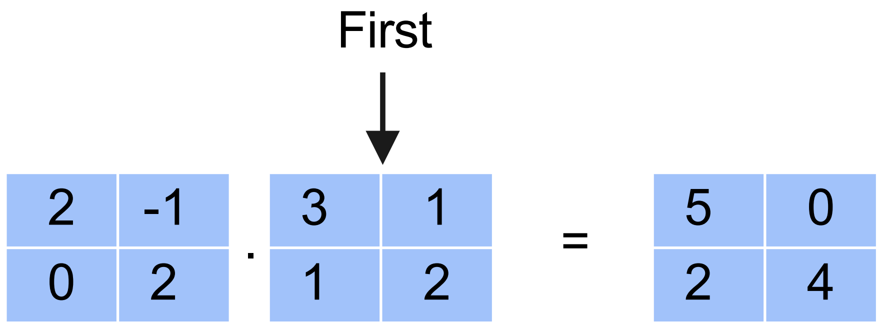


矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

矩阵乘法



矩阵乘法

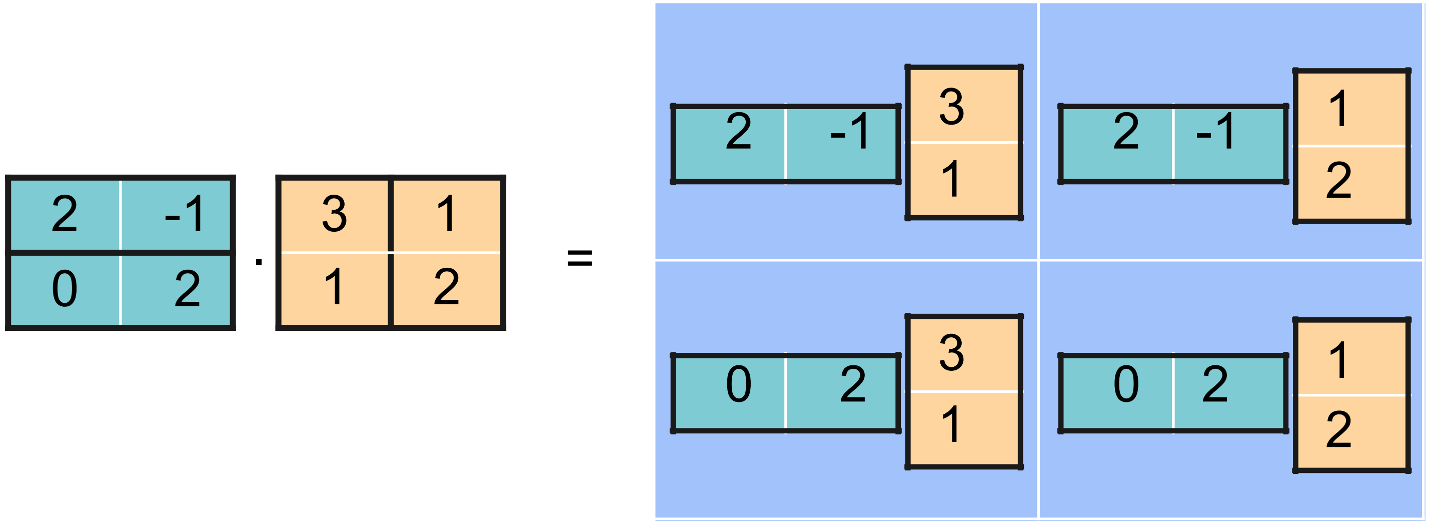
Second　　First

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

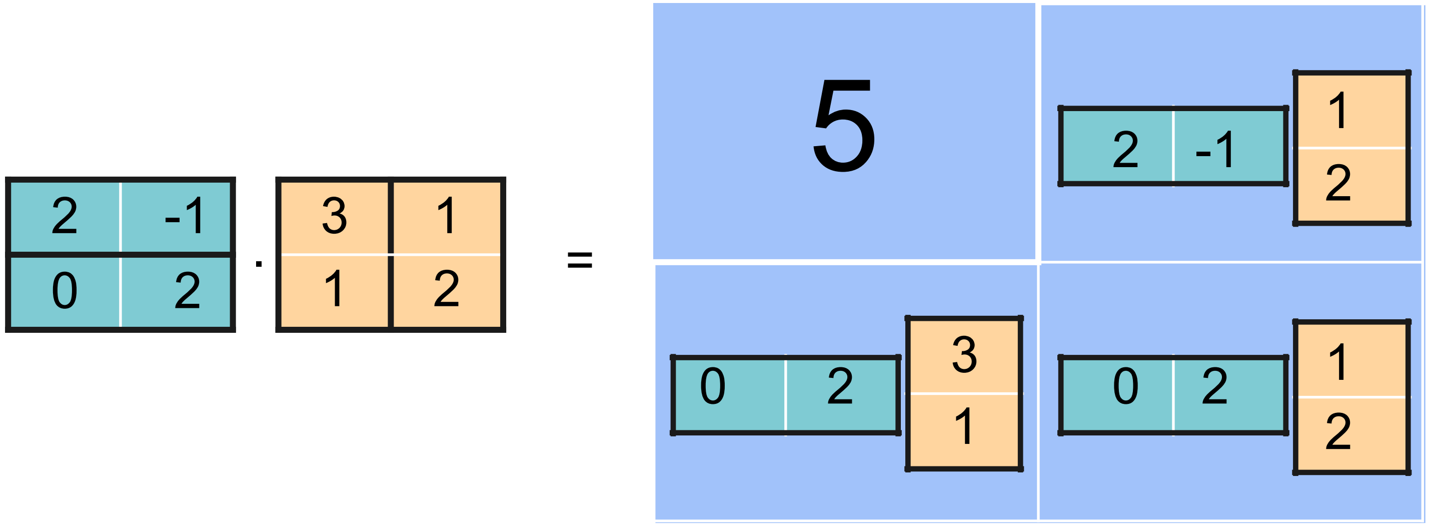
矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} =$$

