



《人工智能数学原理与算法》

第2章：机器学习基础

2.2 线性代数基础

冯福利

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- 01 向量及其属性
- 02 矩阵及其属性
- 03 线性变换和矩阵乘法
- 04 逆矩阵
- 05 机器学习模型实例

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02 矩阵及其属性

03 线性变换和矩阵乘法

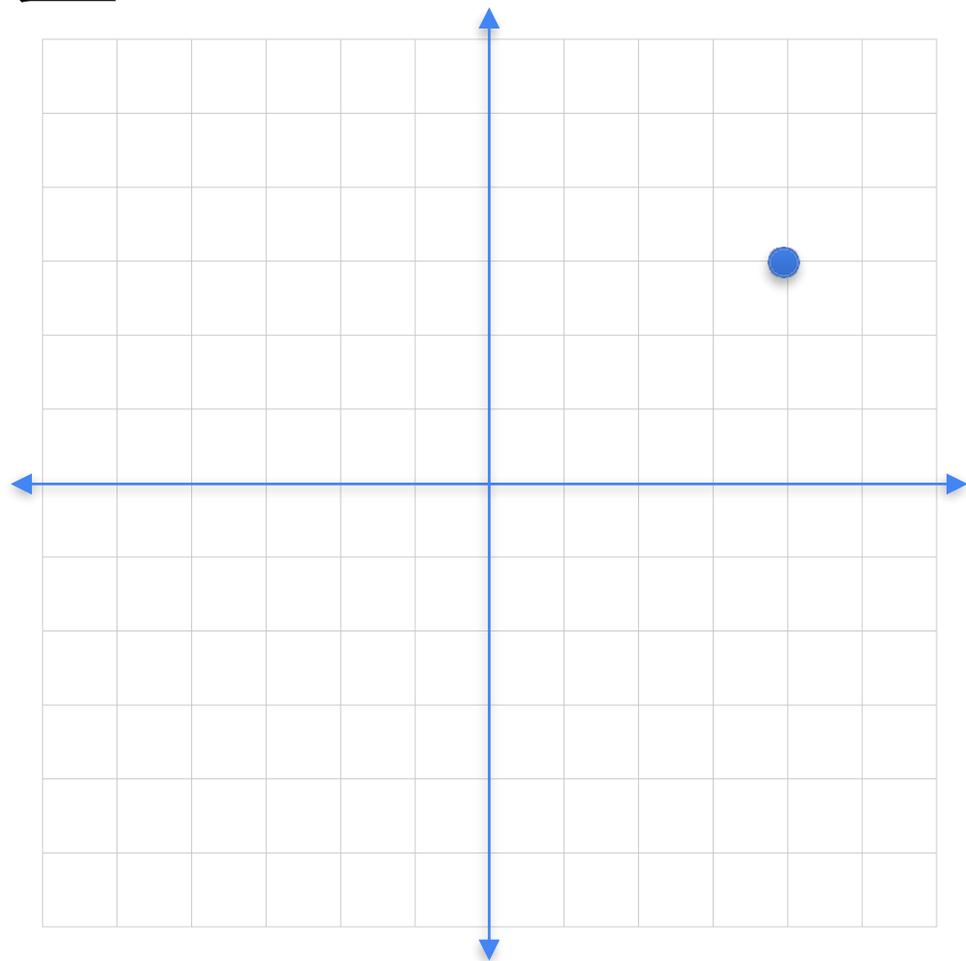
04 逆矩阵

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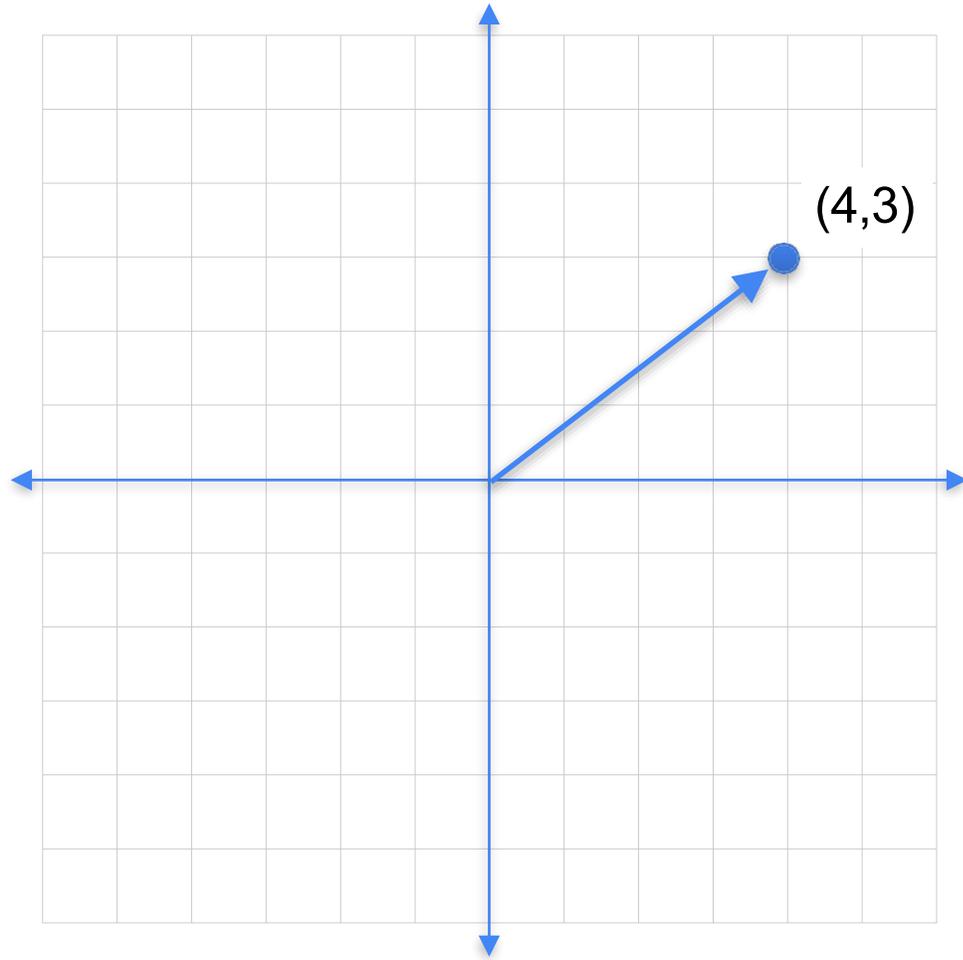
向量



二维

1. 向量及其属性

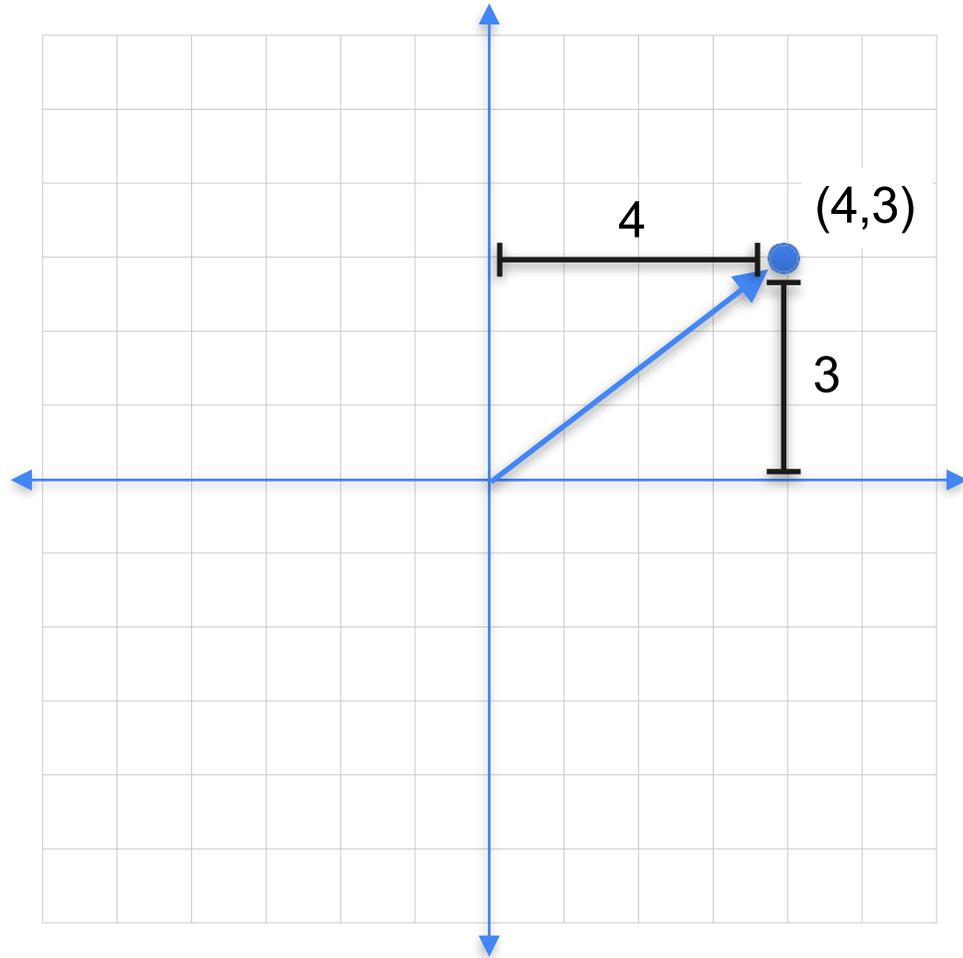
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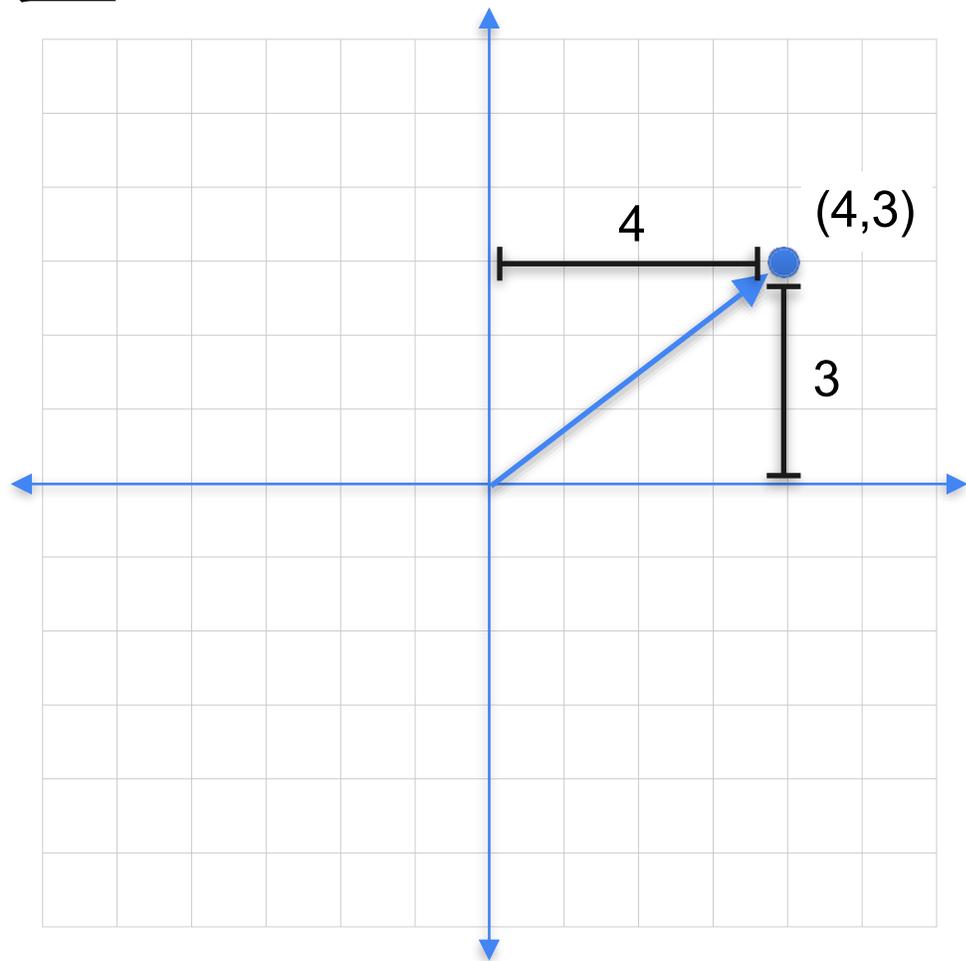
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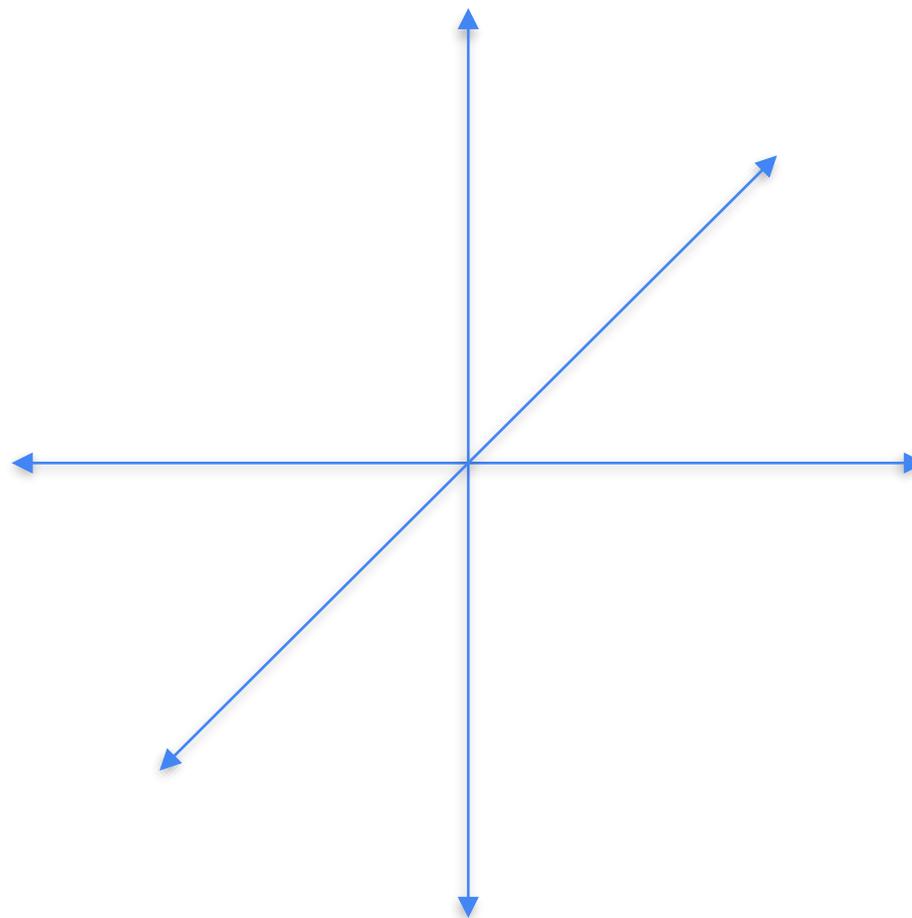
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向量



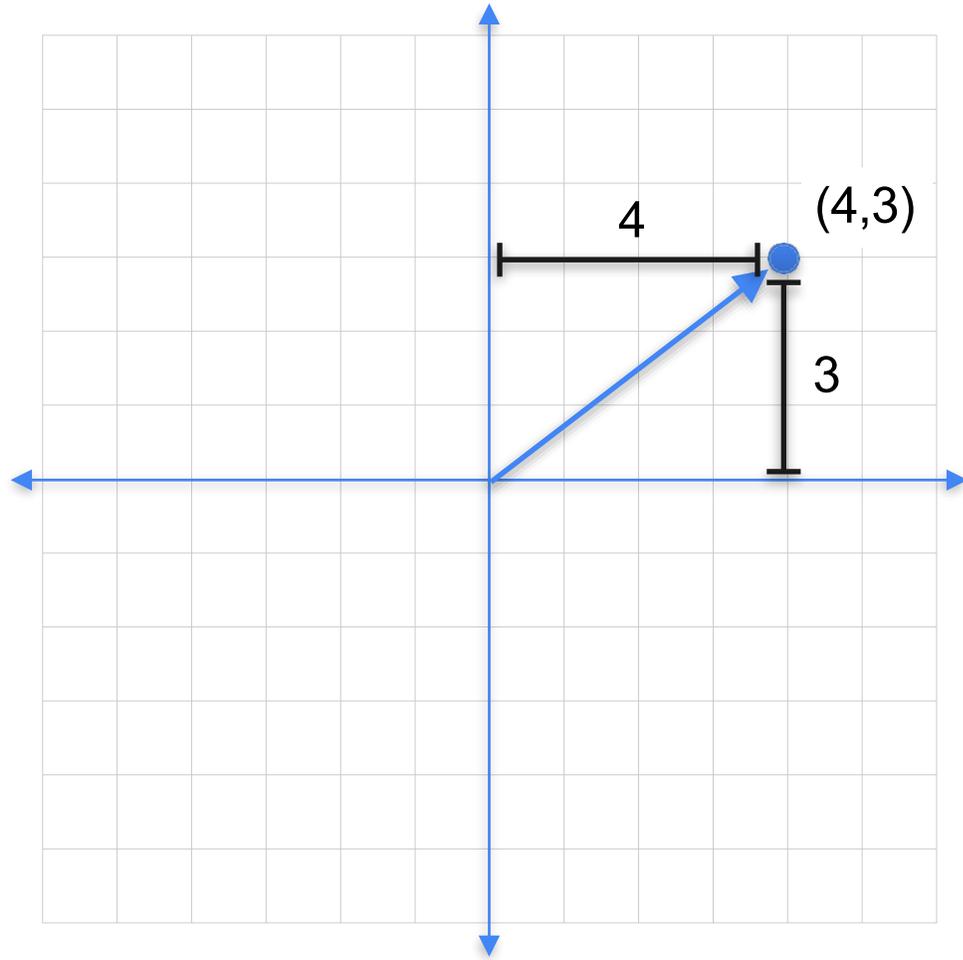
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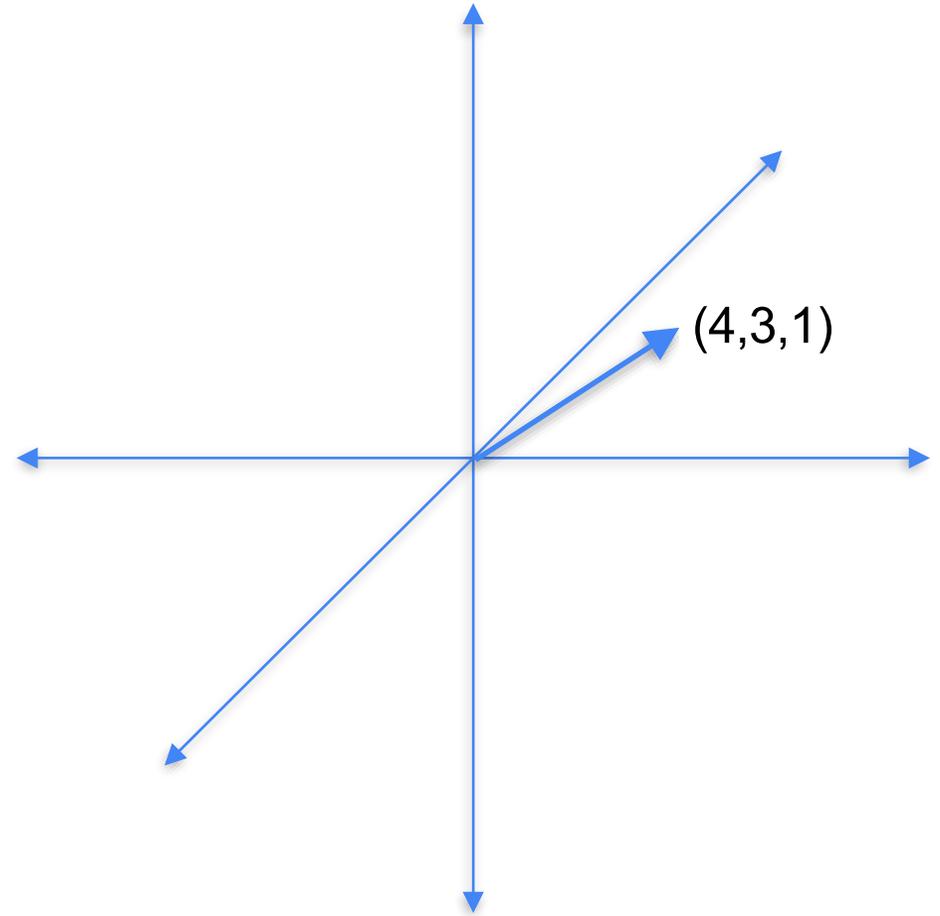
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向量