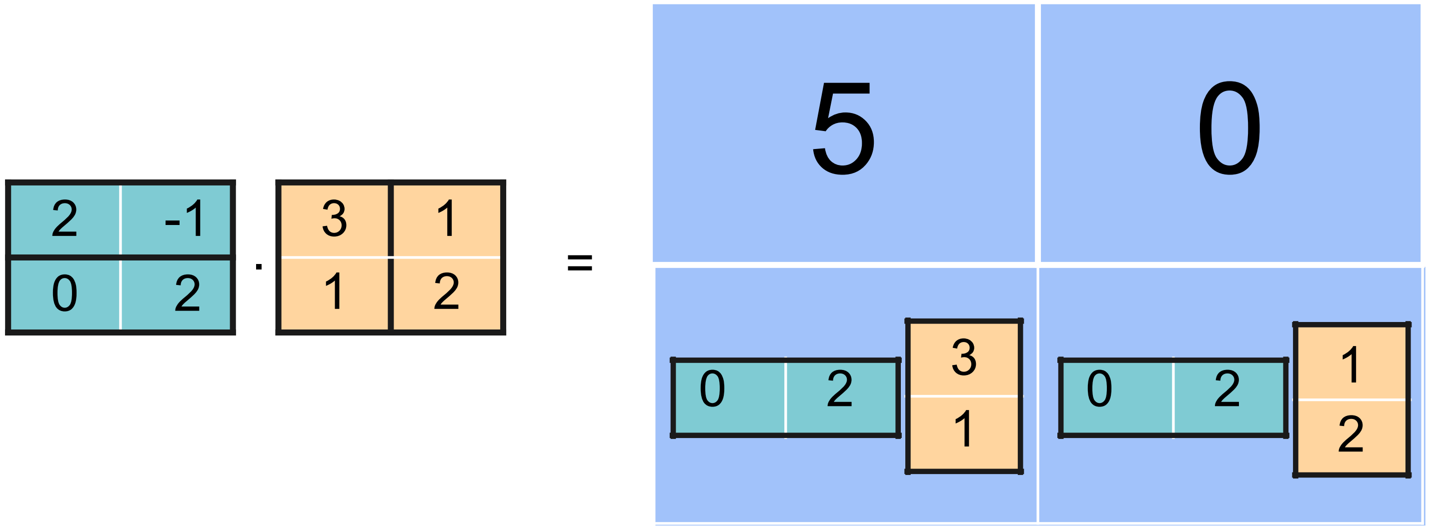
矩阵乘法



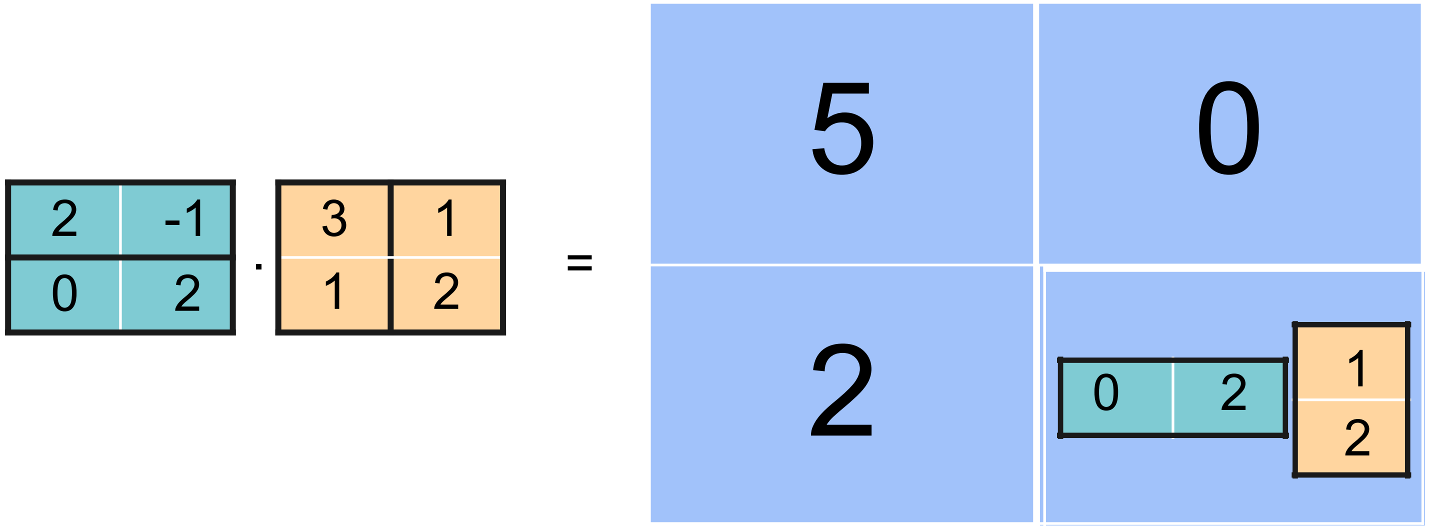
矩阵乘法



矩阵乘法



矩阵乘法



矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

单位矩阵乘法

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

单位矩阵乘法

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

单位矩阵乘法