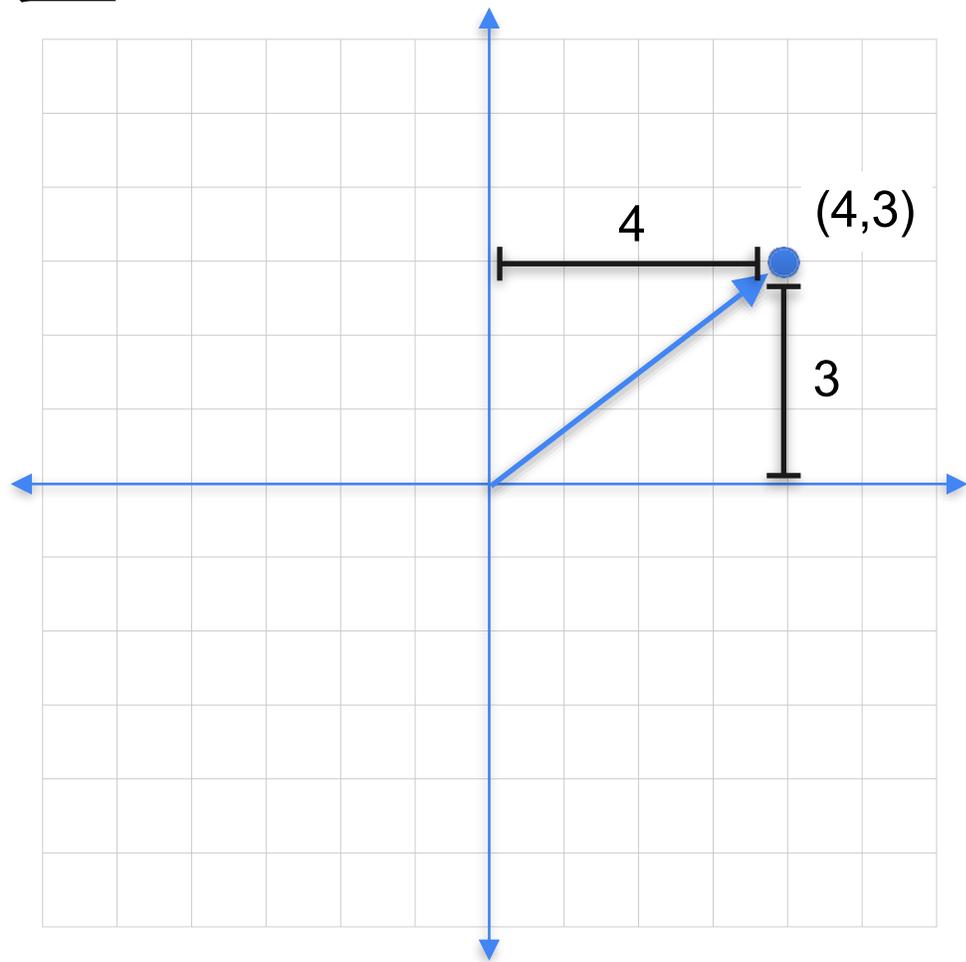
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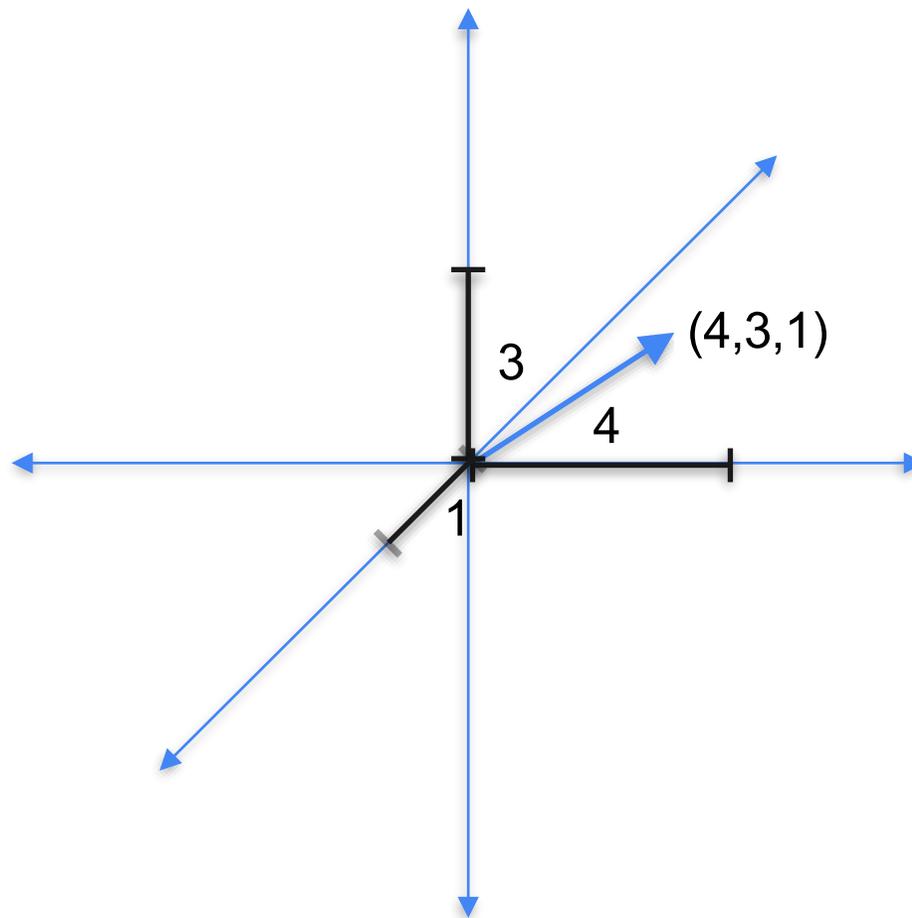
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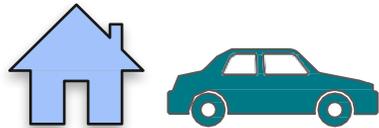
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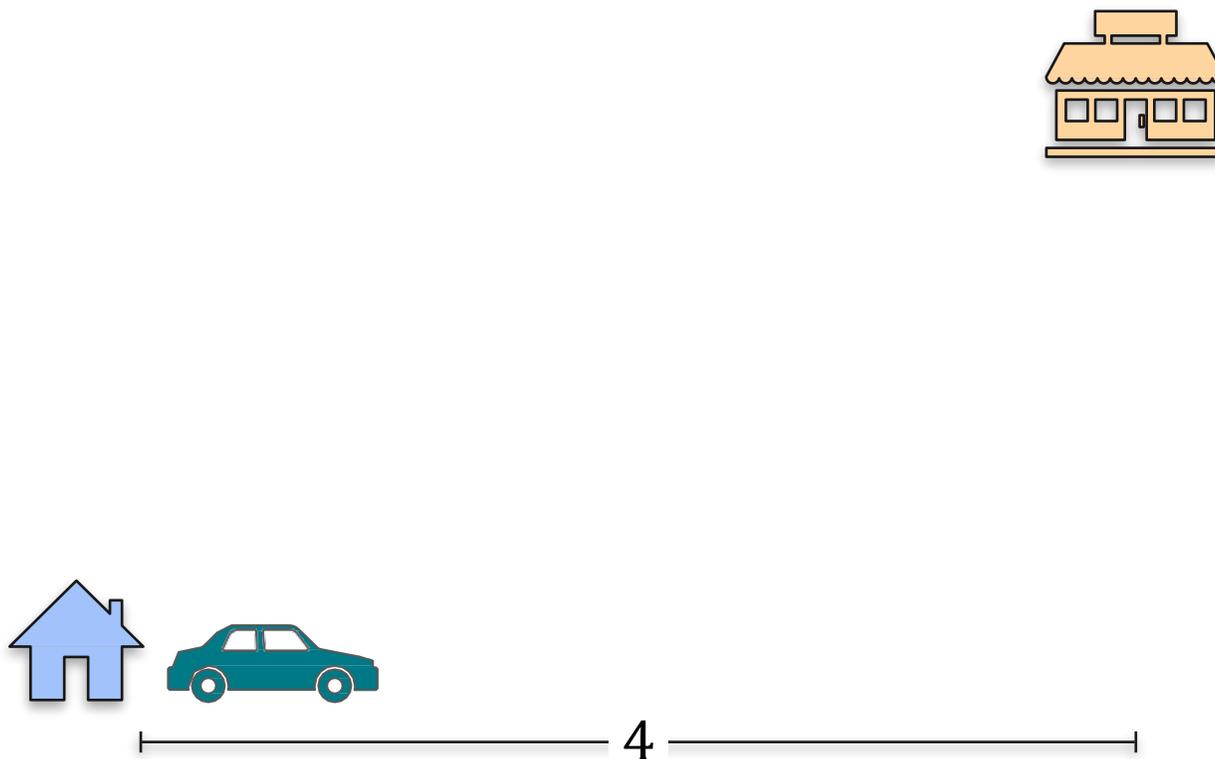
1. 向量及其属性

如何从下面的点走到上面的点？



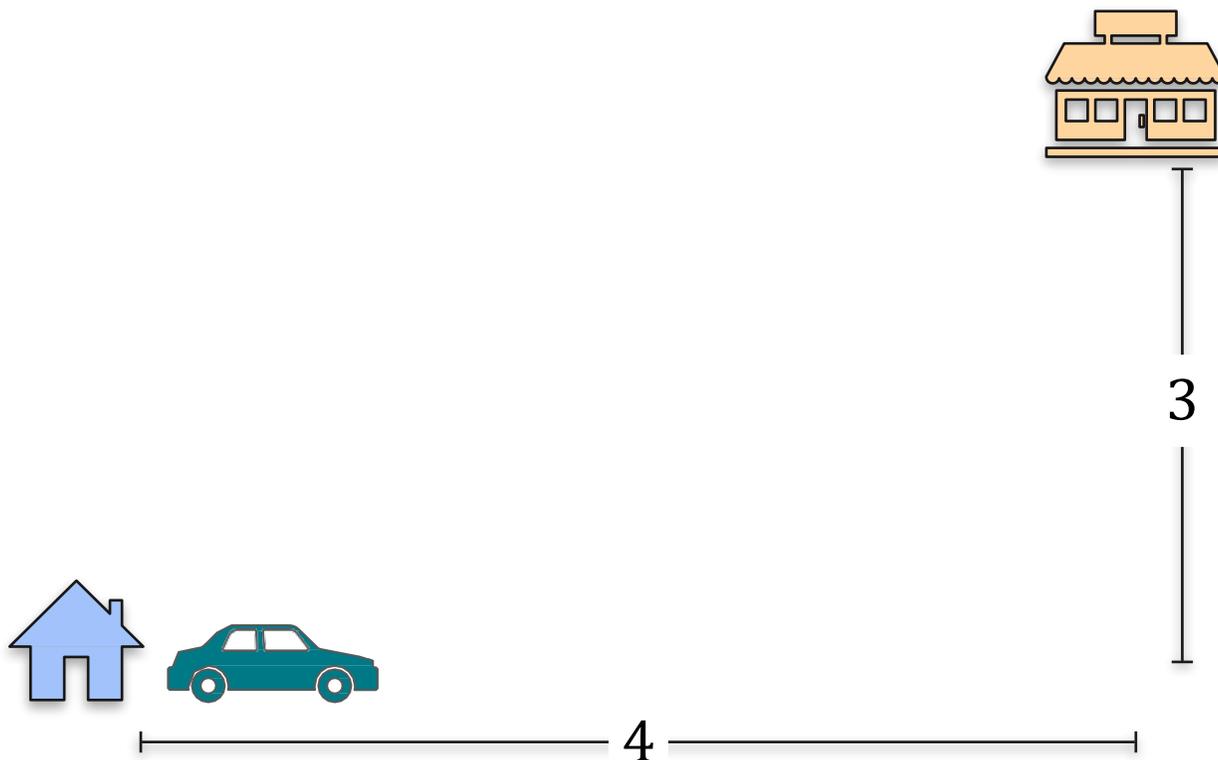
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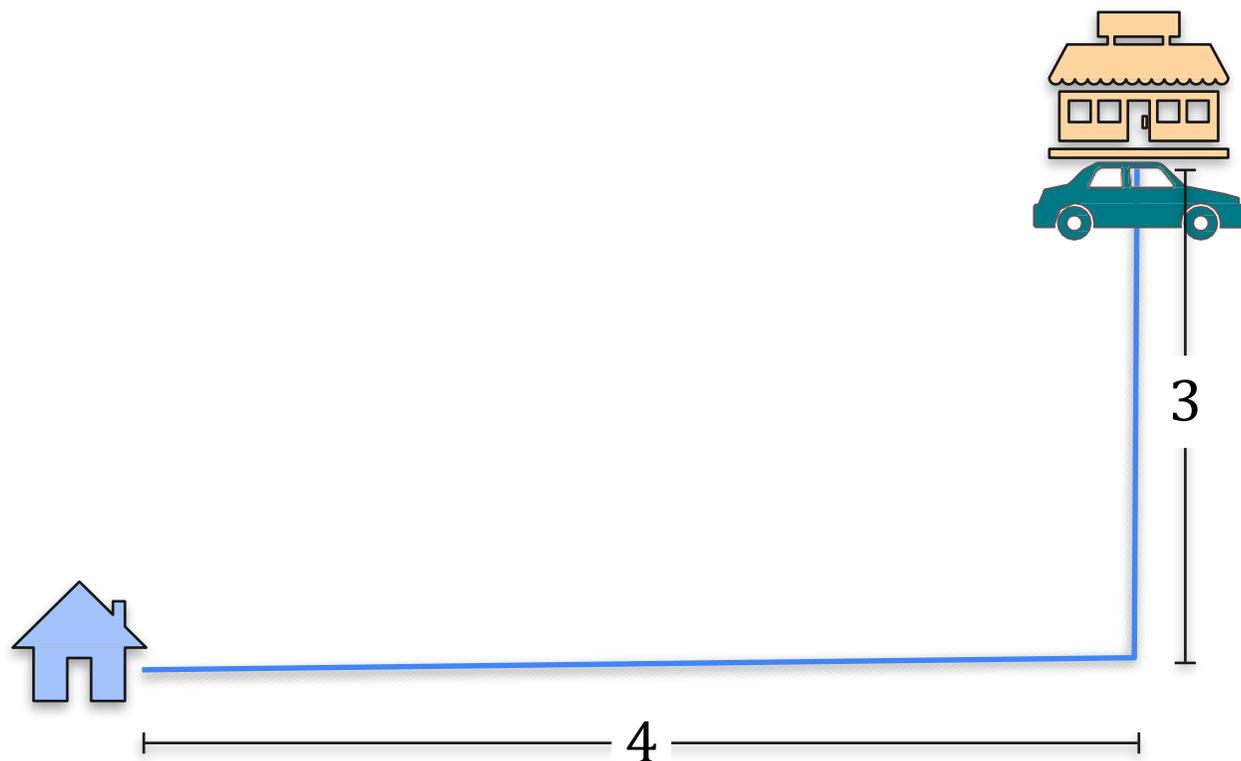
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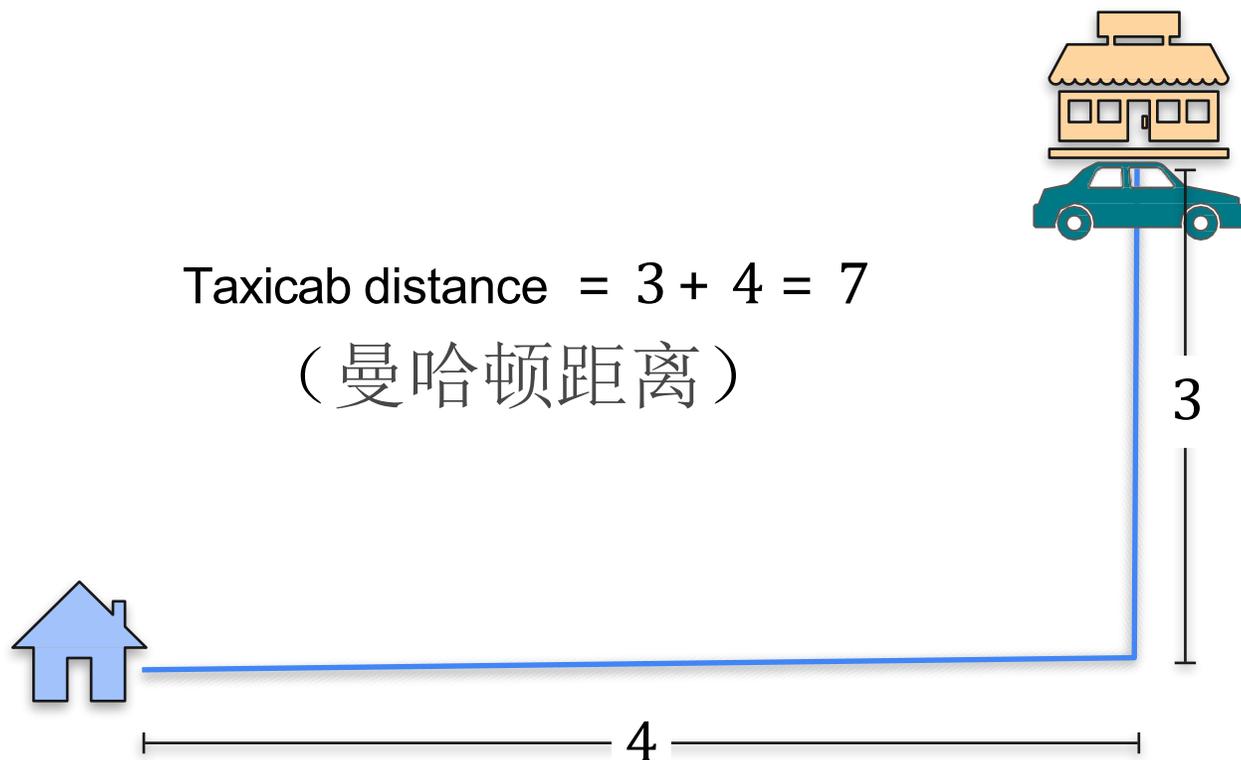
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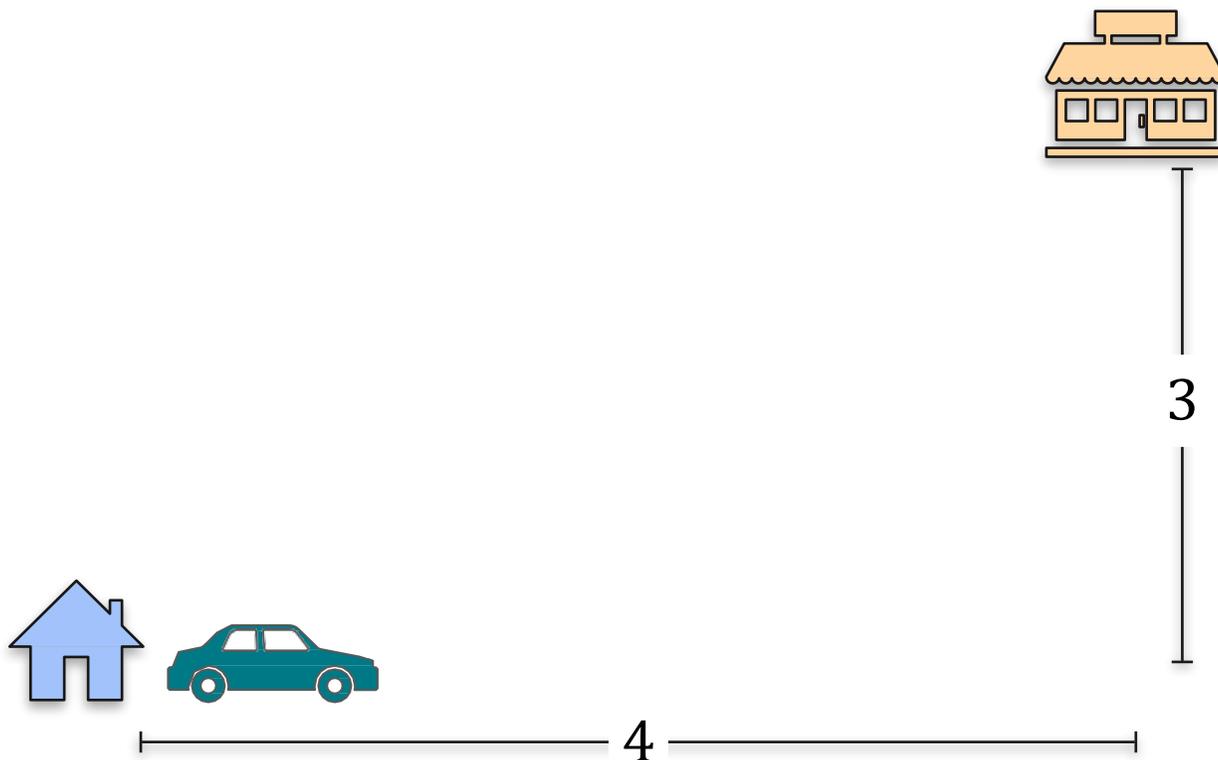
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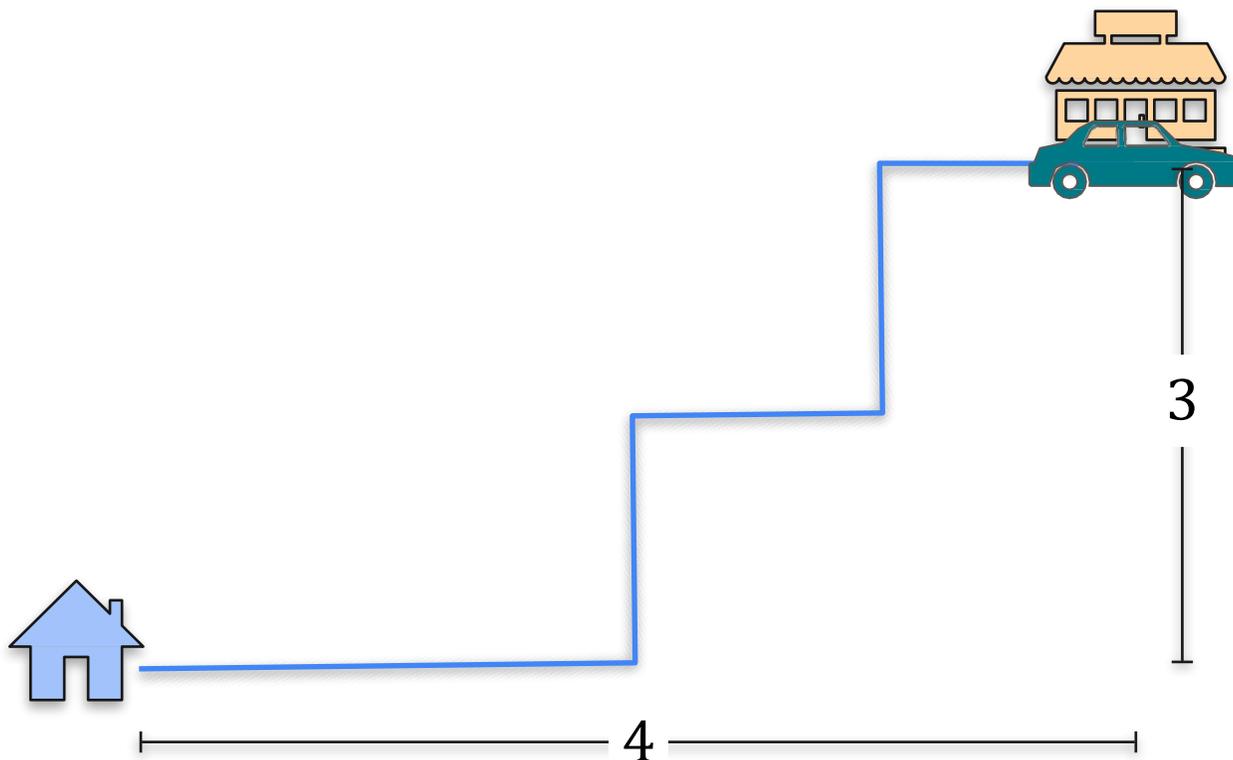
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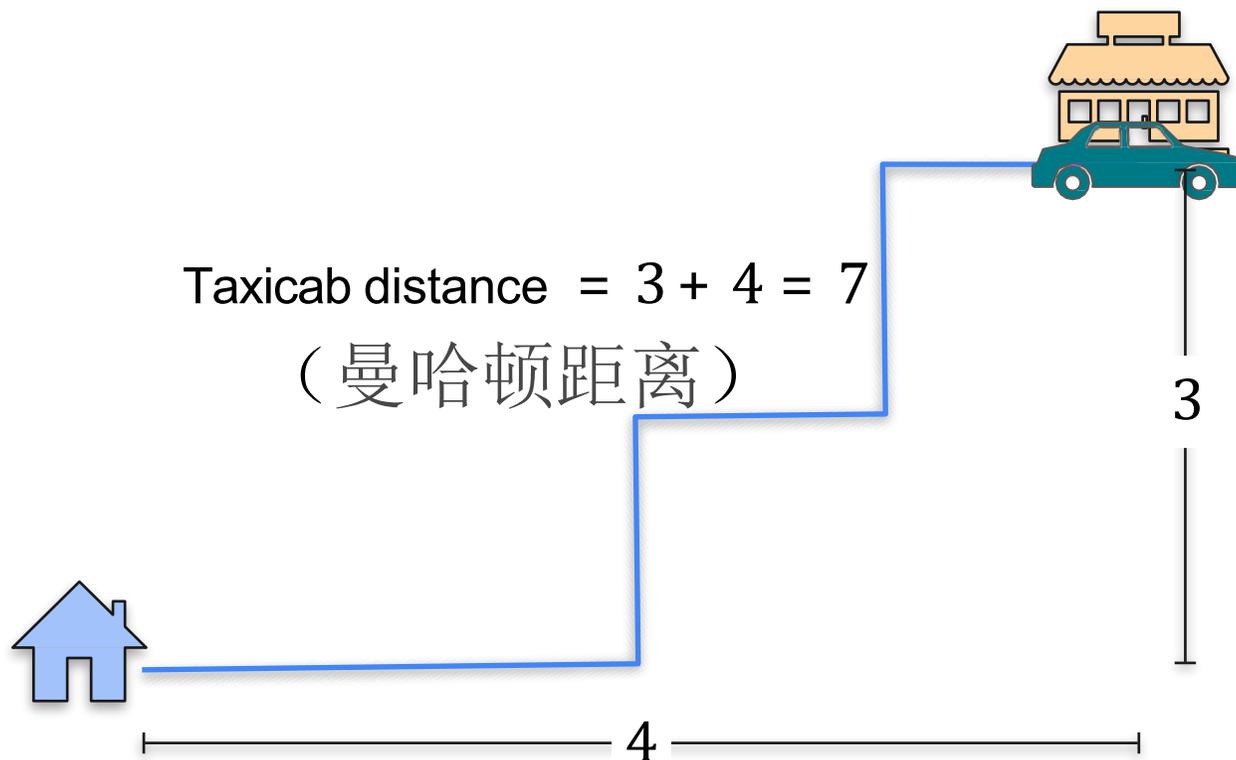
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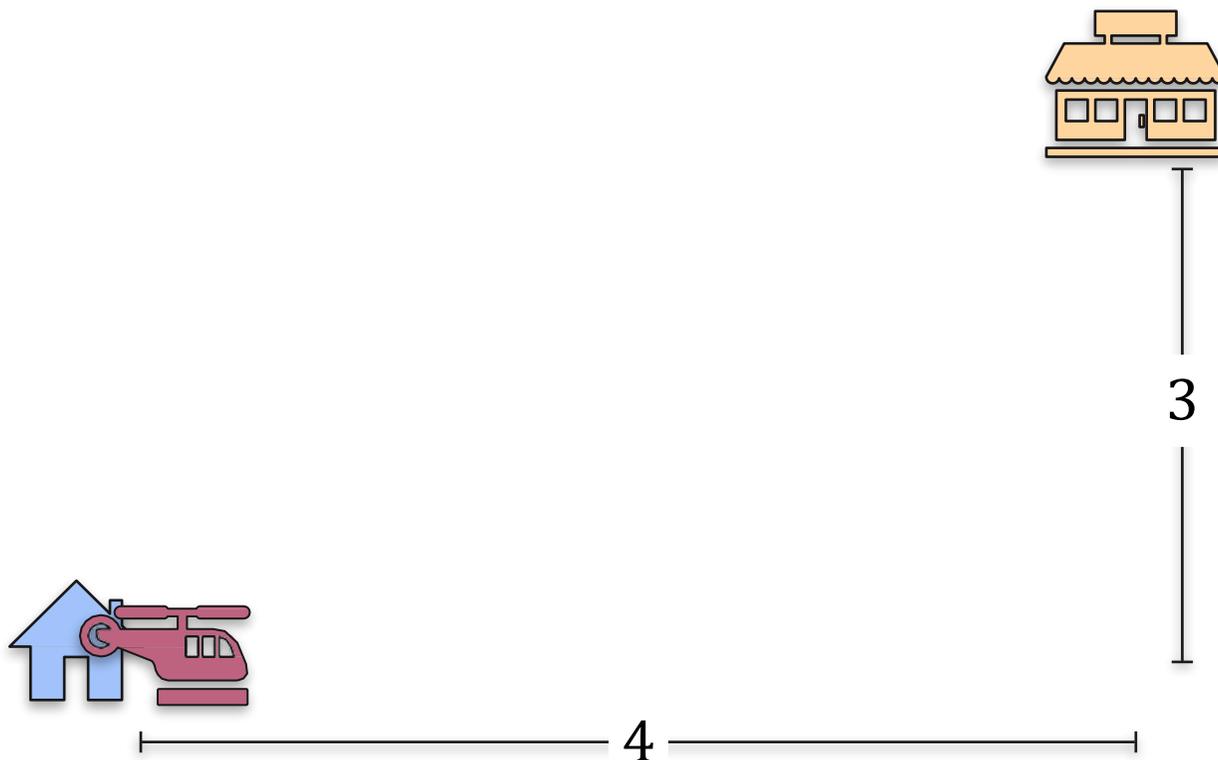
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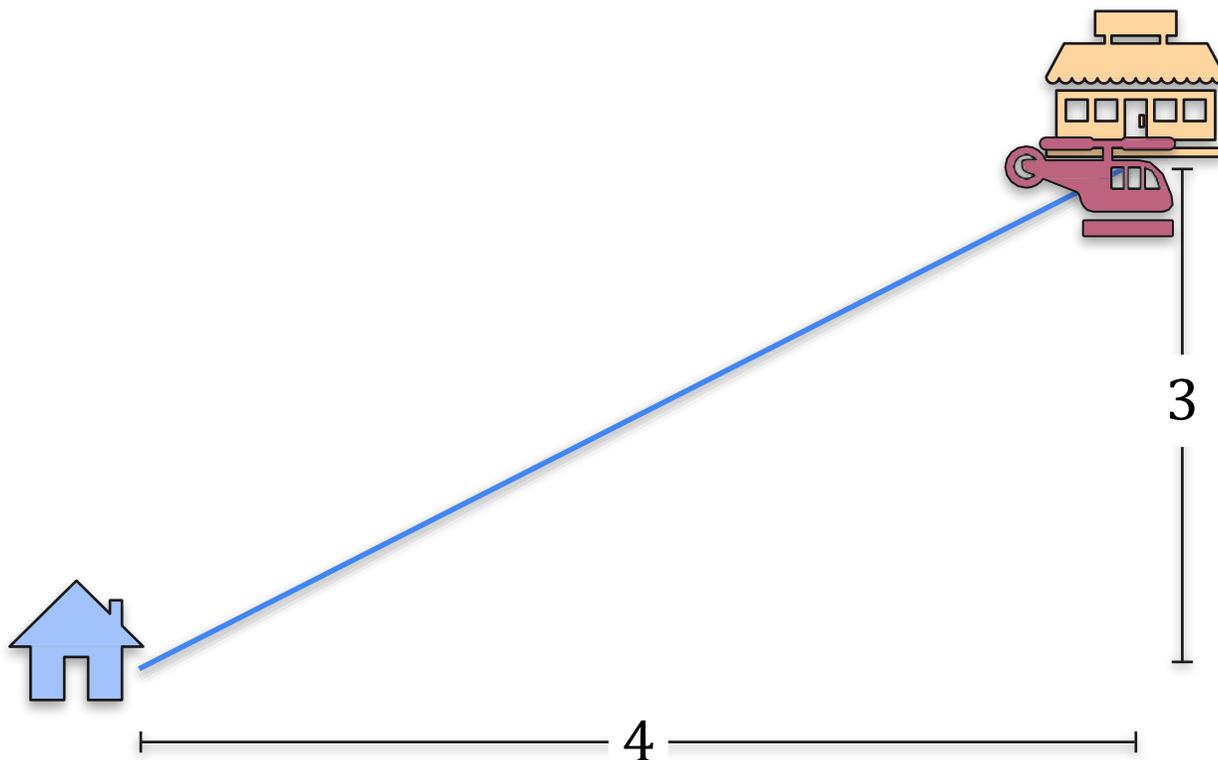
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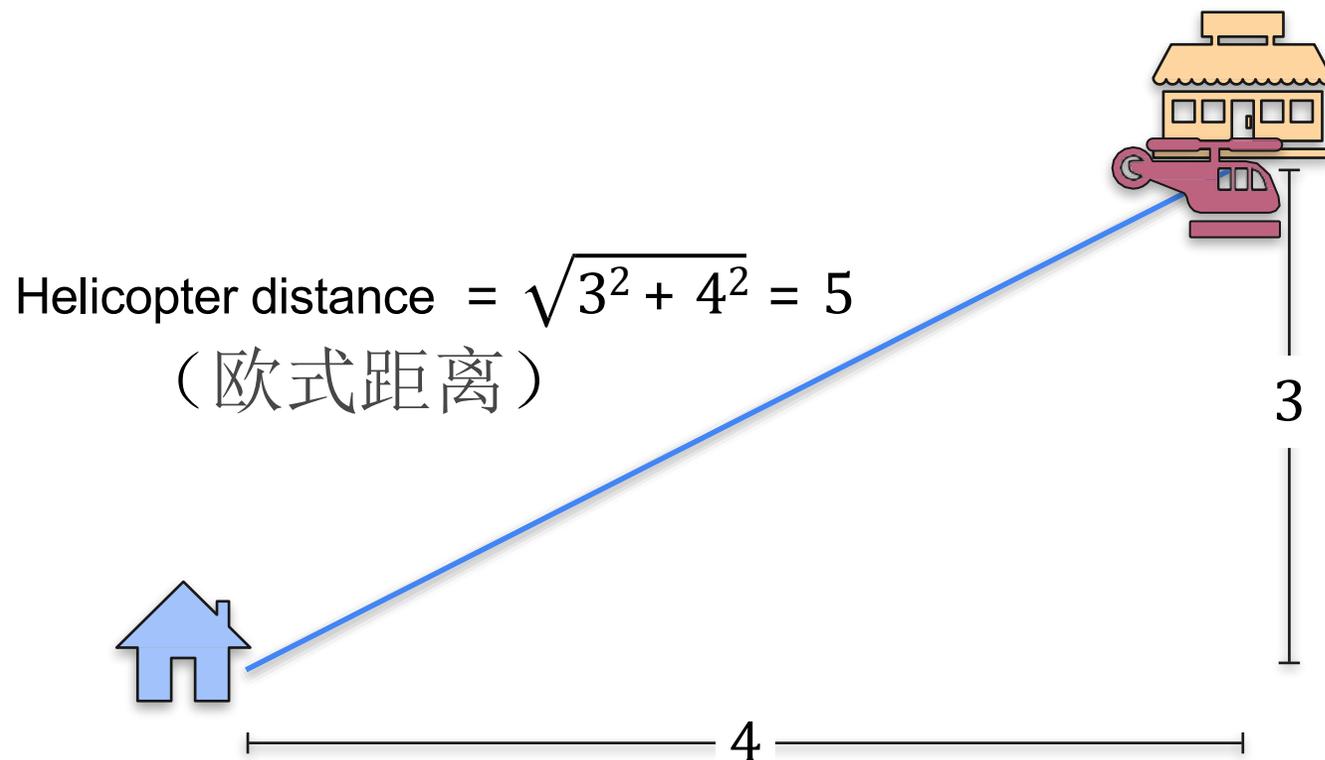
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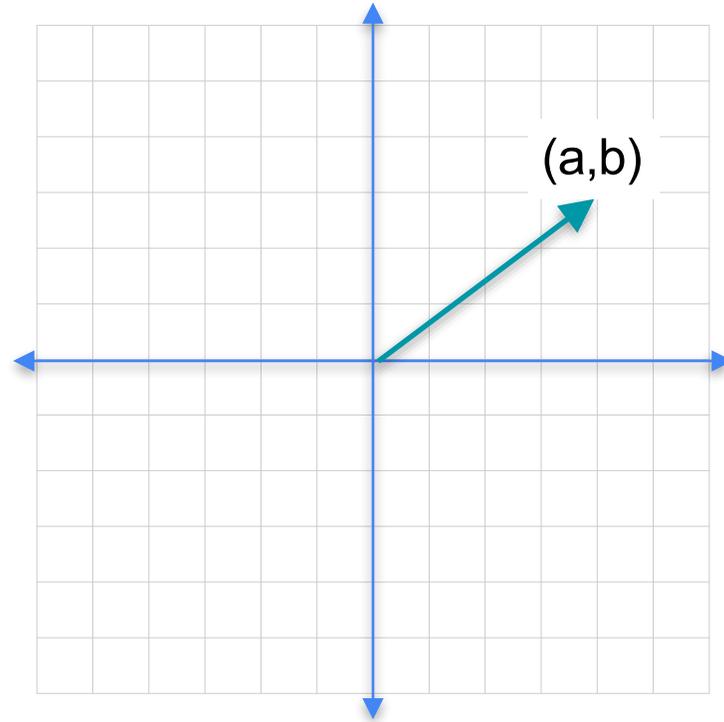
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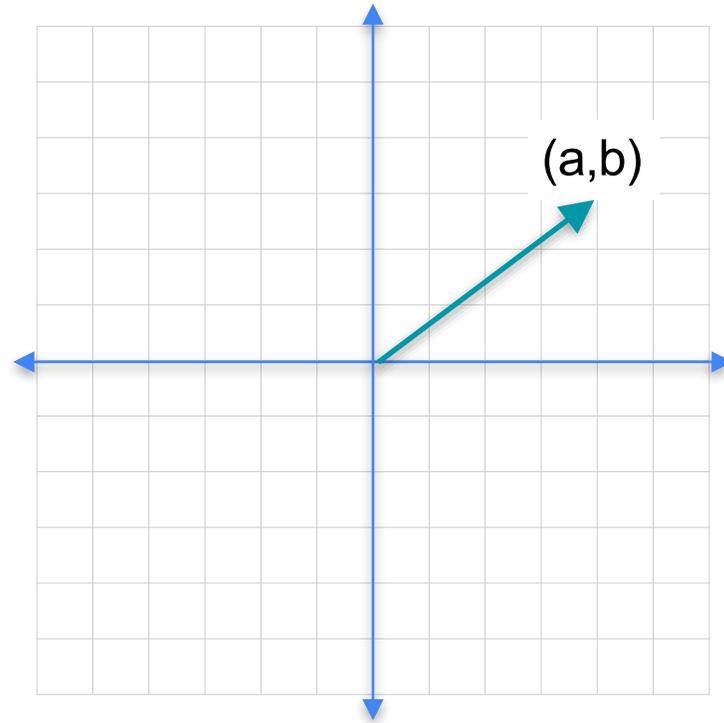
1. 向量及其属性

范数 (Norm)



1. 向量及其属性

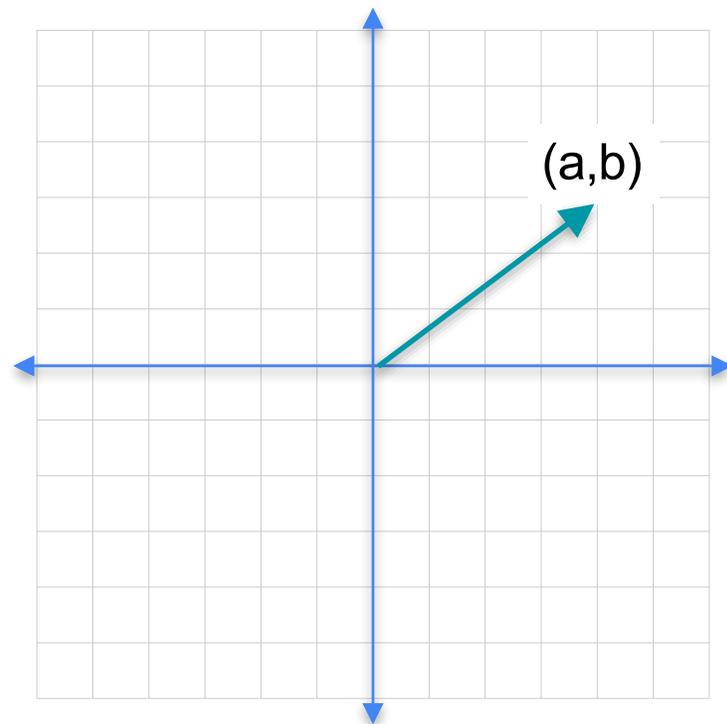
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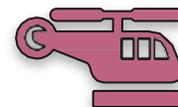
$$\text{L1-范数} = |(a, b)|_1 = |a| + |b|$$

1. 向量及其属性

范数 (Norm)