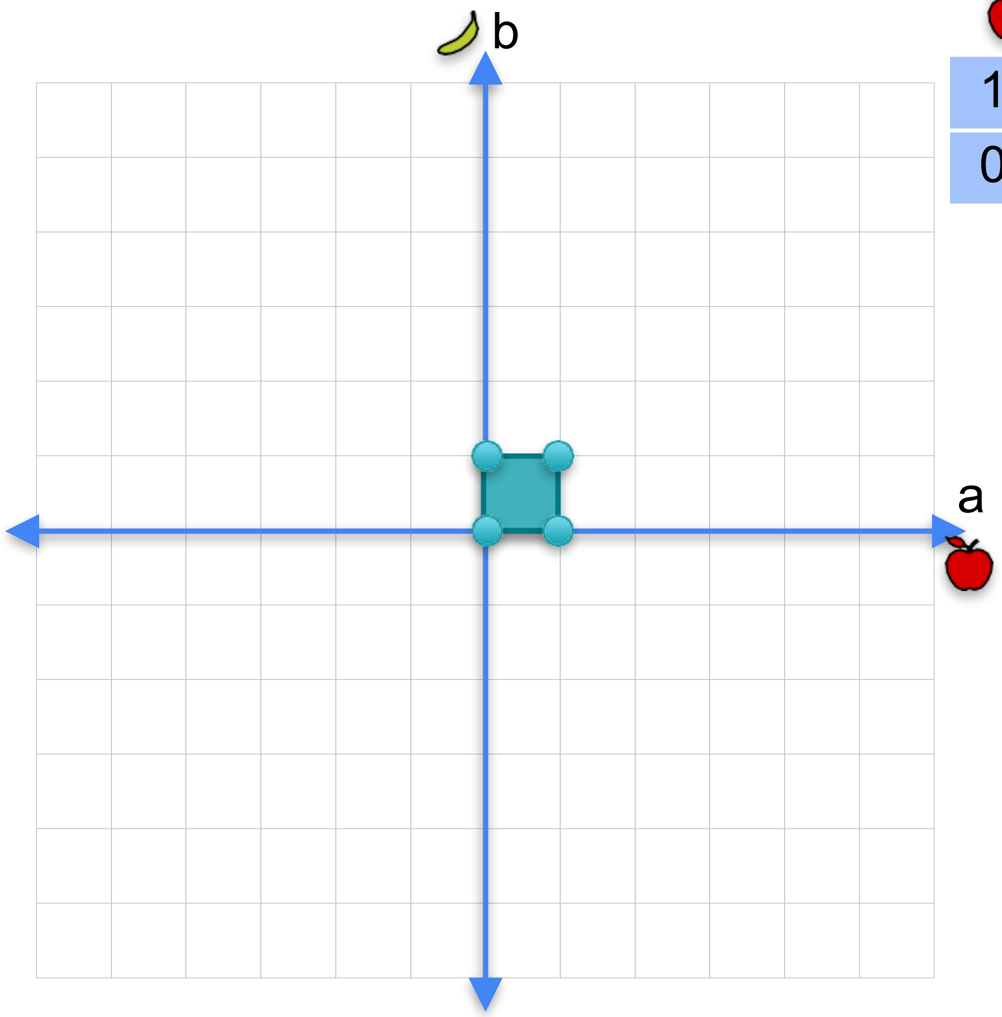
$$
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}
=
\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}
$$

单位矩阵乘法

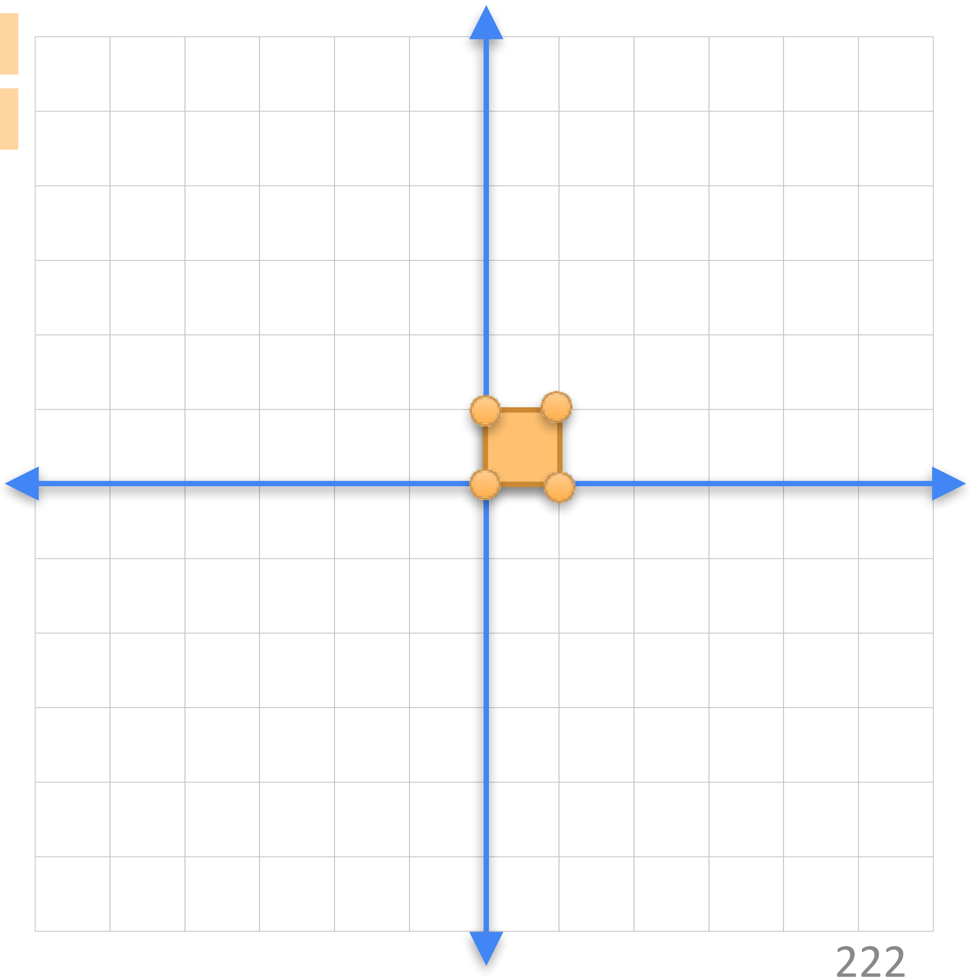
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = a$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