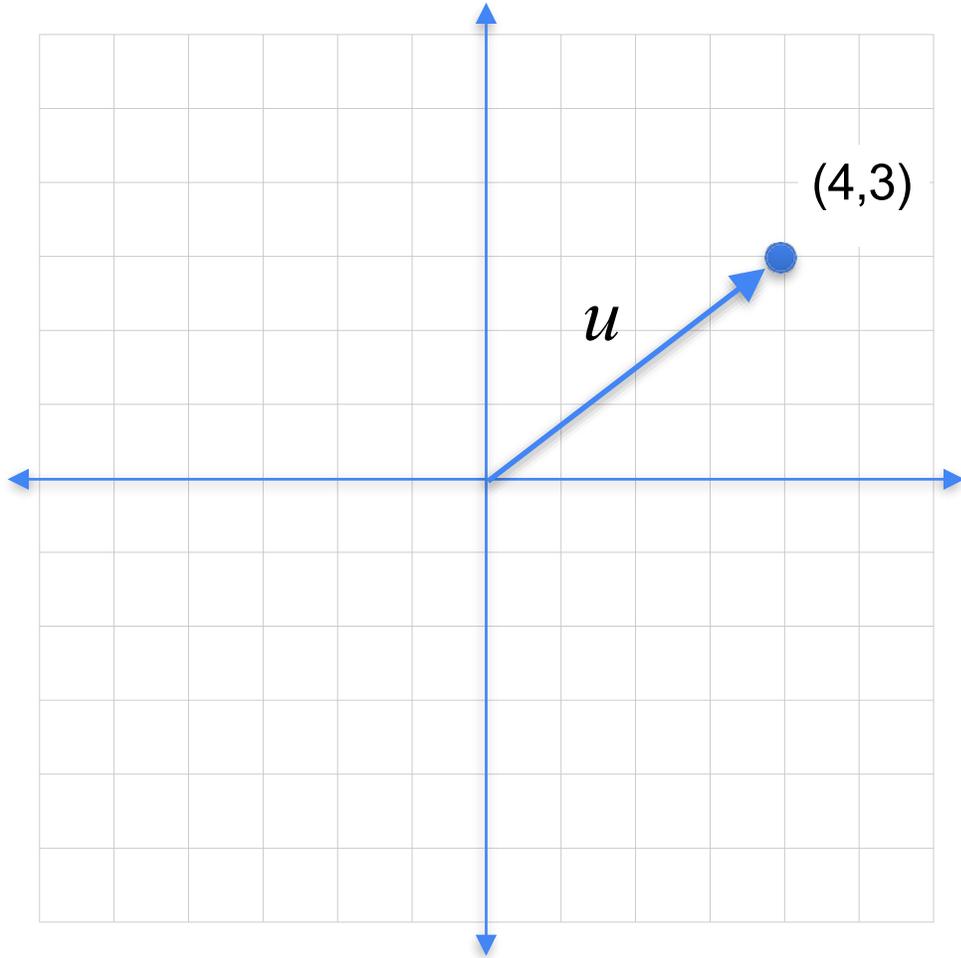
$$\text{L1-范数} = |(a, b)|_1 = |a| + |b|$$



$$\text{L2-范数} = |(a, b)|_2 = \sqrt{a^2 + b^2}$$

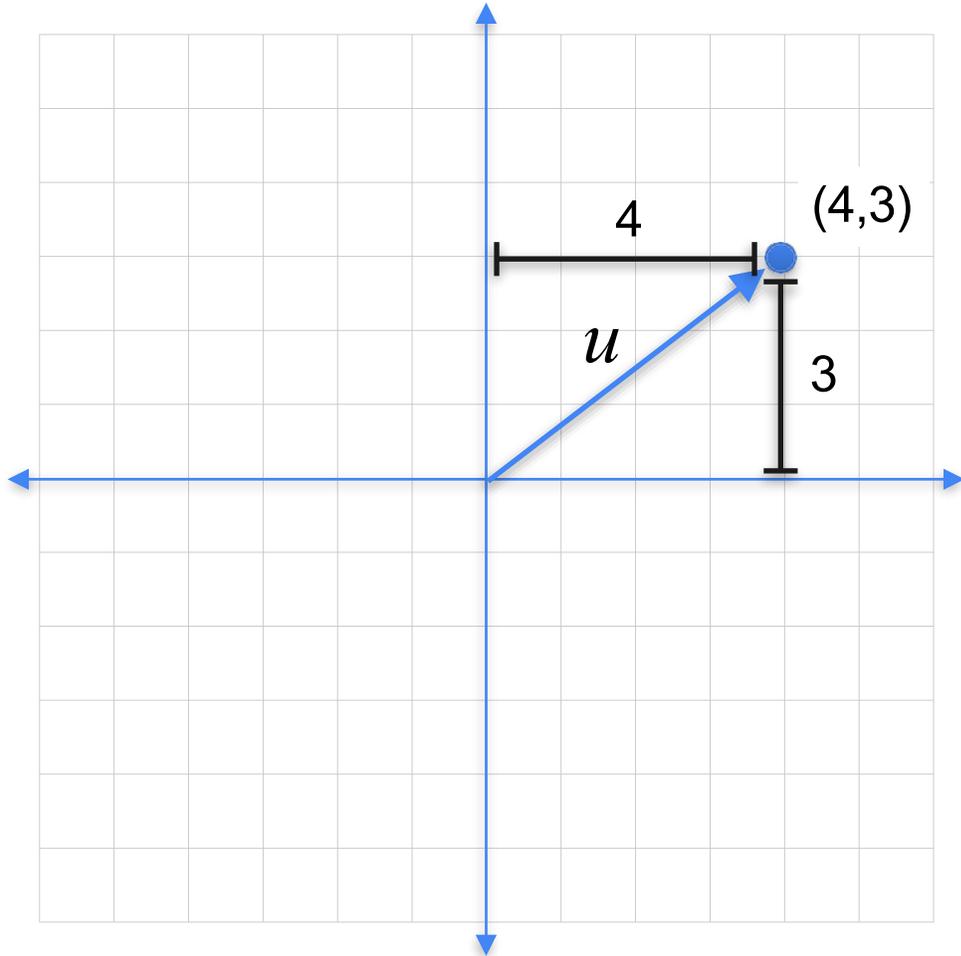
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范数例子



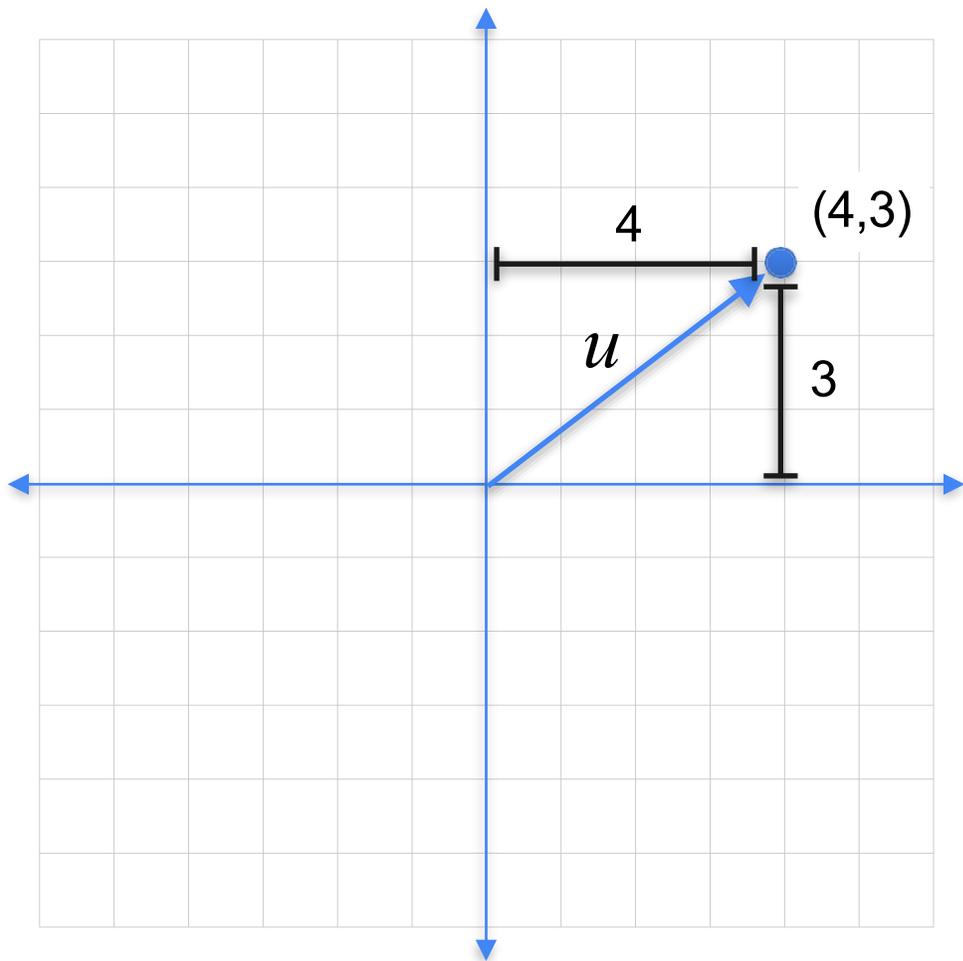
1. 向量及其属性

范数例子



1. 向量及其属性

范数例子

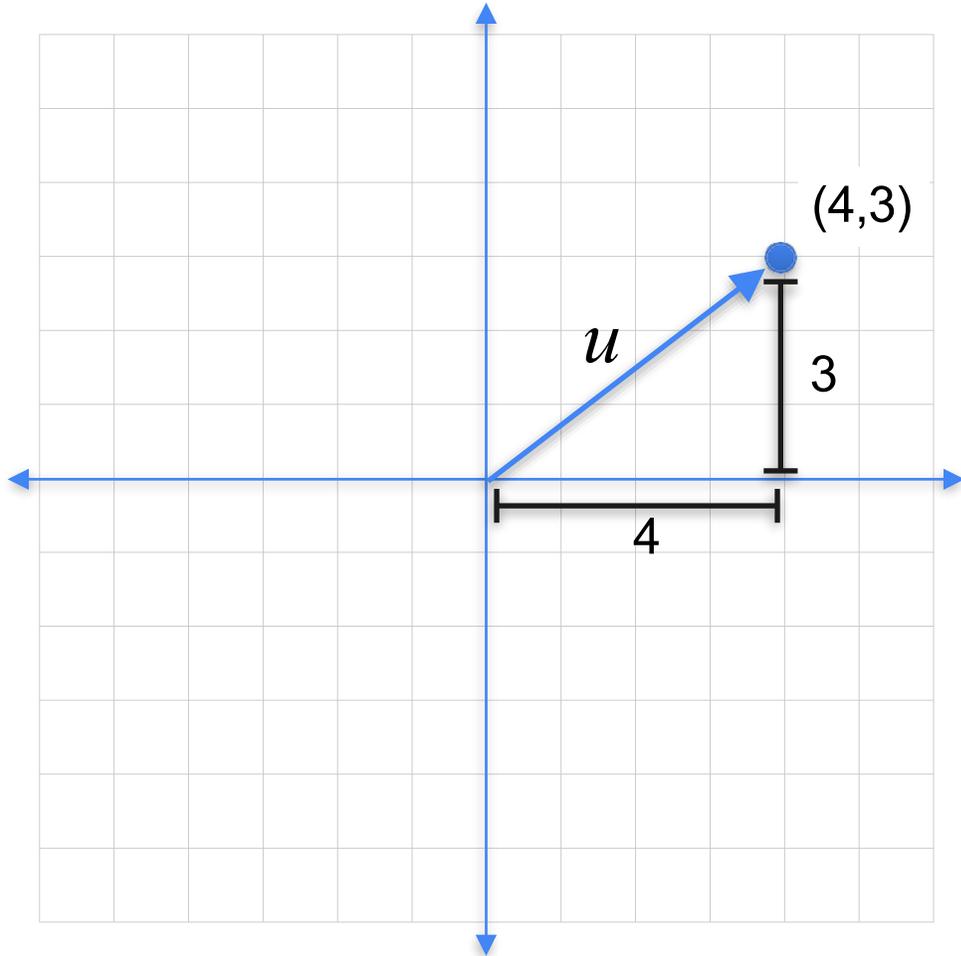


$$\text{L1-范数} = |4| + |3| = 7$$

$$\text{L2-范数} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

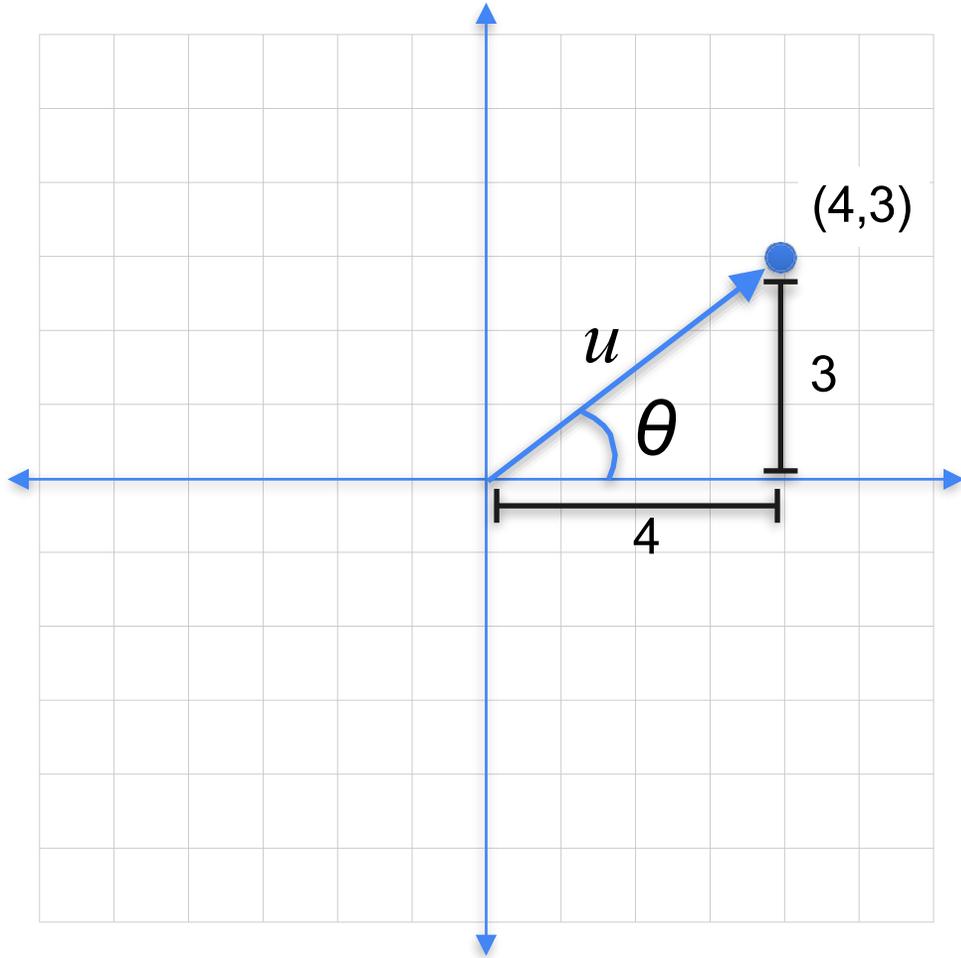
1. 向量及其属性

向量的方向



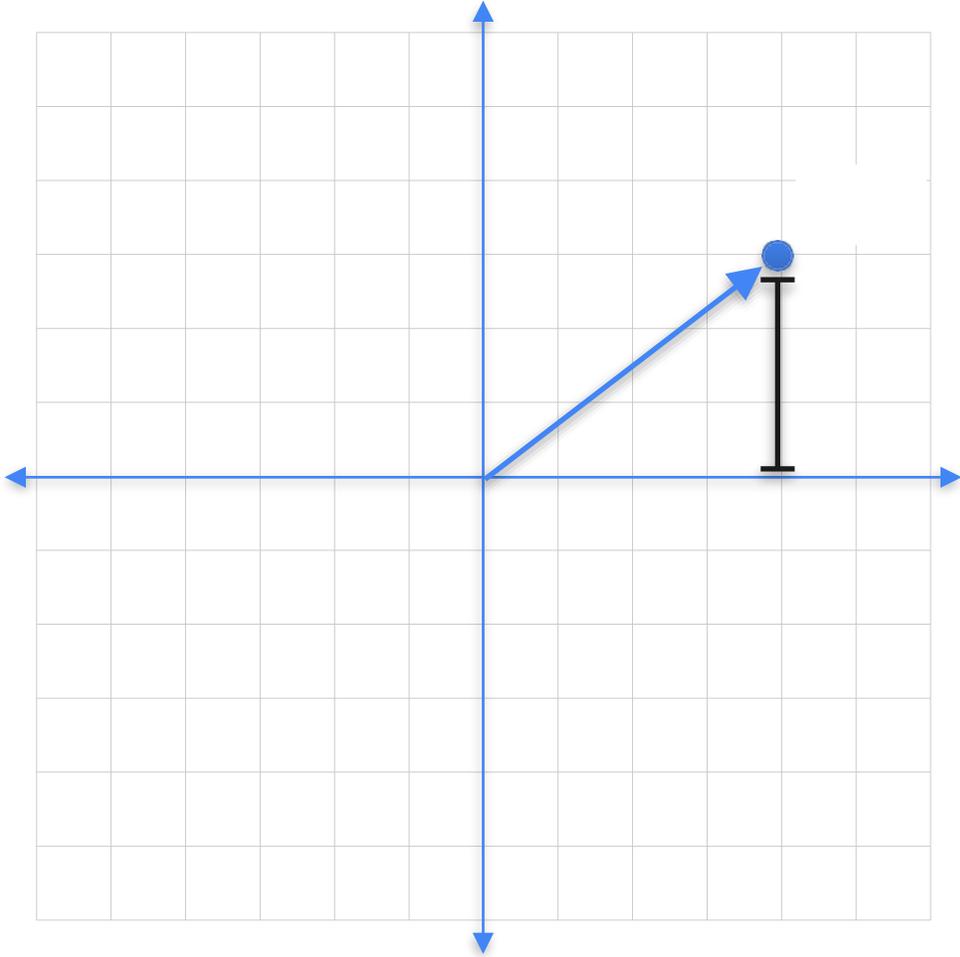
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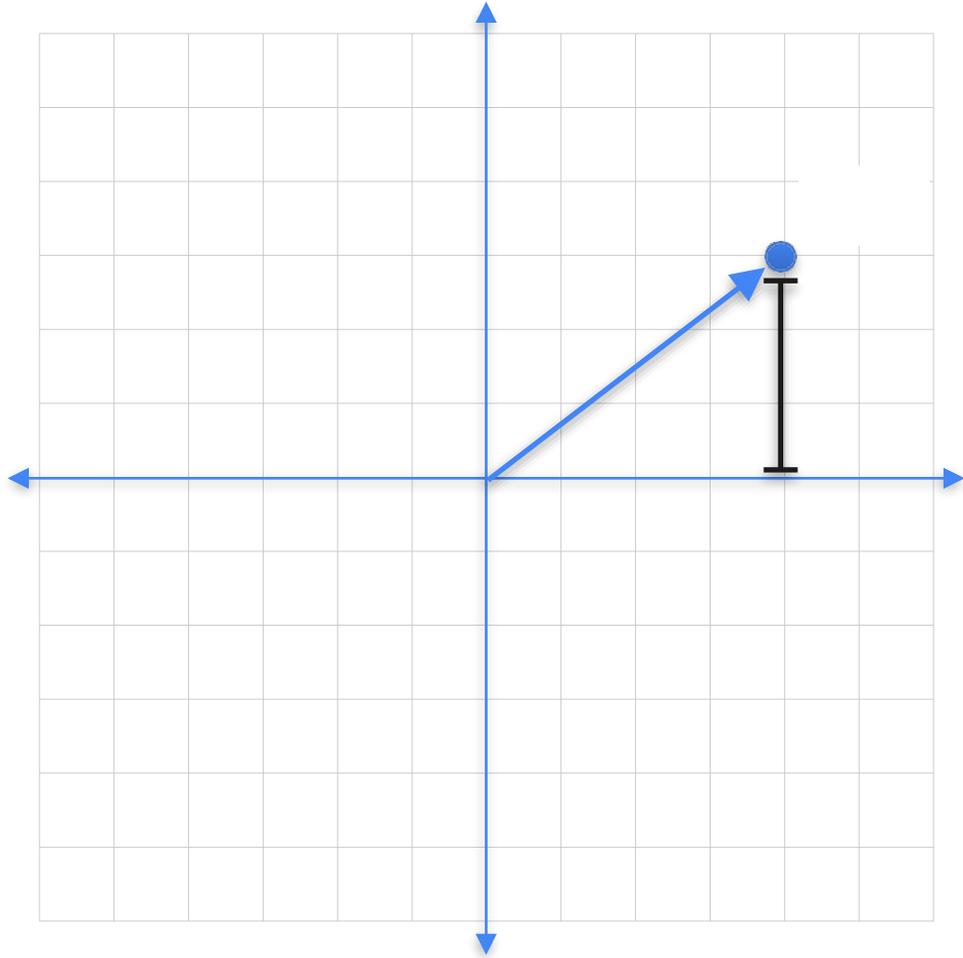
向量的方向



$$\tan(\theta) = 3/4$$

1. 向量及其属性

向量的方向

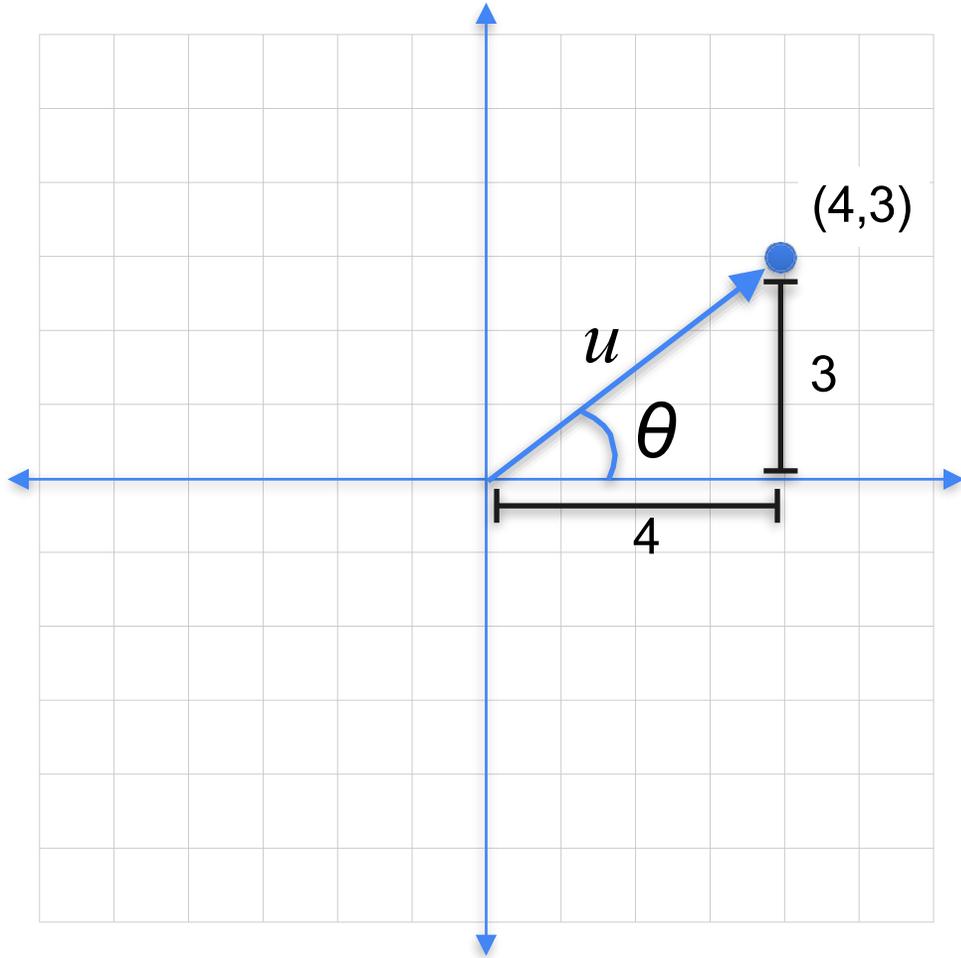


$$\tan(\theta) = 3/4$$

$$\theta = \arctan(3/4) = 0.64$$

1. 向量及其属性

向量的方向

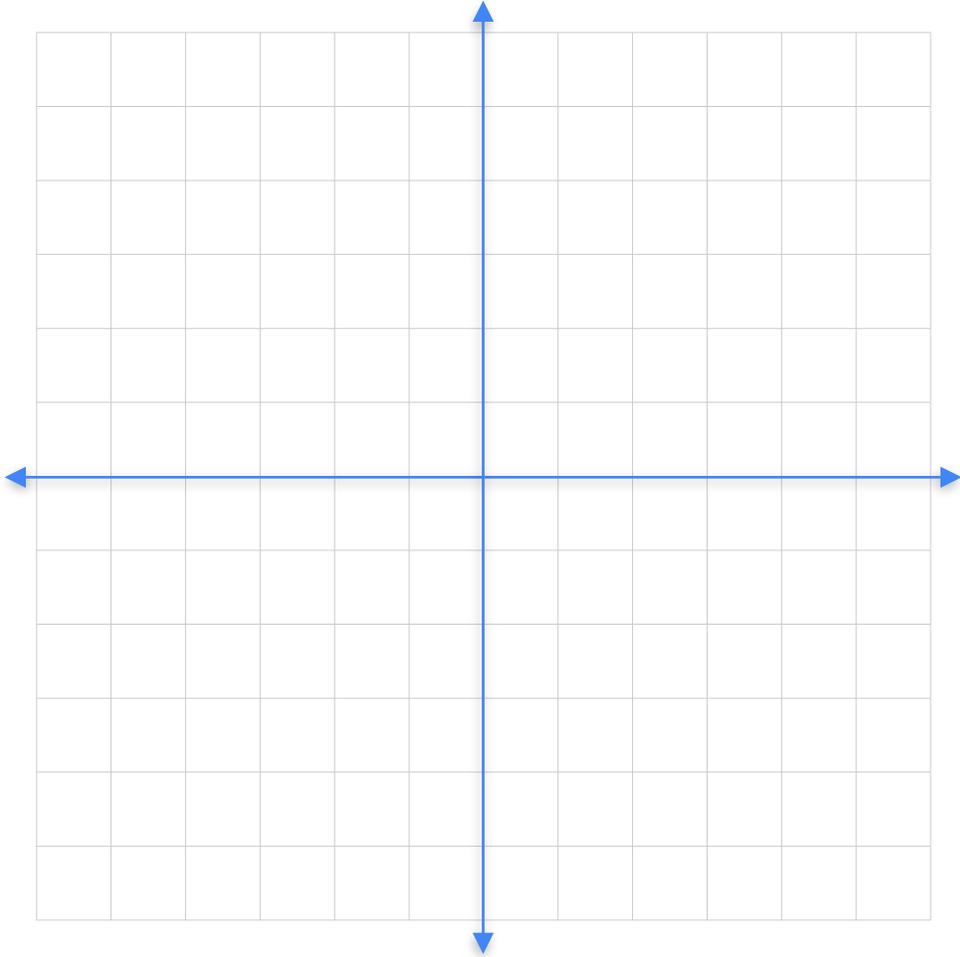


$$\tan(\theta) = 3/4$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$

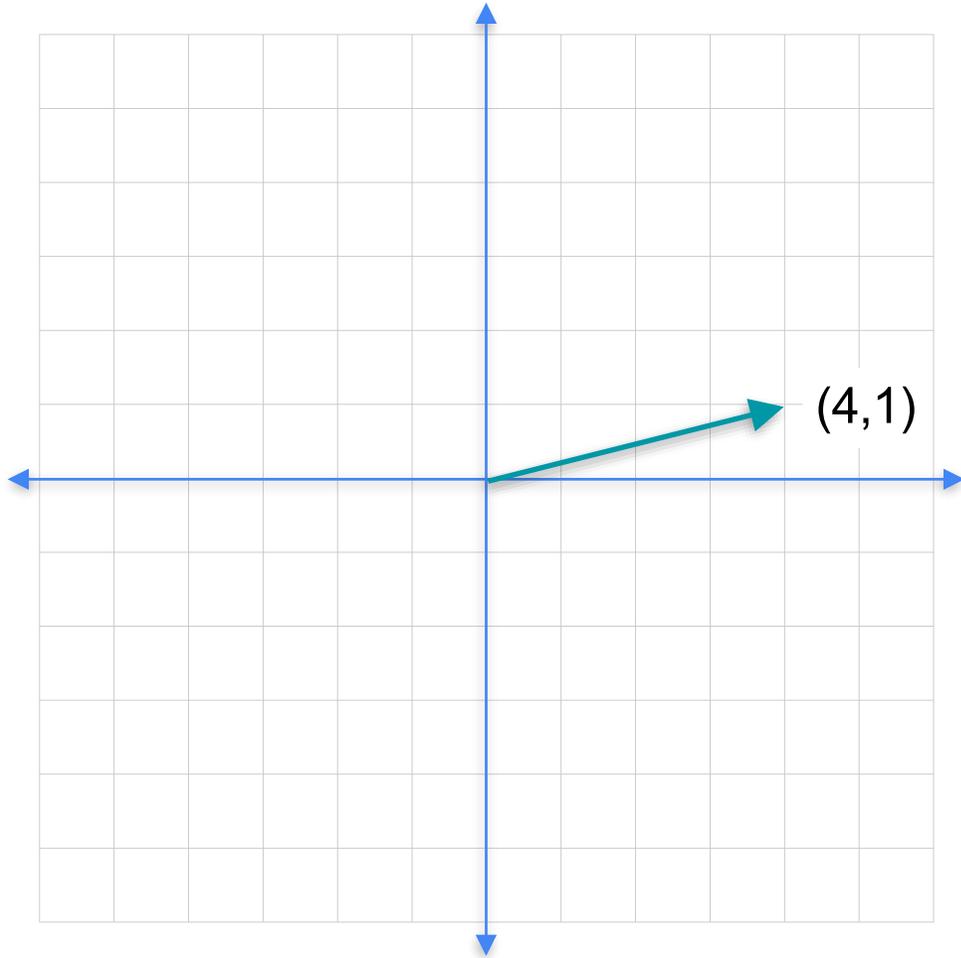
1. 向量及其属性

向量相加



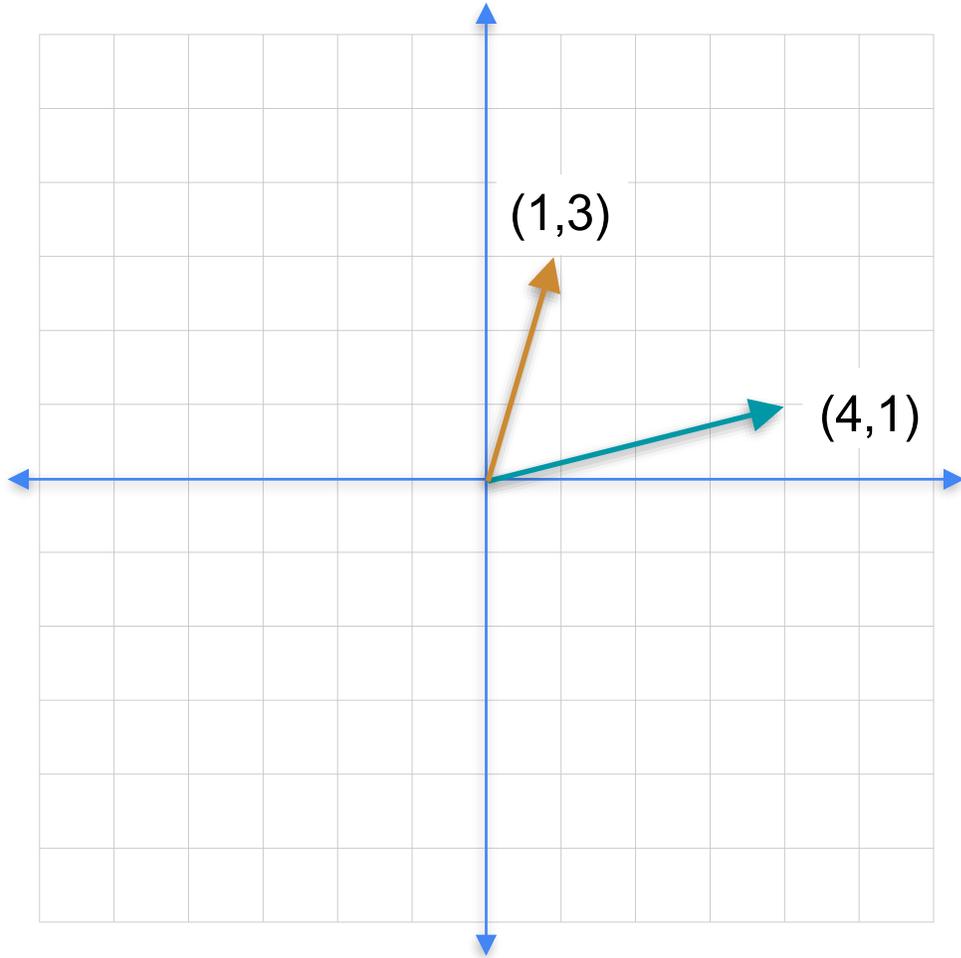
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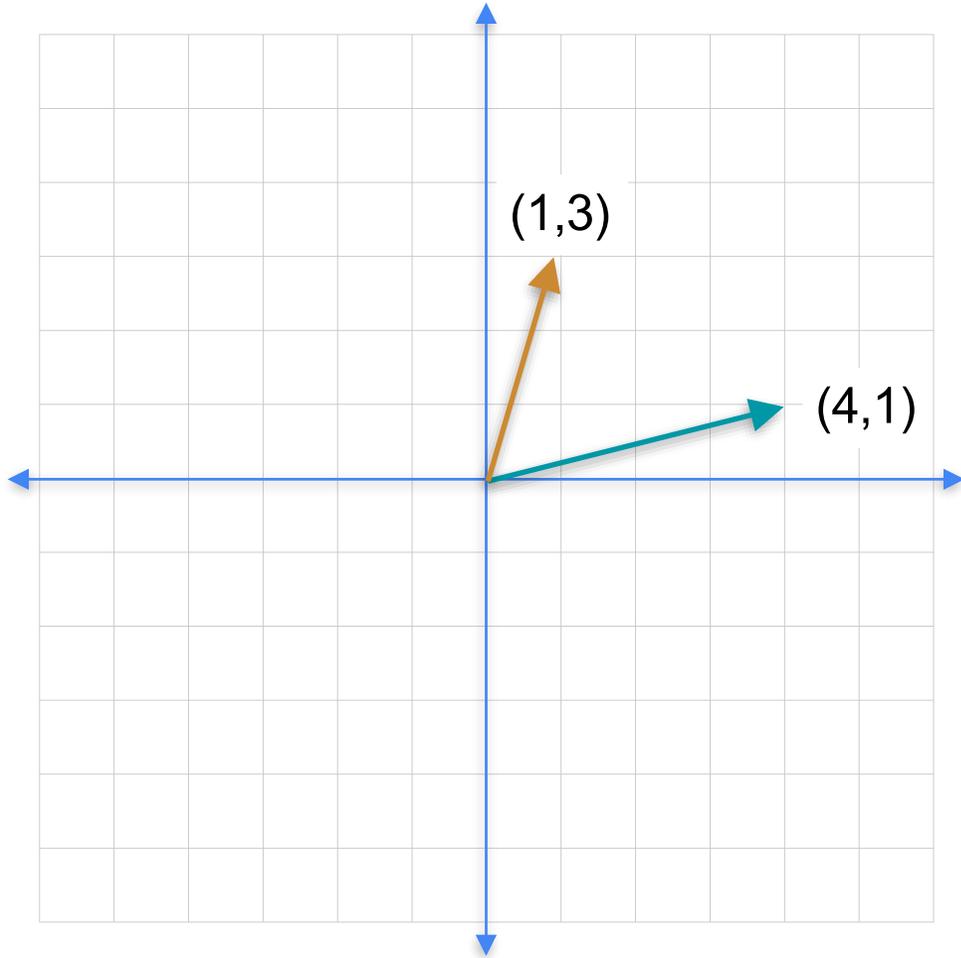
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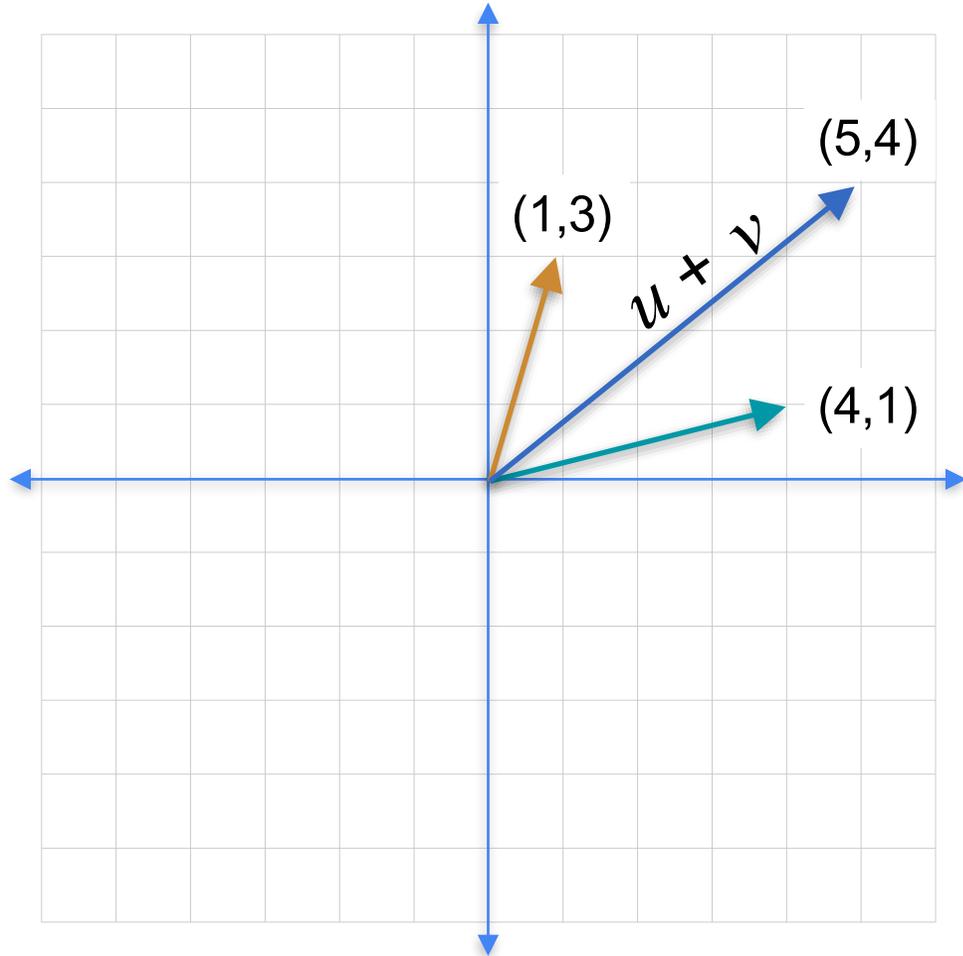
向量相加



$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

1. 向量及其属性

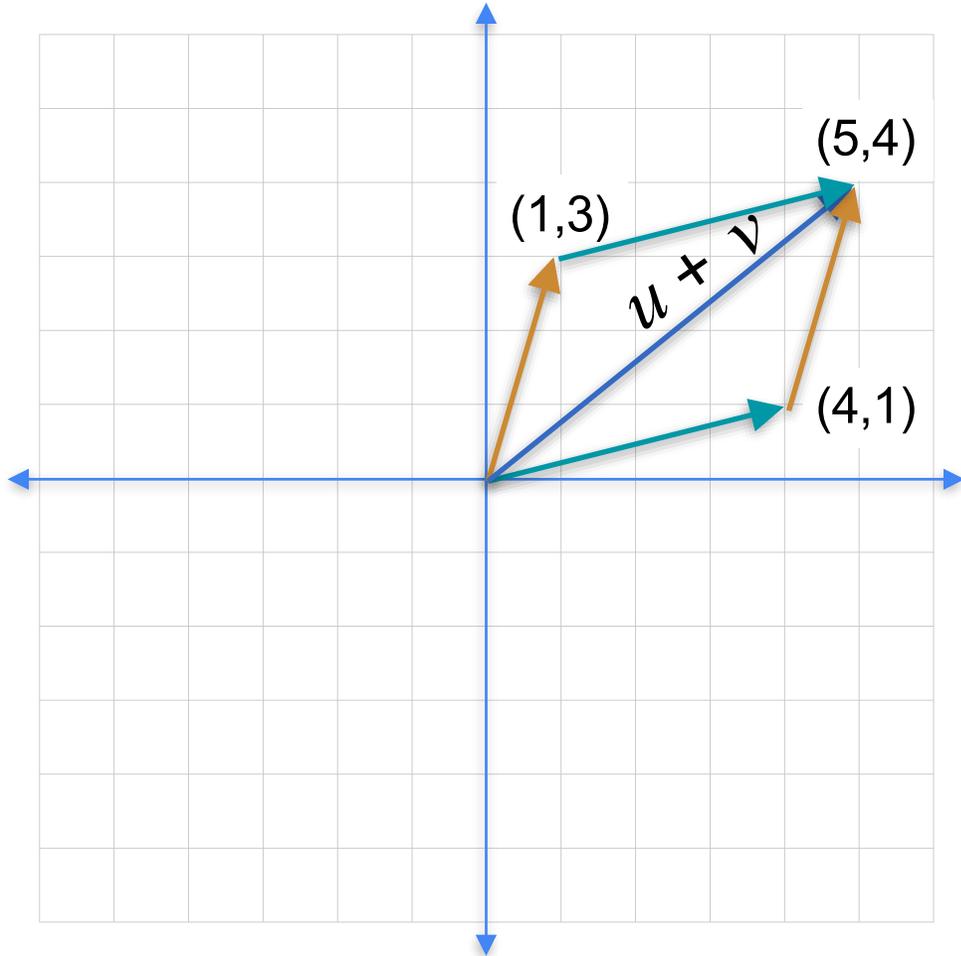
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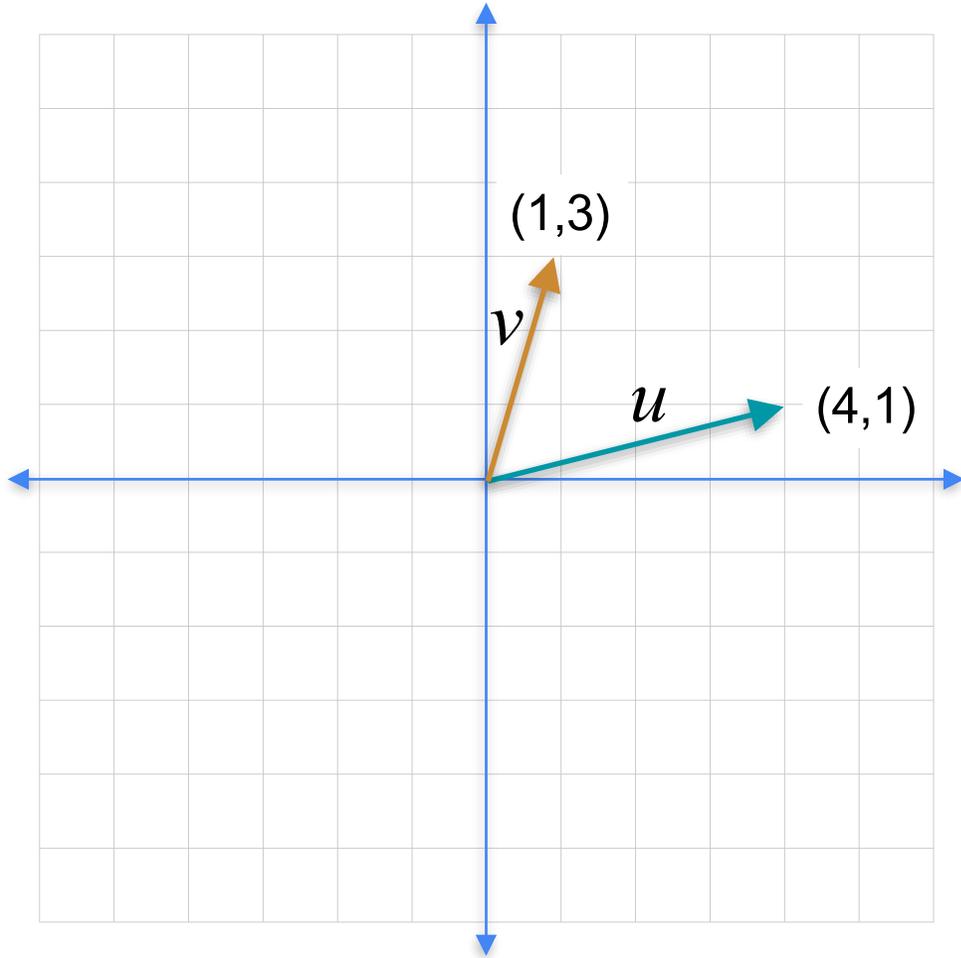
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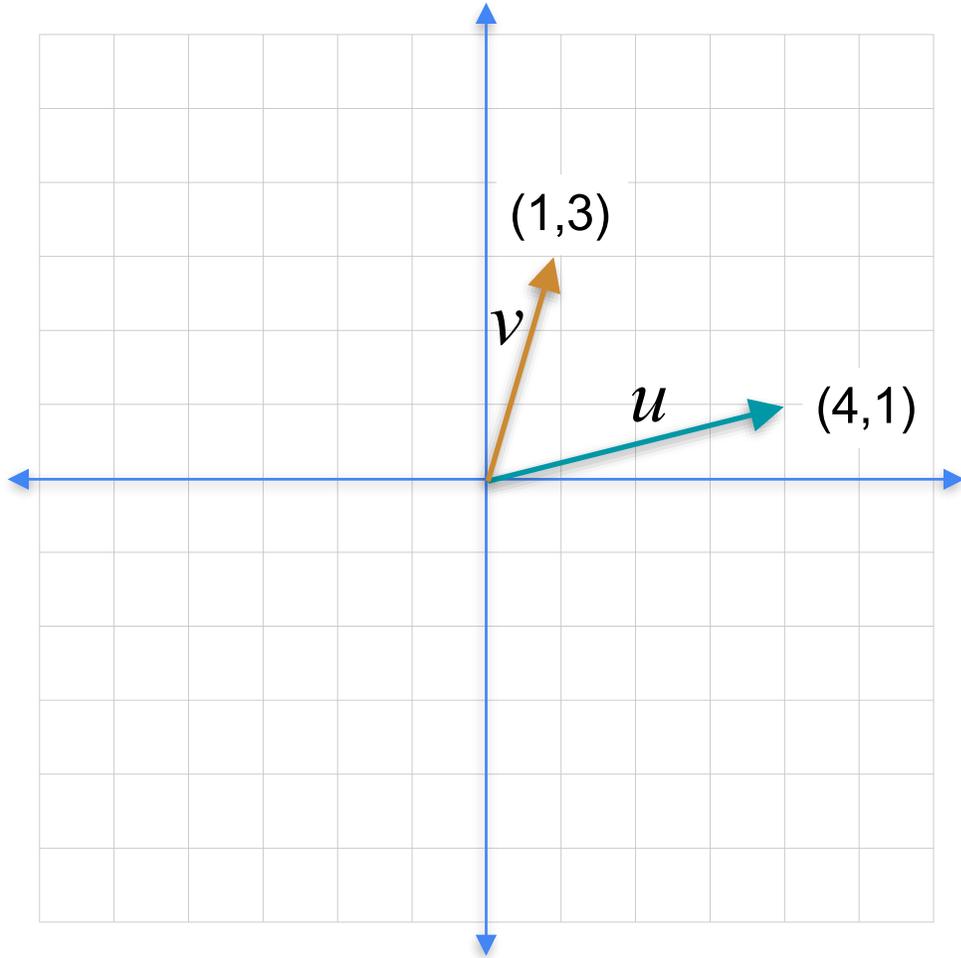
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1. 向量及其属性