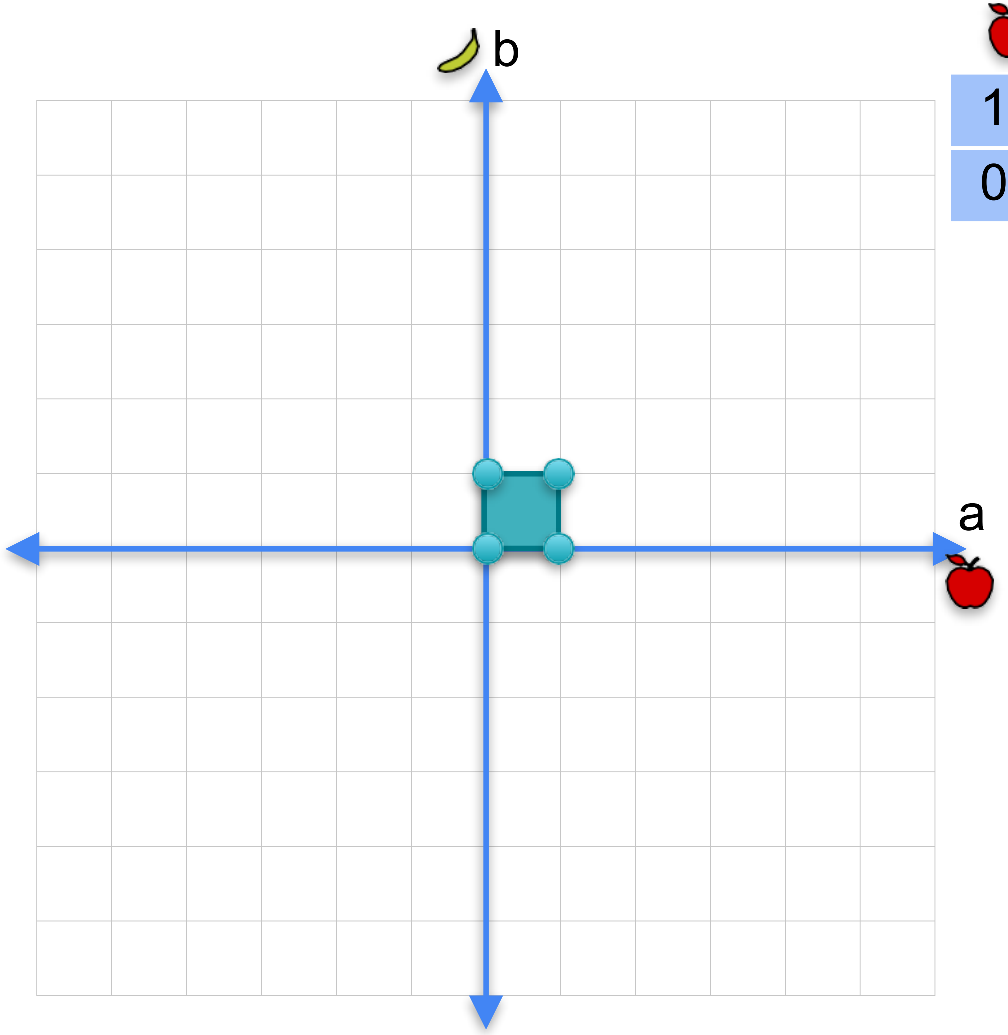
单位矩阵乘法



1	0	x	=	x
0	1	y		y



单位矩阵乘法



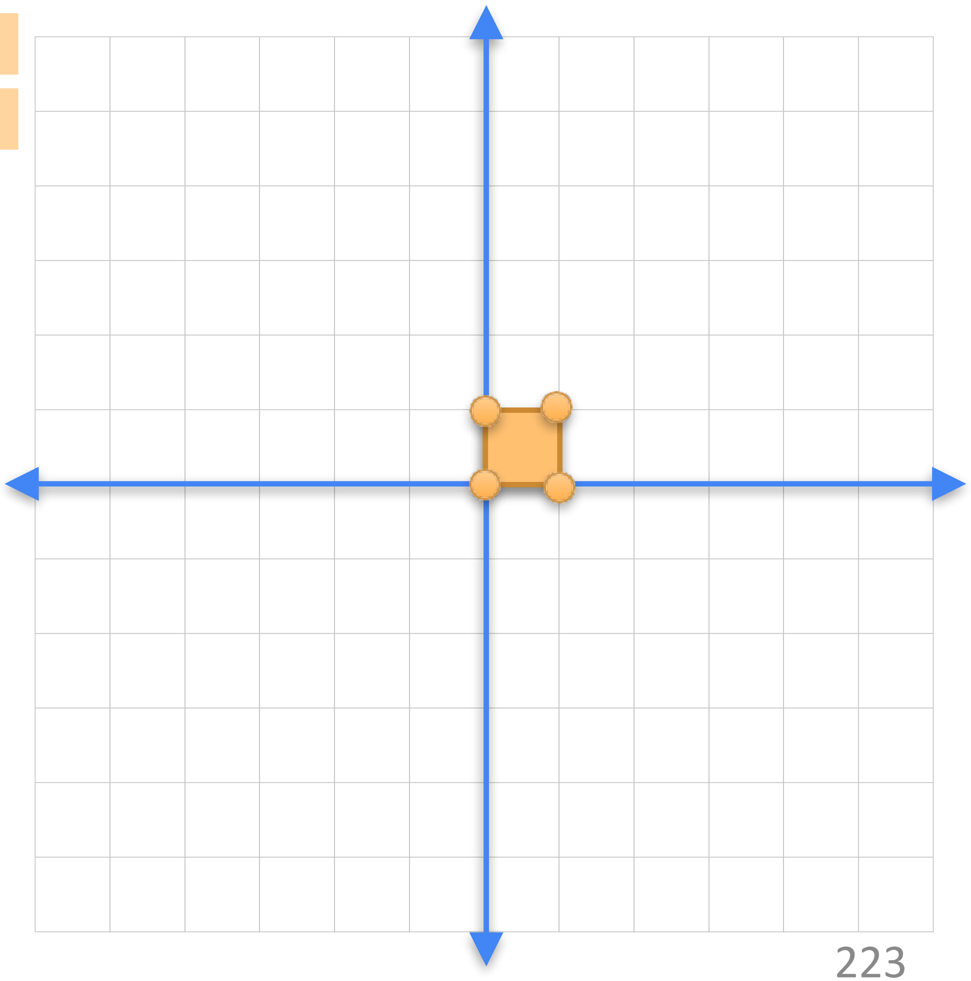
$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