向量相减

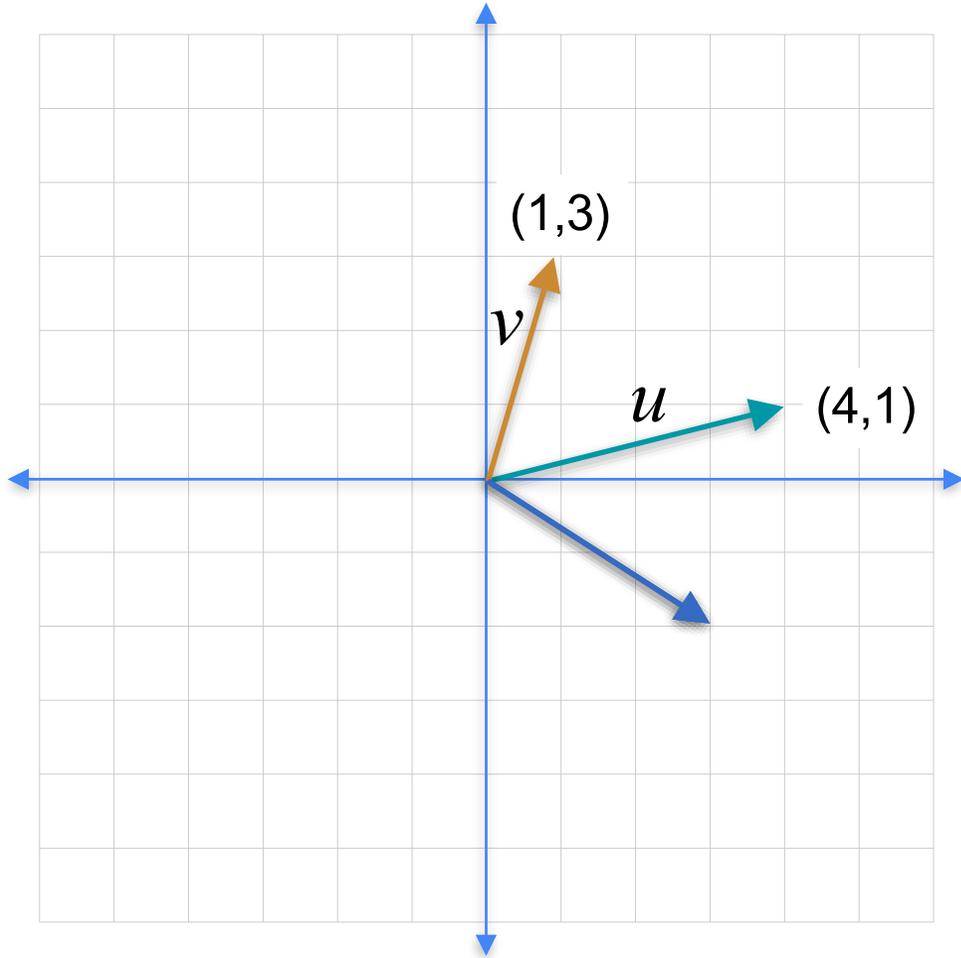


向量相减



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

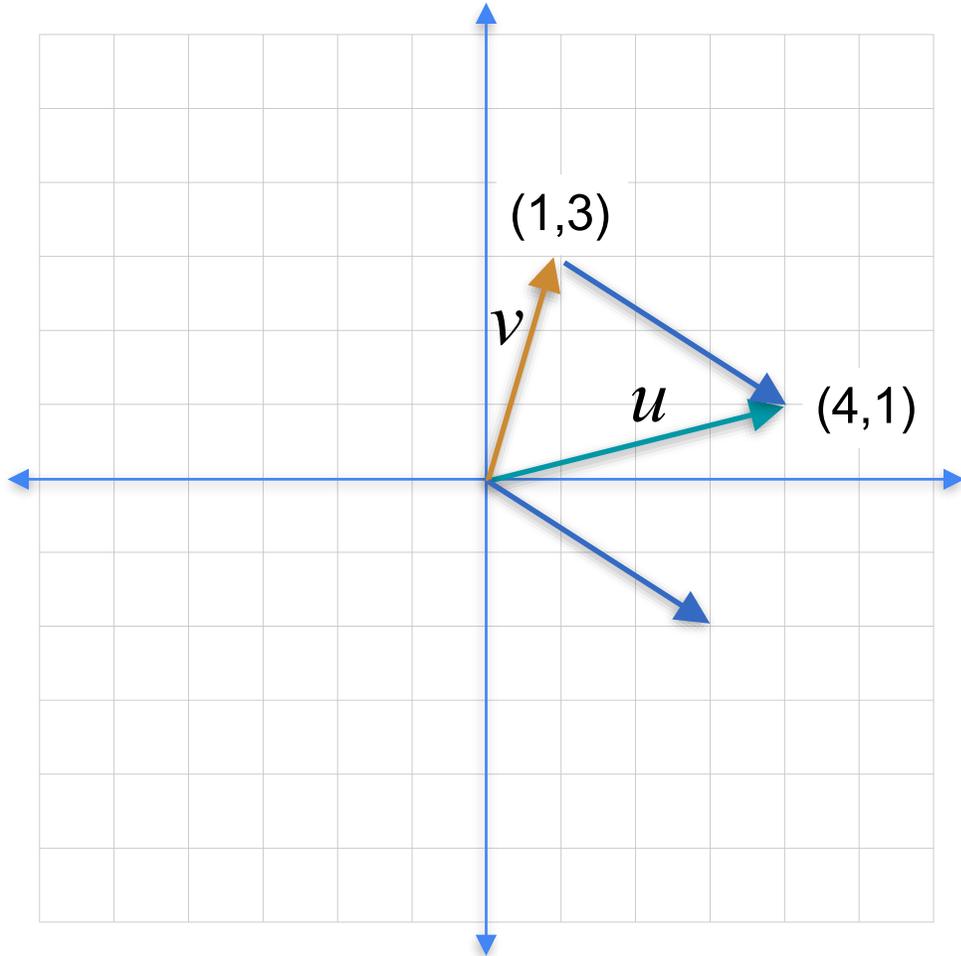
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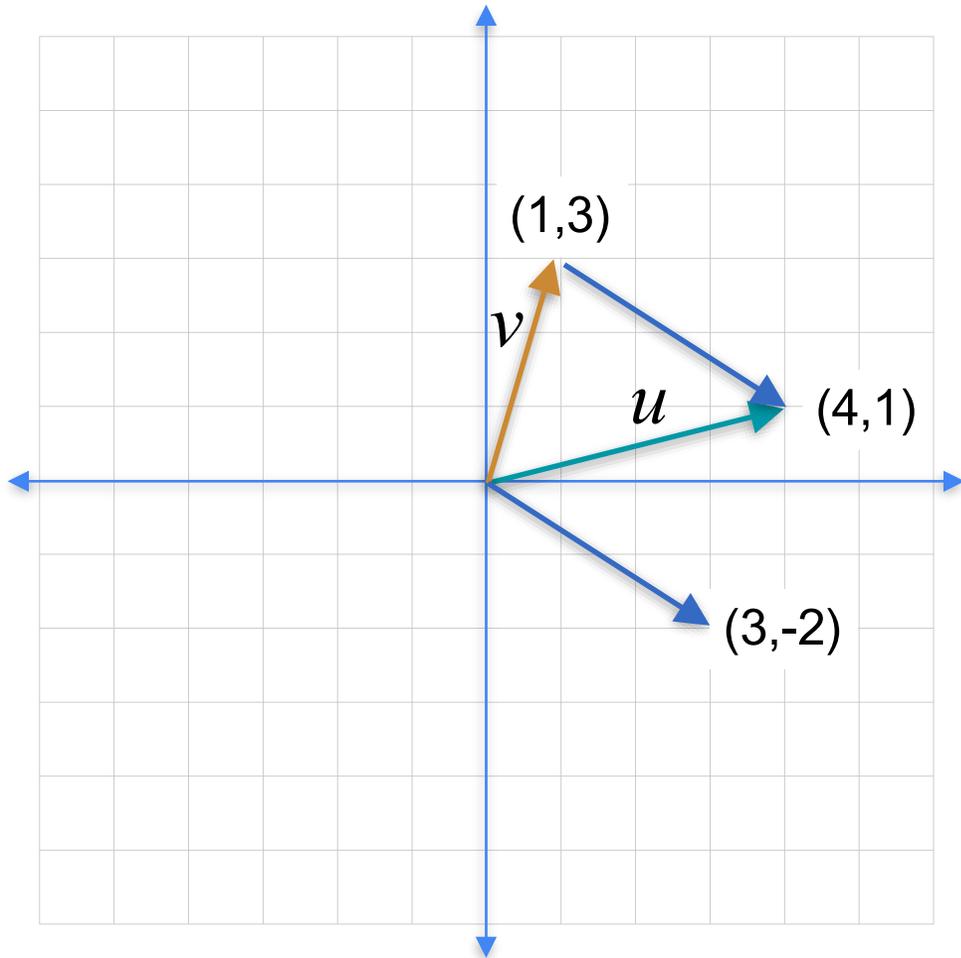
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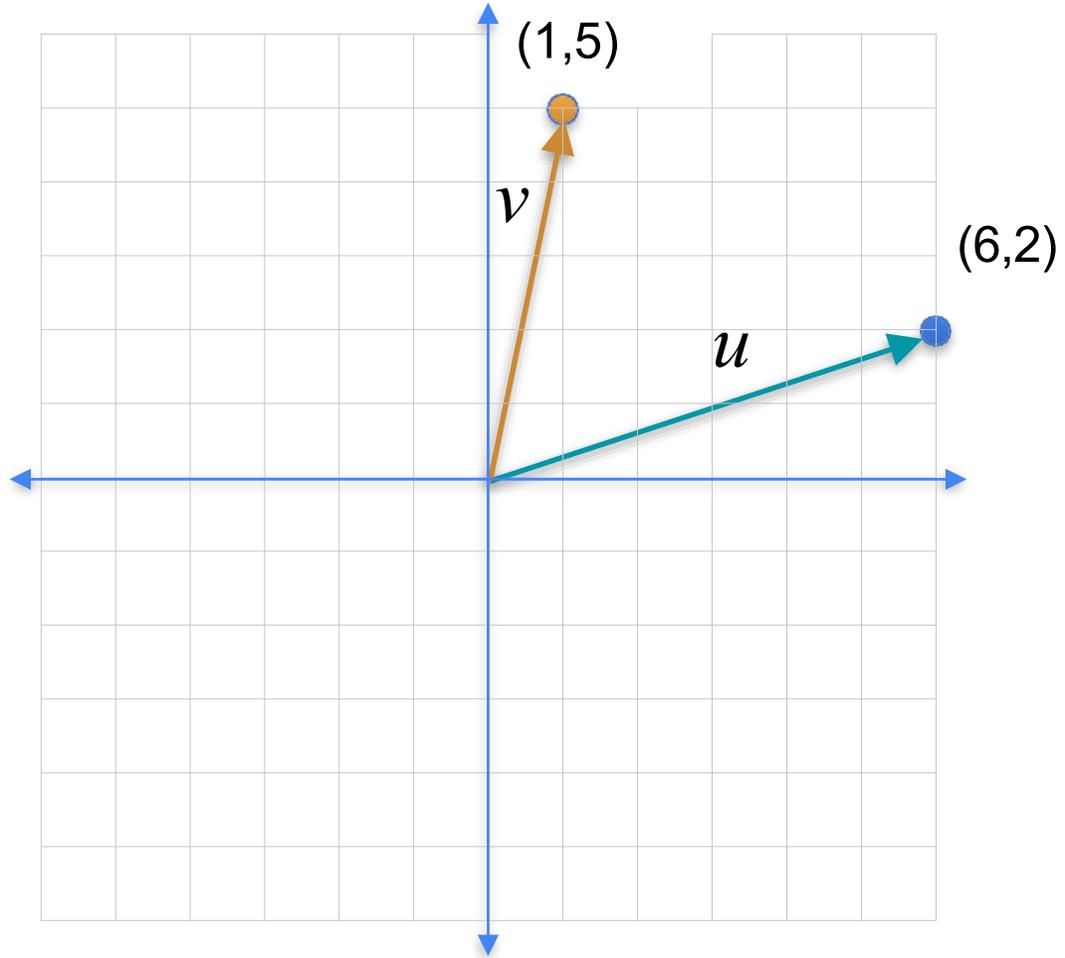
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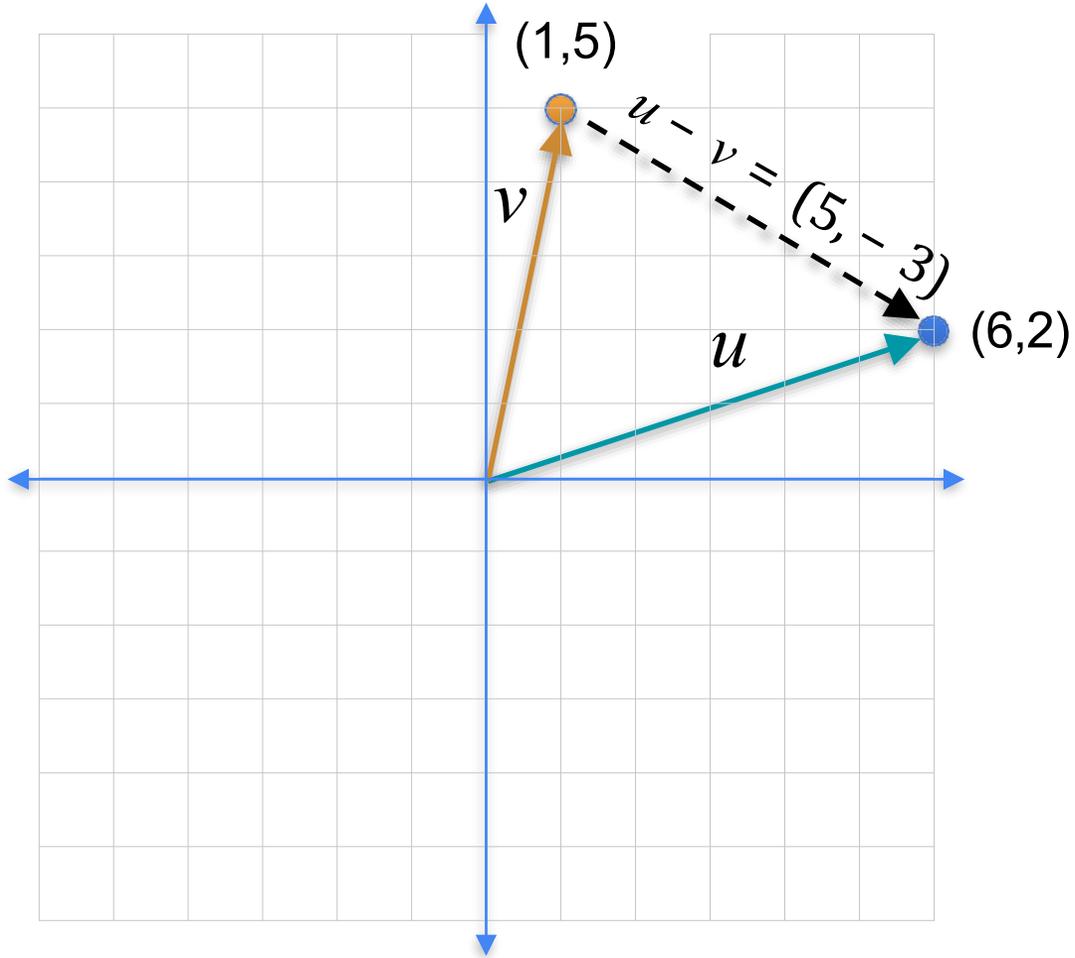
1. 向量及其属性

向量距离



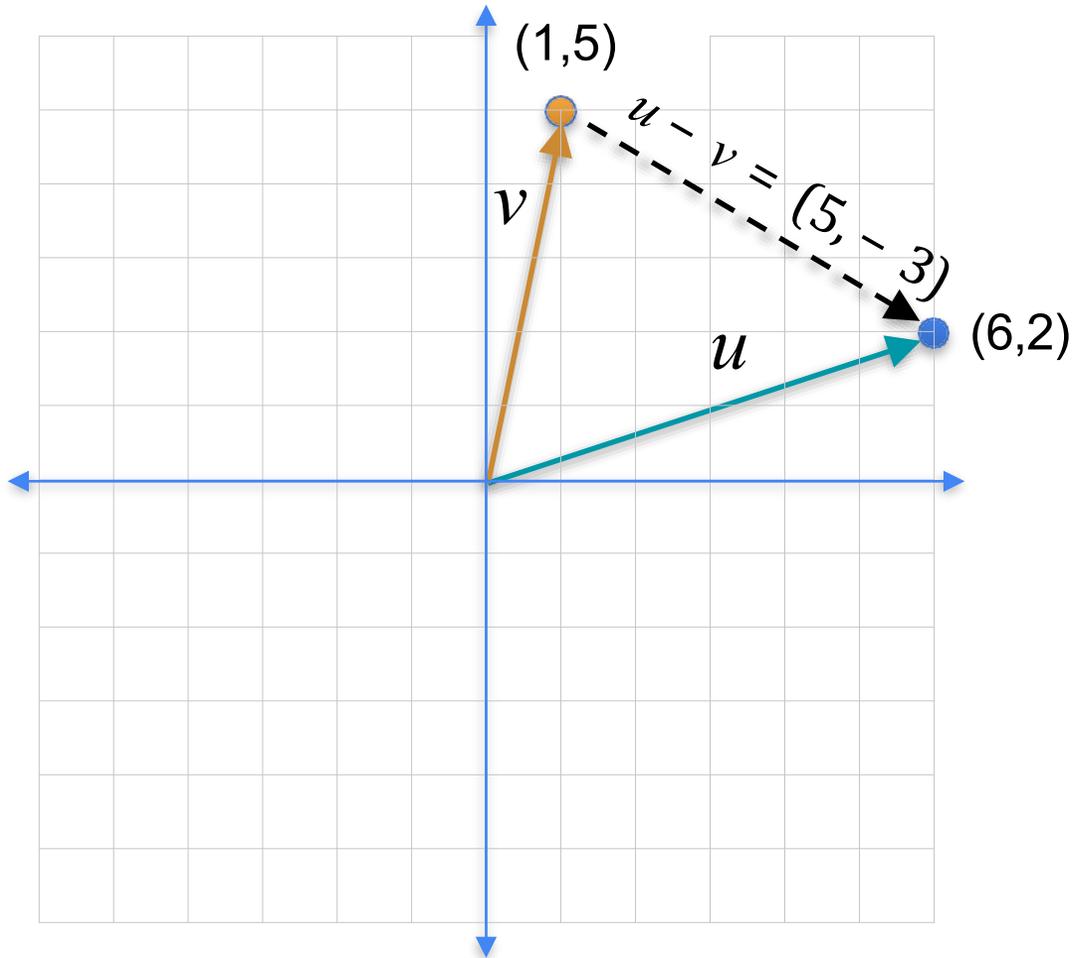
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向量距离