}x\\ y\end{bmatrix}$$

(0,0) → (0,0)

(1,0) → (1,0)

(0,1) → (0,1)

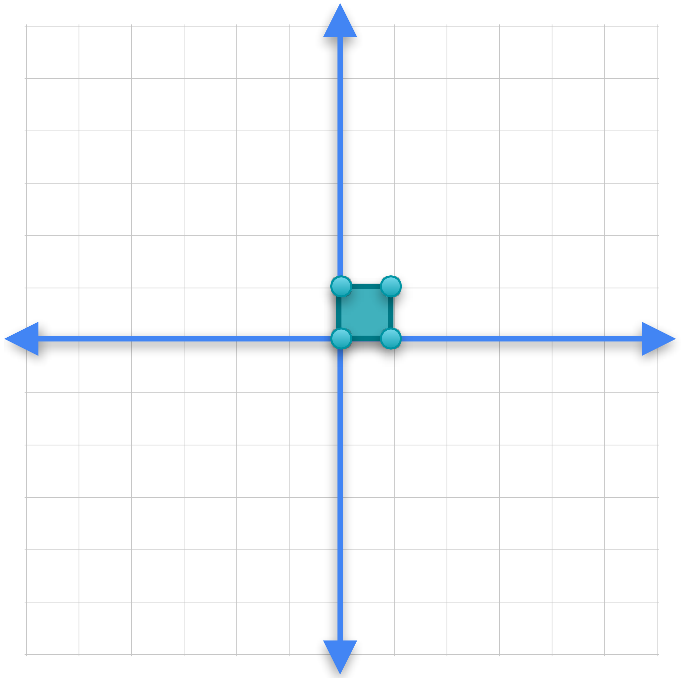
(1,1) → (1,1)