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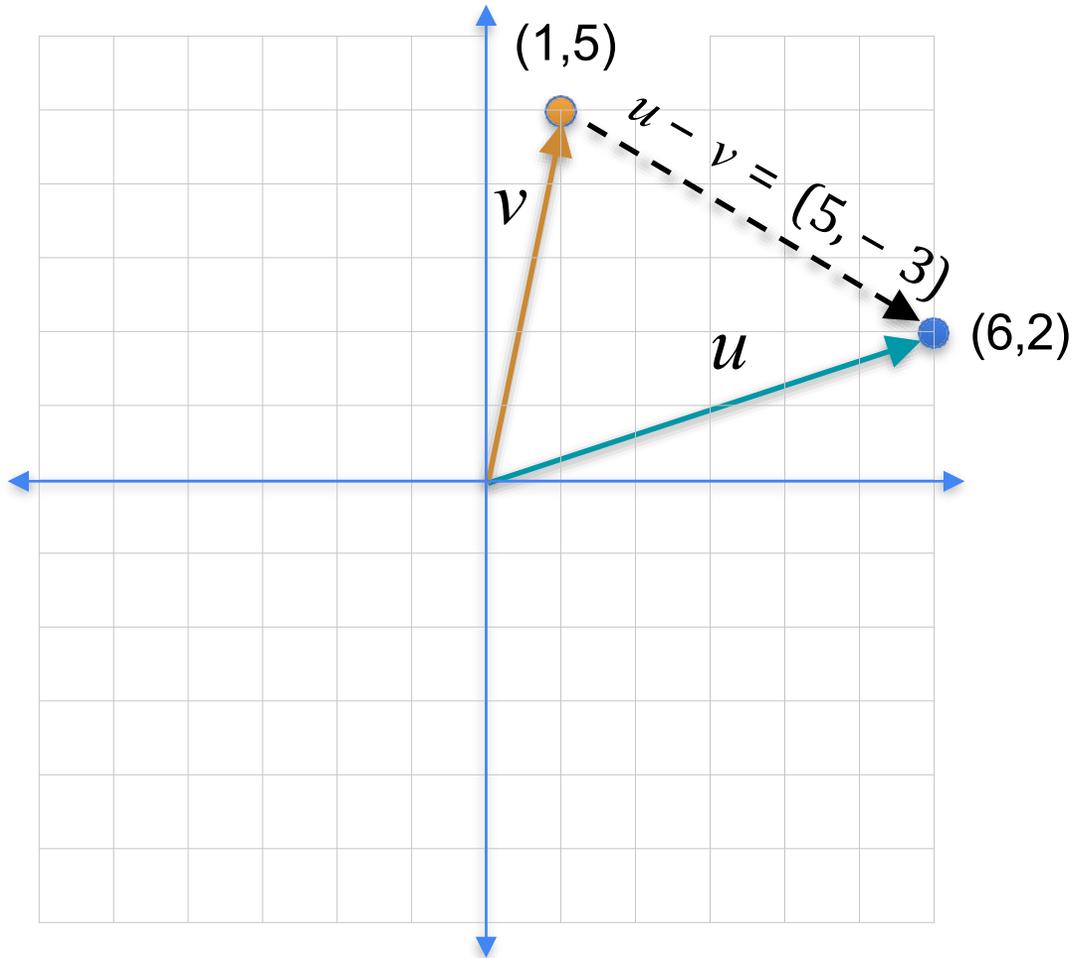
向量距离



 L1-distance $|u - v|_1 = |5| + |-3| = 8$

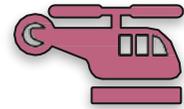
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向量距离



L1-distance

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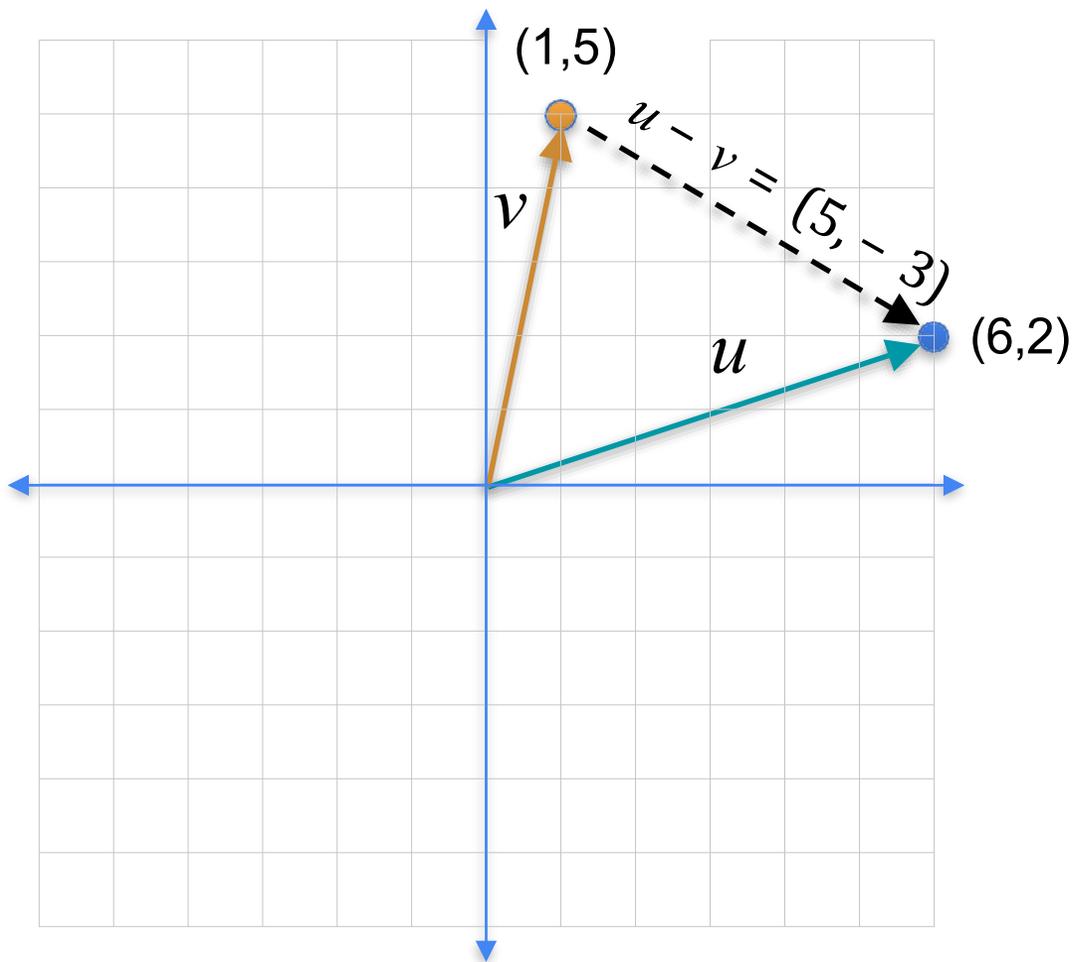


L2-distance

$$|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$$

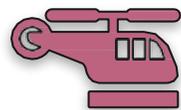
1. 向量及其属性

向量距离



L1-distance

$$|u - v|_1 = |5| + |-3| = 8$$

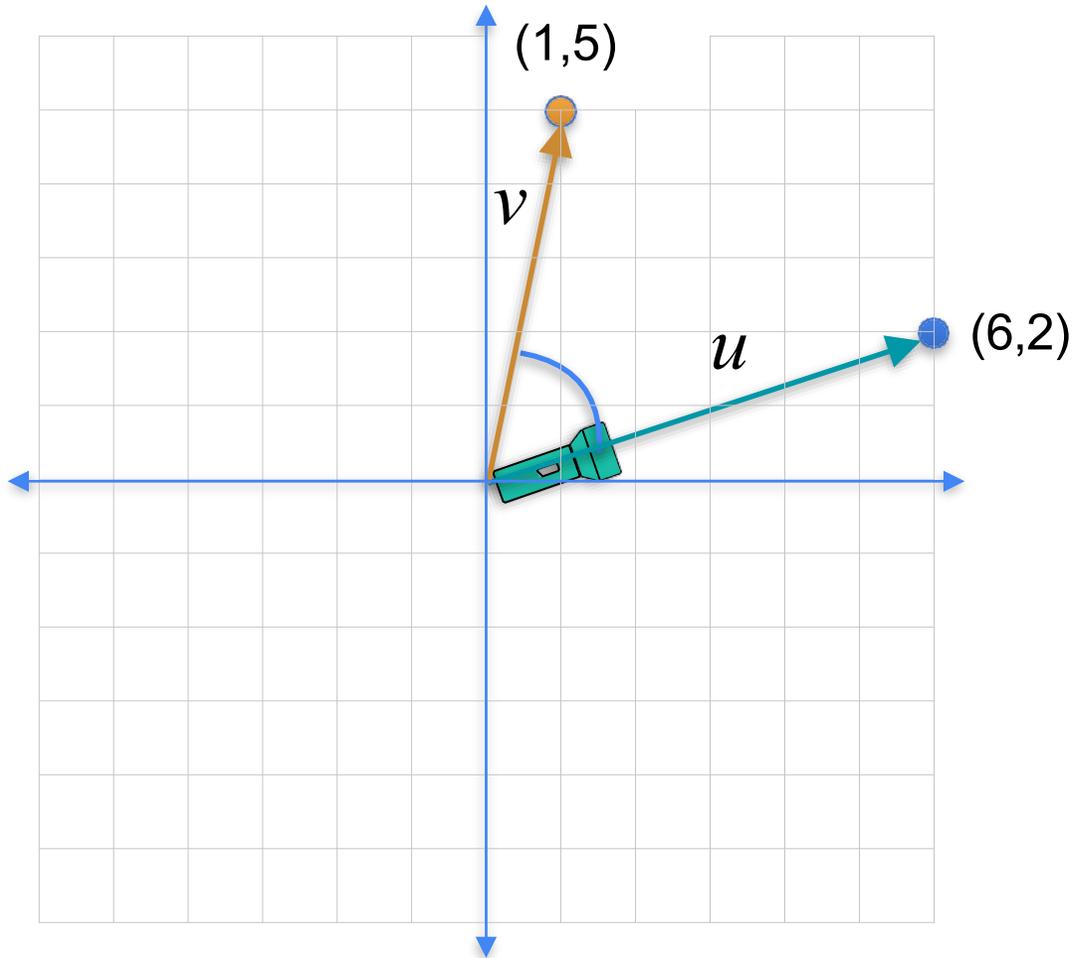


L2-distance

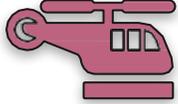
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向量距离

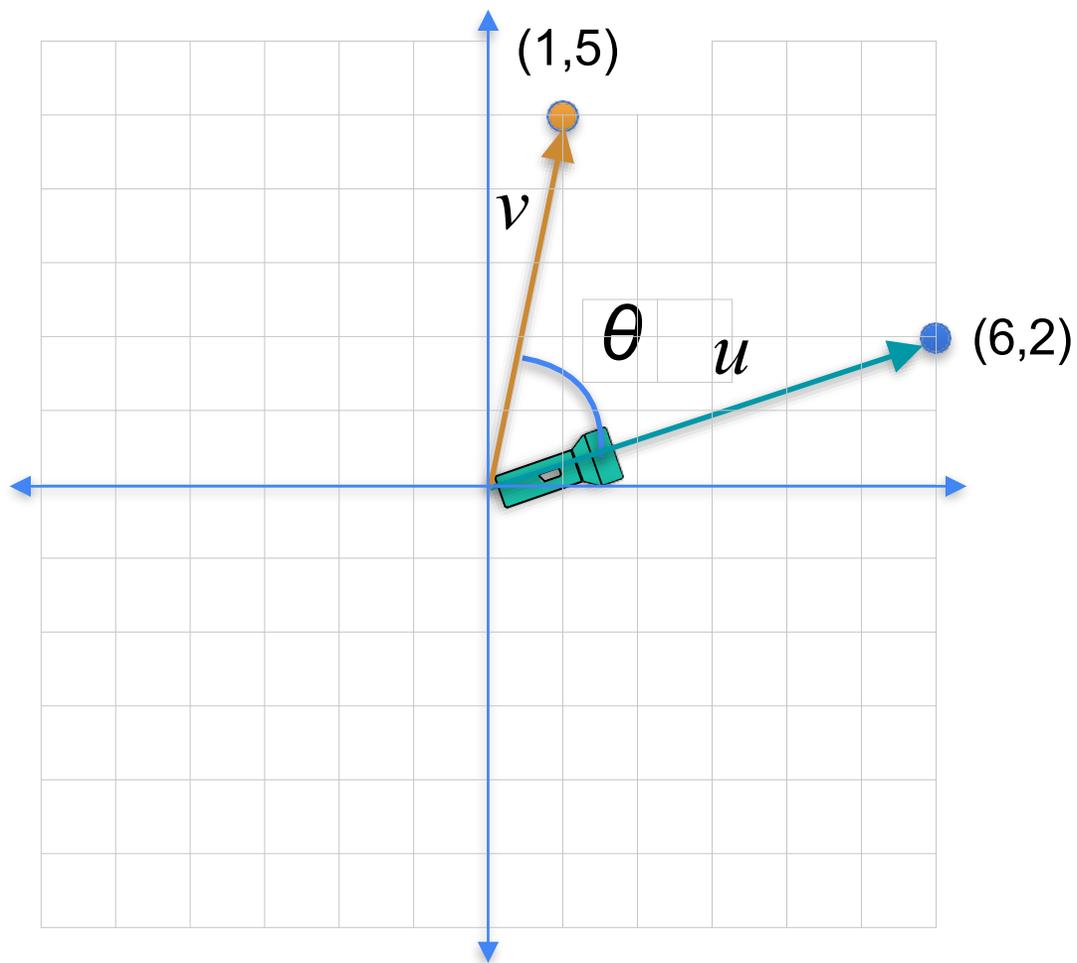


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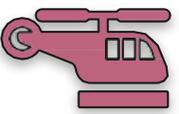
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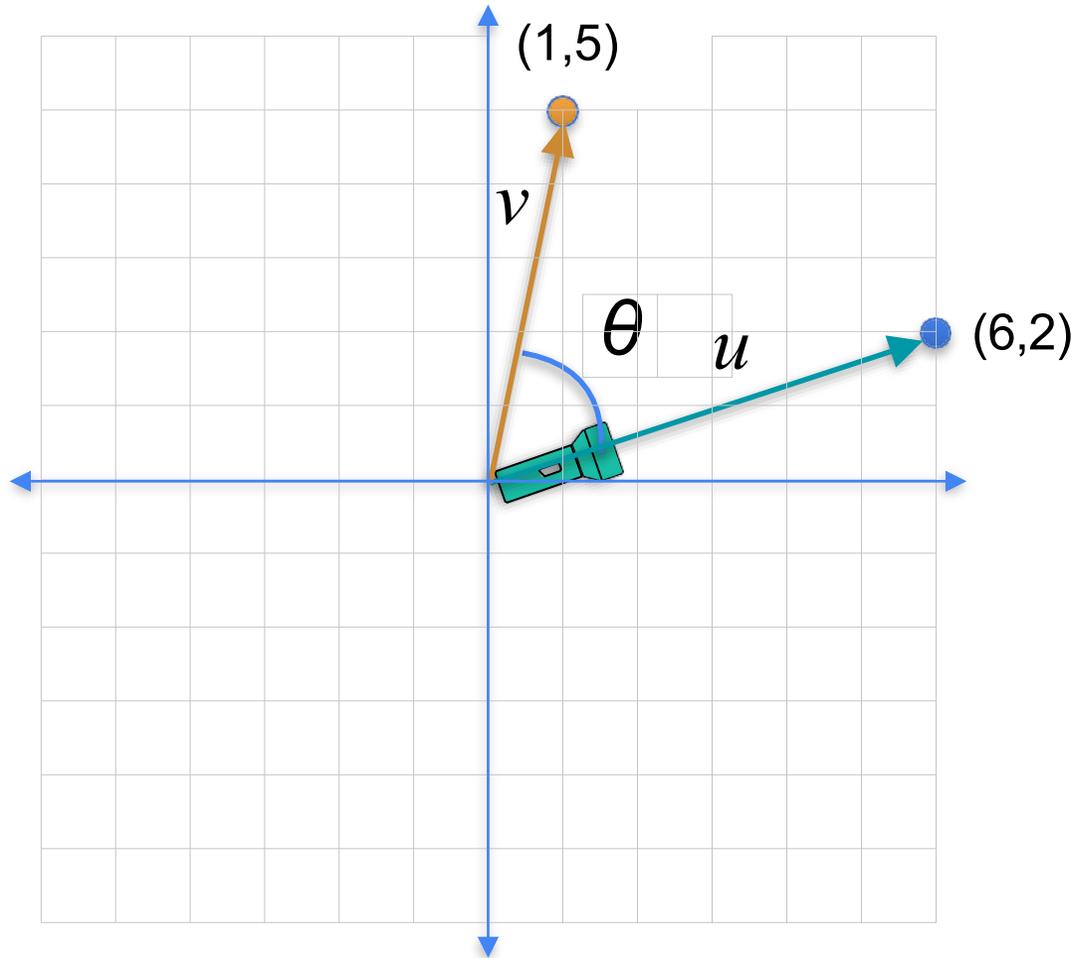


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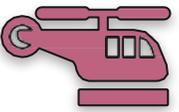
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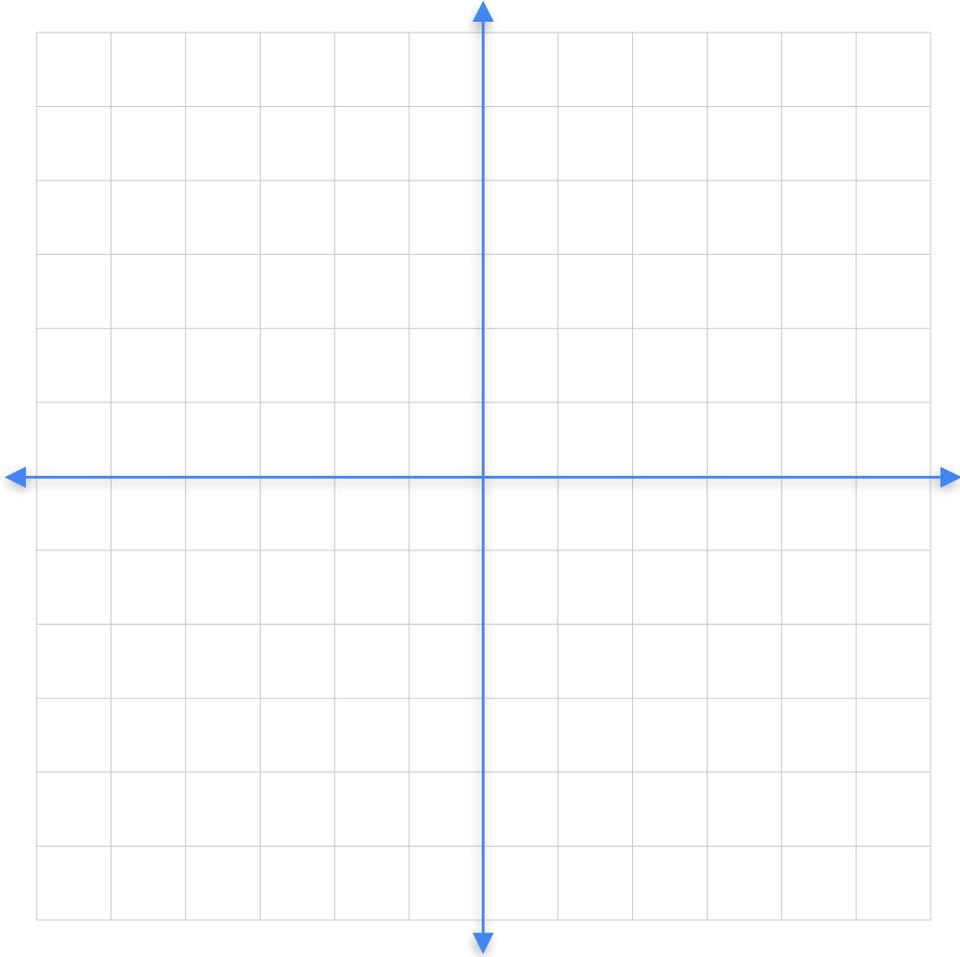
 L1-distance $|u - v|_1 = |5| + |-3| = 8$