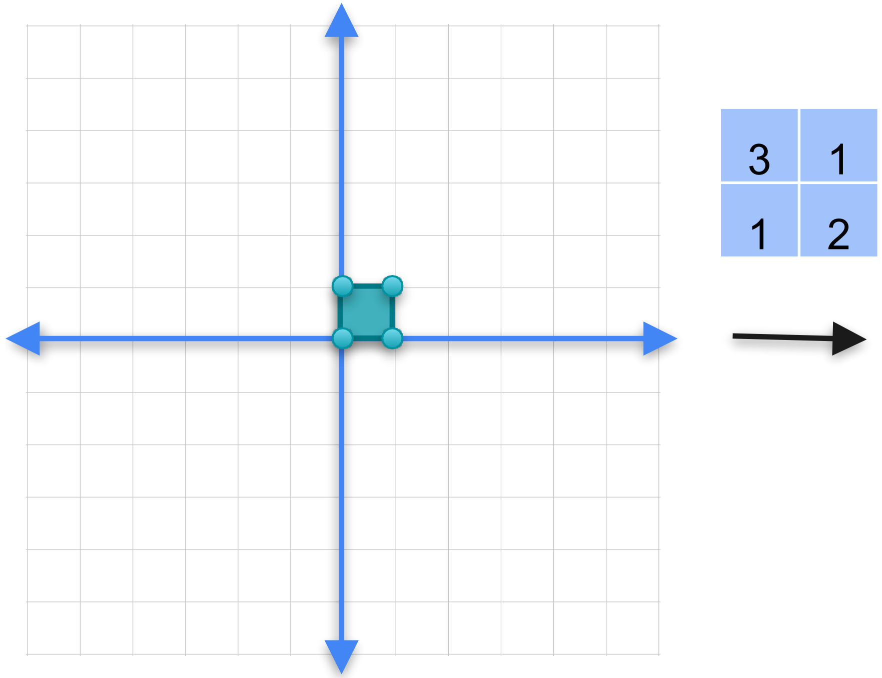
目录

逆矩阵

逆矩阵



逆矩阵



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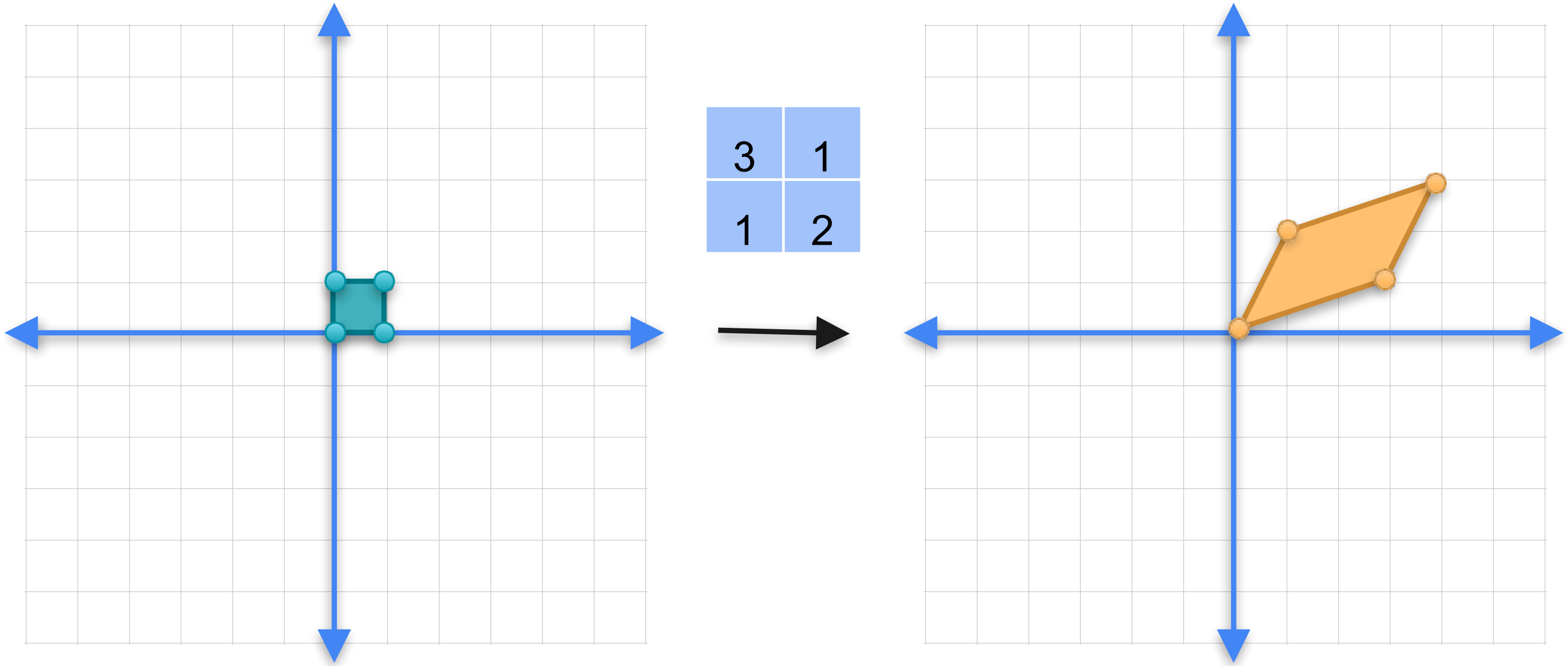

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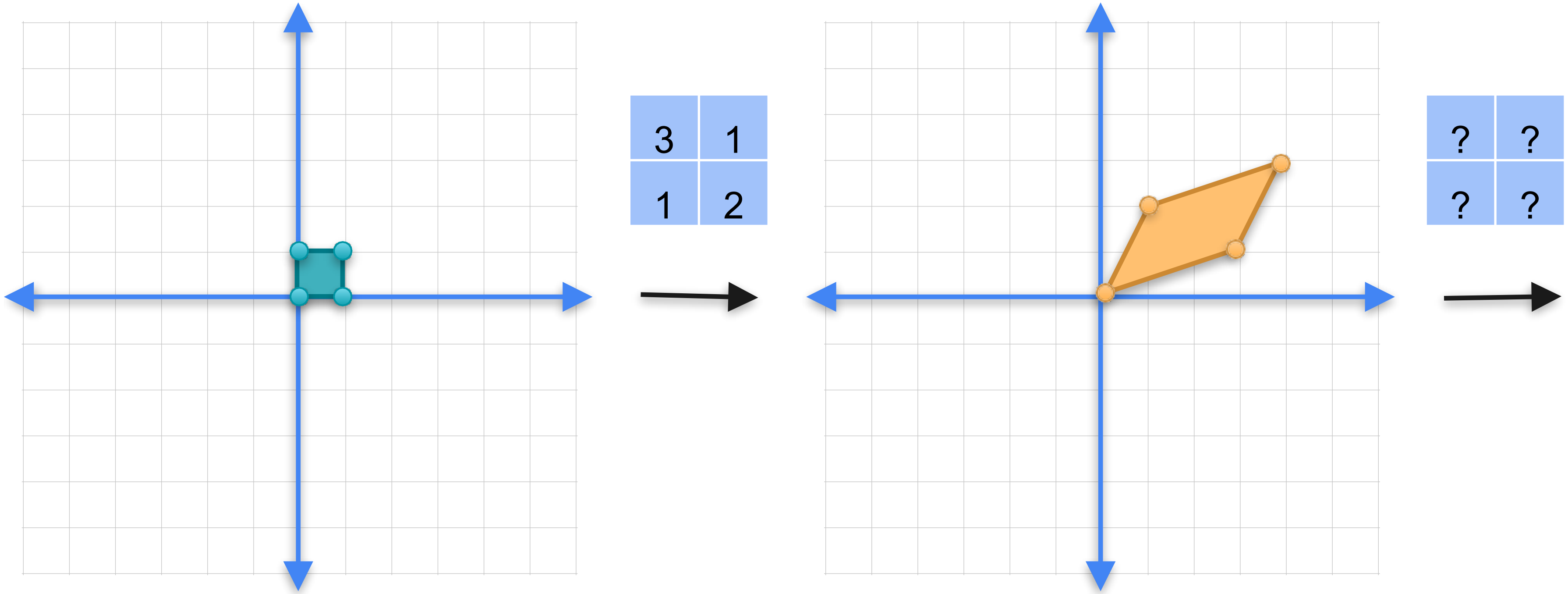
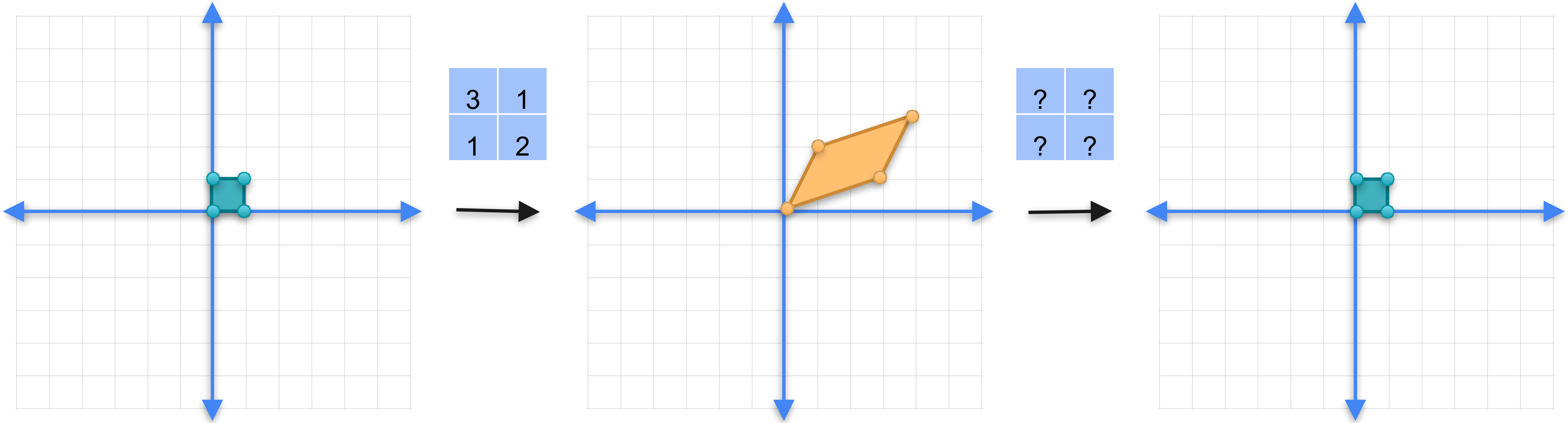


逆矩阵



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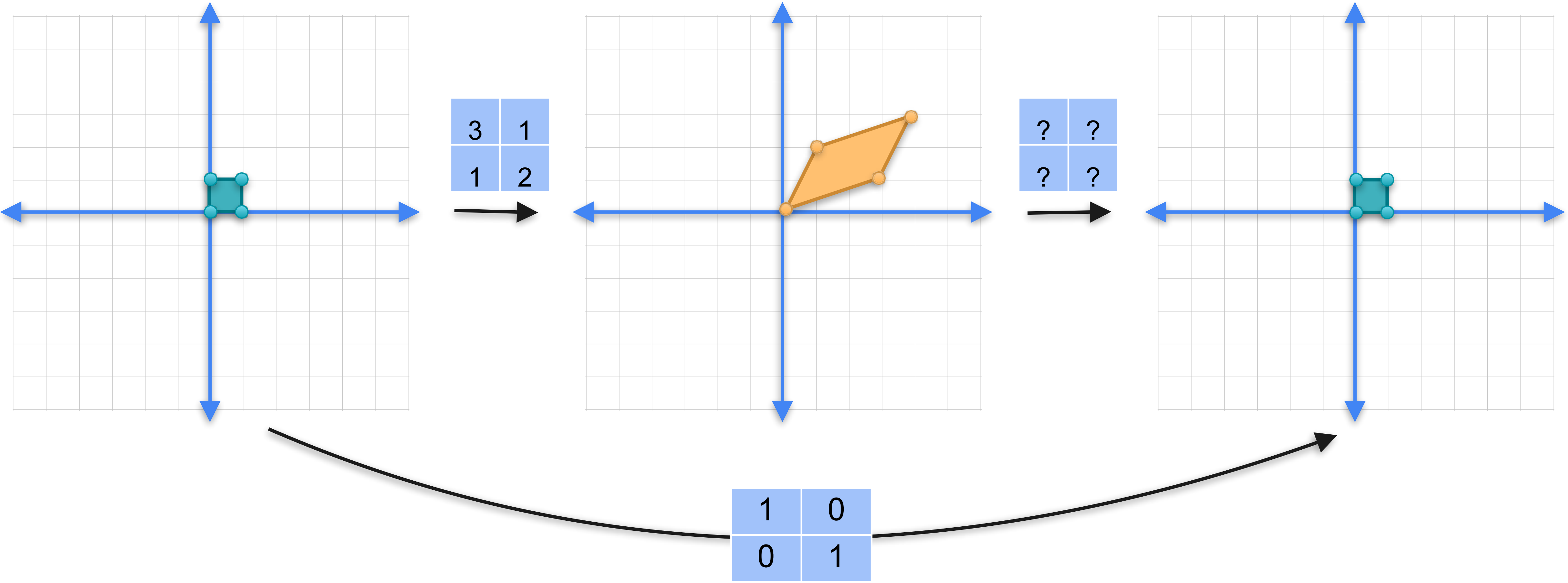

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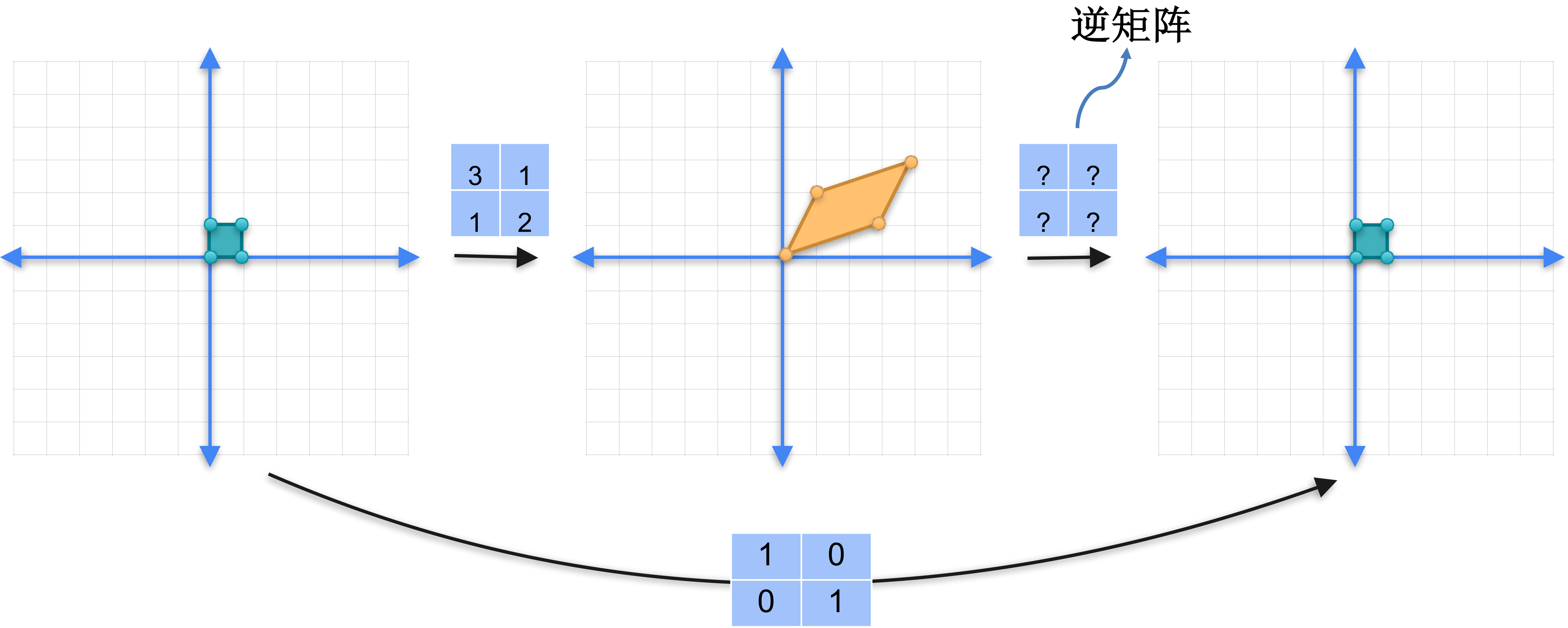

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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如果存在另一个方阵B，使得$AB = I_n$成立，则方阵A称为可逆的。这时候可以证明也$BA = I_n$成立，可将矩阵B称为A的逆矩阵。一个矩阵A的逆矩阵如果存在的话，就是唯一的，通常记作A^{-1}

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详细计算方法可以参考Strang G. Introduction to linear algebra[M]. Wellesley-Cambridge Press, 2022.

目录


AI , ML , DL , RL
MATRIX
MULTIPLICATION

用矩阵重写西瓜分类模型

用矩阵重写西瓜分类模型

数据

色泽（x_1）	根蒂（x_2）	敲声（x_3）	好瓜（y）
0（青绿）	0（蜷缩）	0（浊响）	0（是）
1（乌黑）	0（蜷缩）	0（浊响）	0（是）
0（青绿）	1（硬挺）	1（清脆）	1（否）
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$$y = ax_1 + bx_2 + cx_3 + d$$

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$\hat{y}$ 是预测值

代入x_1，x_2，x_3
- 若$y \le 0$，$\hat{y} = 0$
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用矩阵重写西瓜分类模型

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$$y = \mathbf{x}\mathbf{w}^T$$

延伸阅读（非必需）

- 吴恩达《机器学习数学基础（线性代数/微积分）》：
  https://www.coursera.org/specializations/mathematics-for-machine-learning-and-data-science
- 线性代数的本质（Essense of Linear Algebra）系列
  https://www.bilibili.com/video/BV1Ys411k7yQ

中国科学技术大学
University of Science and Technology of China

谢谢！

Slides are inspired by DeepLearning.AI