 L2-distance $|u - v|_2 = \sqrt{5^2 + 3^2} = 5.83$

 Cosine distance $\cos(\theta)$

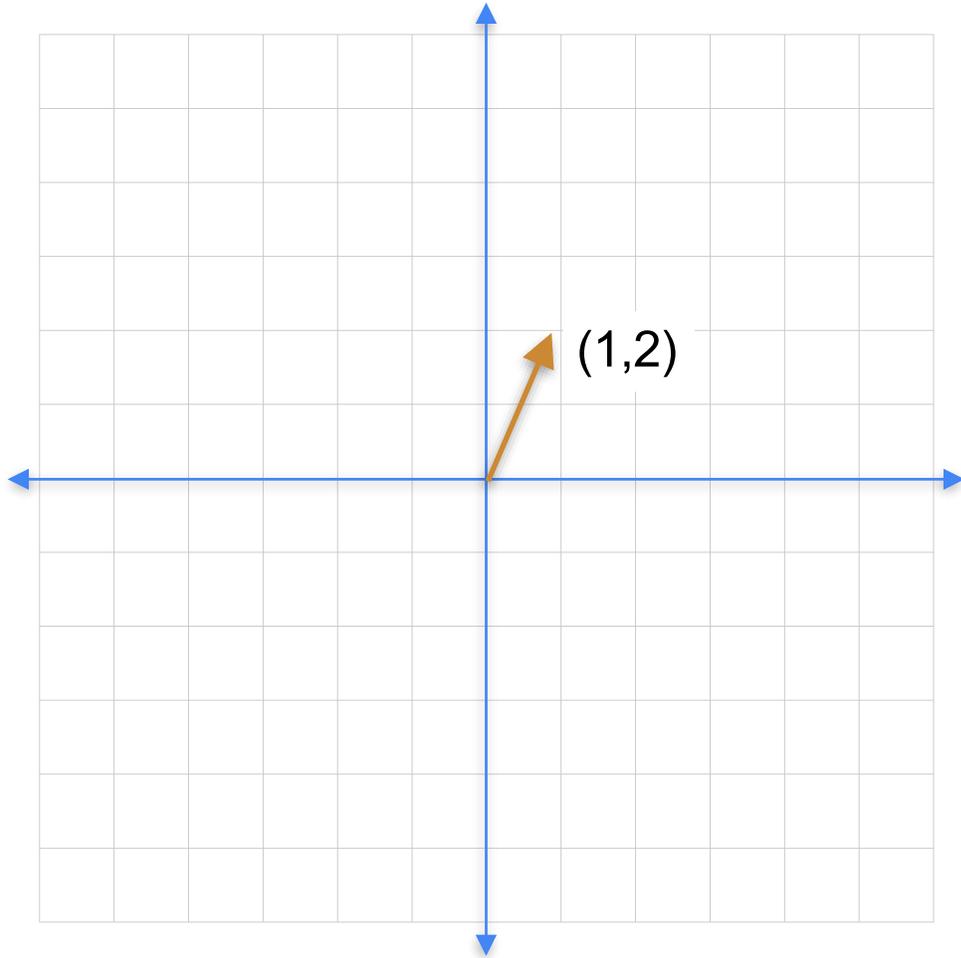
1. 向量及其属性

向量的数乘



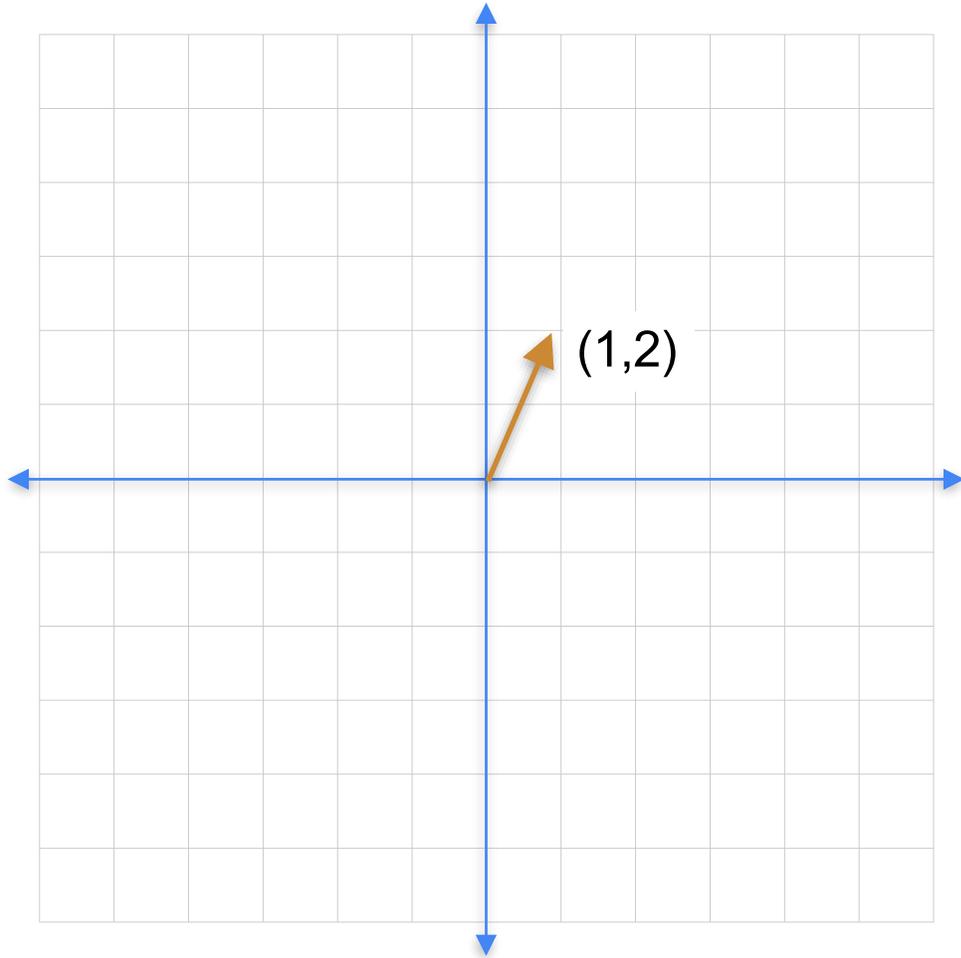
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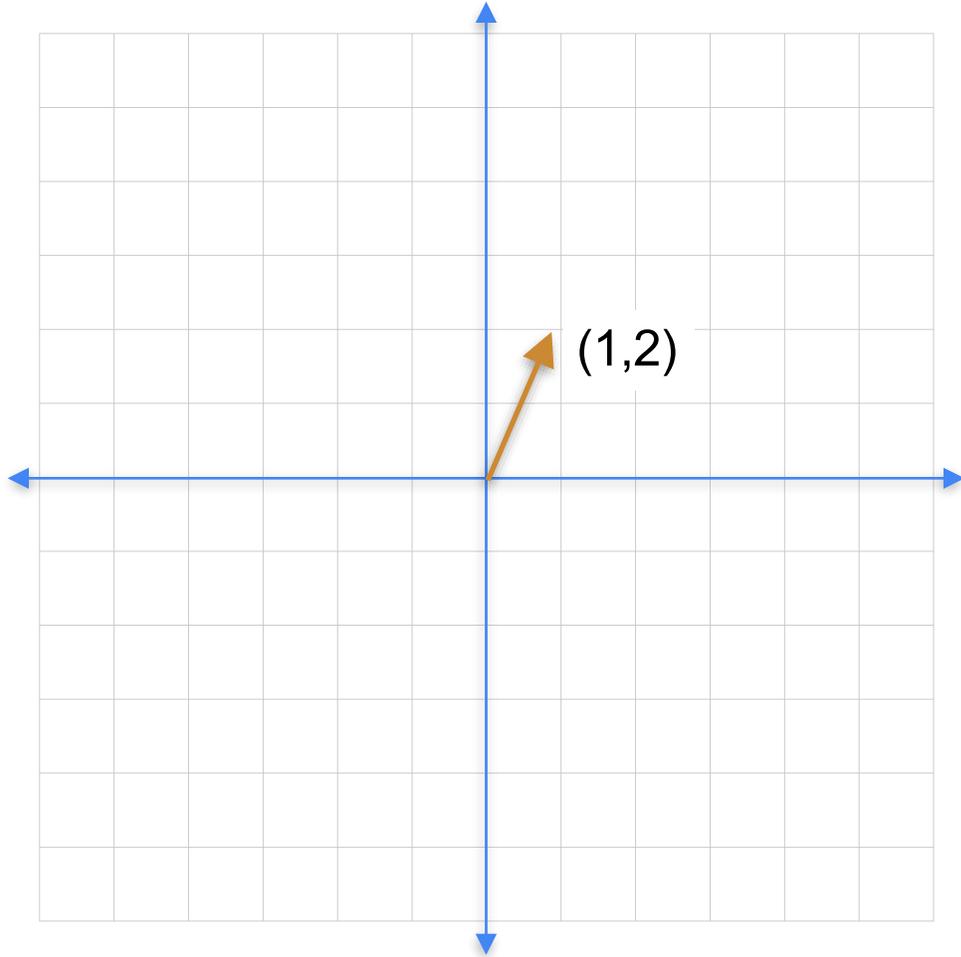
向量的数乘



$$u = (1,2)$$

1. 向量及其属性

向量的数乘

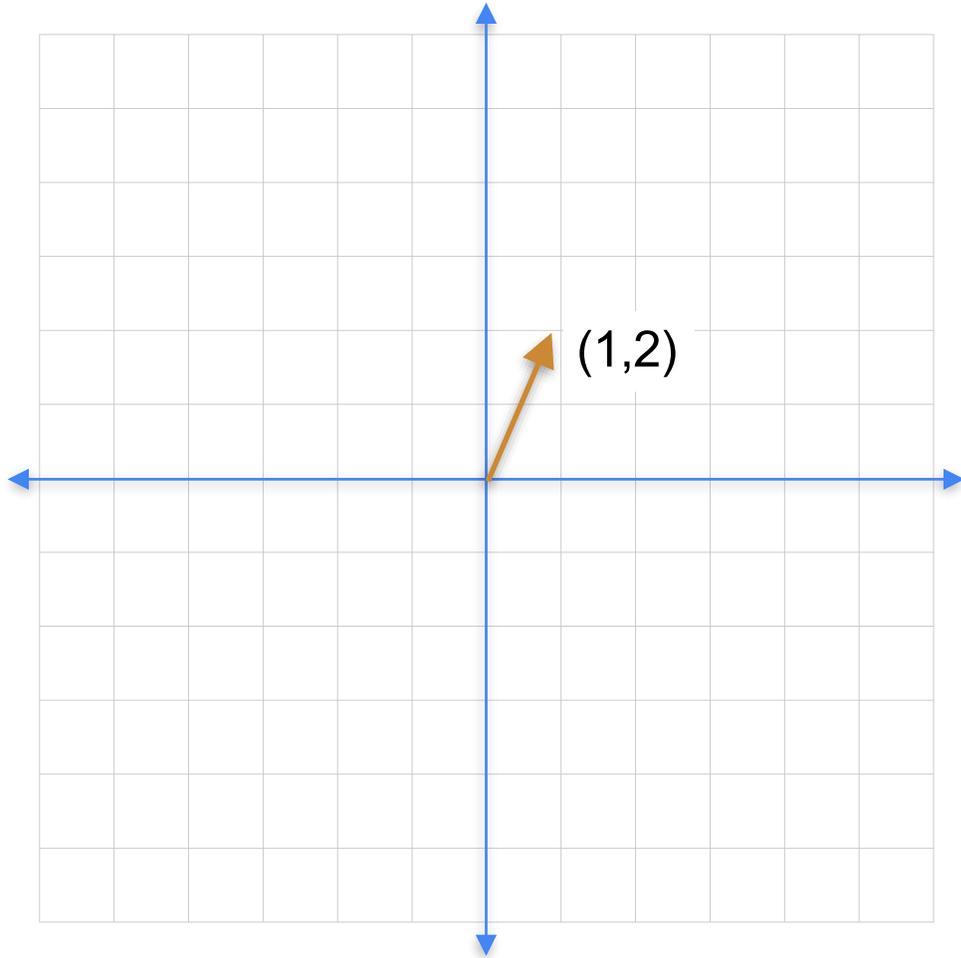


$$u = (1,2)$$

$$\lambda = 3$$

1. 向量及其属性

向量的数乘



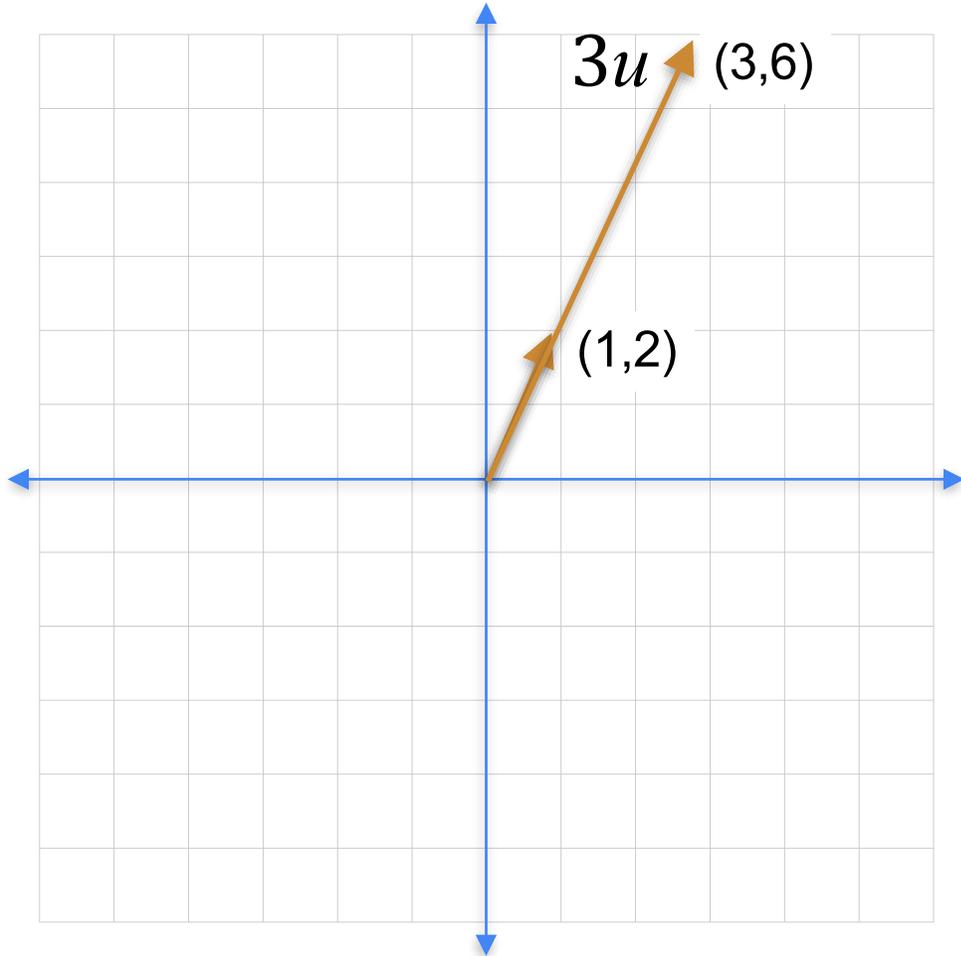
$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

1. 向量及其属性

向量的数乘



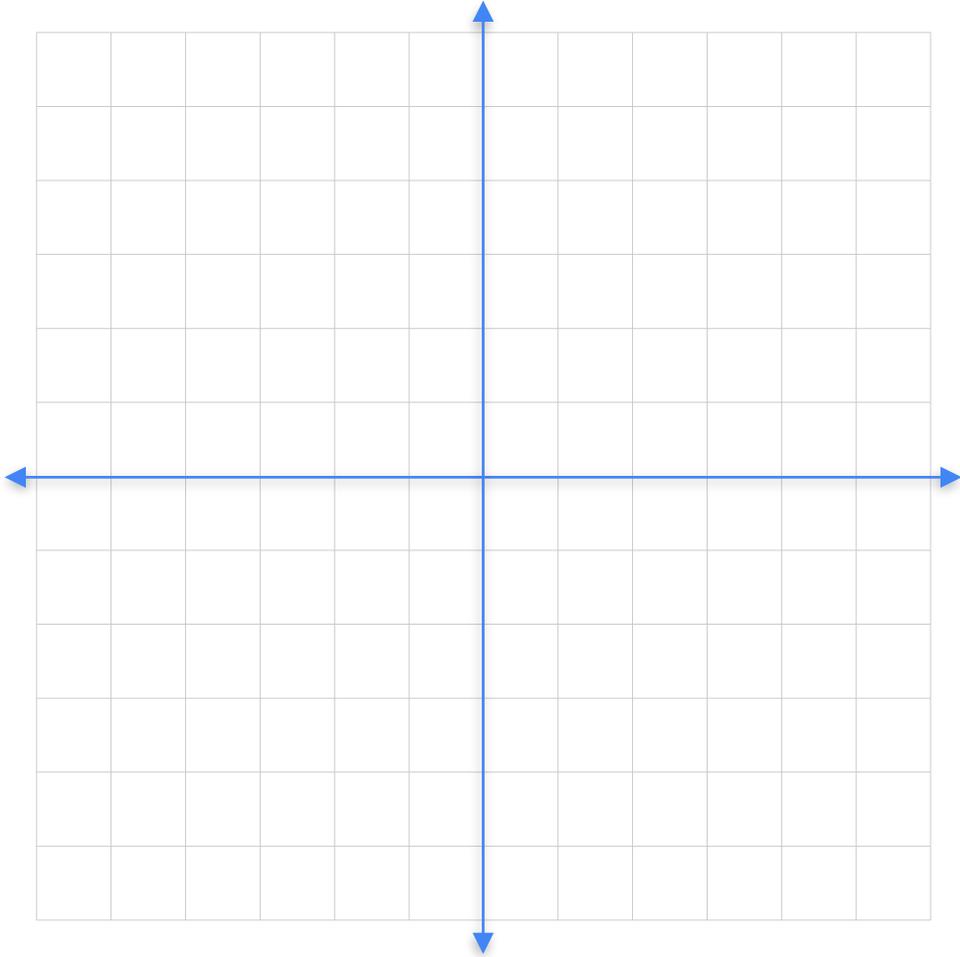
$$u = (1,2)$$

$$\lambda = 3$$

$$\lambda u = (3,6)$$

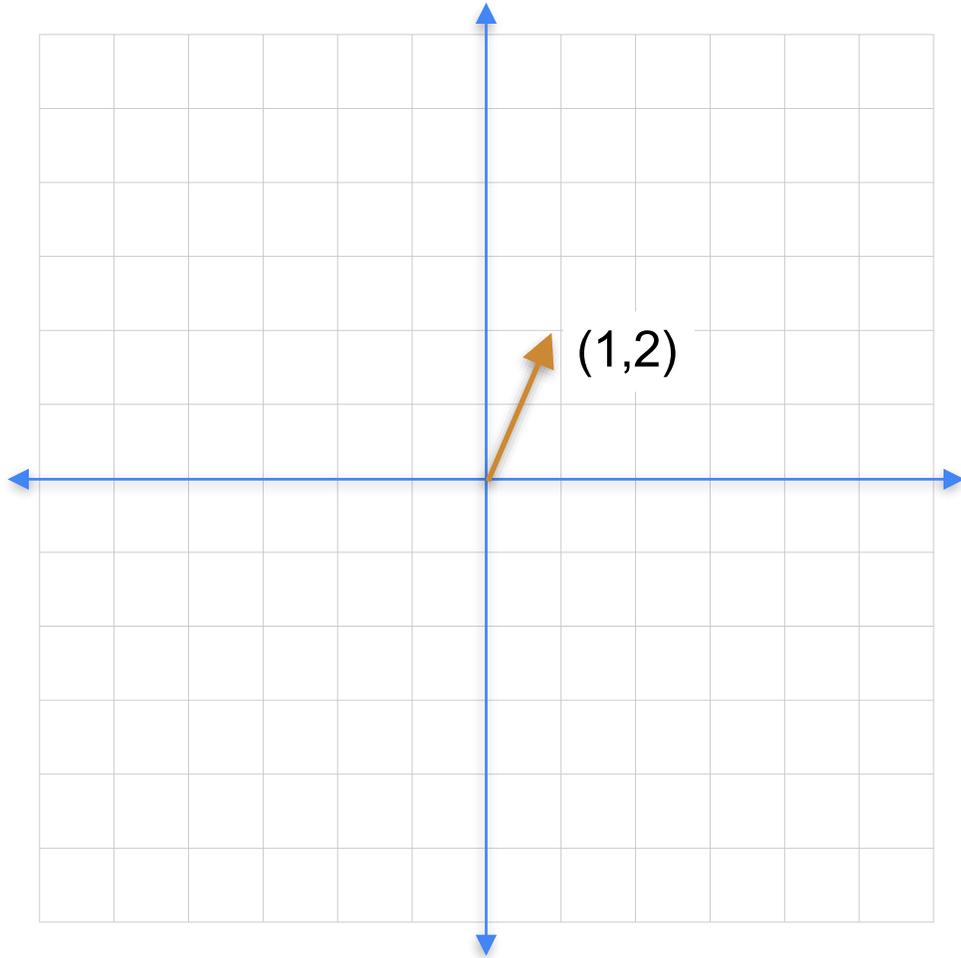
1. 向量及其属性

如果系数为负？



1. 向量及其属性

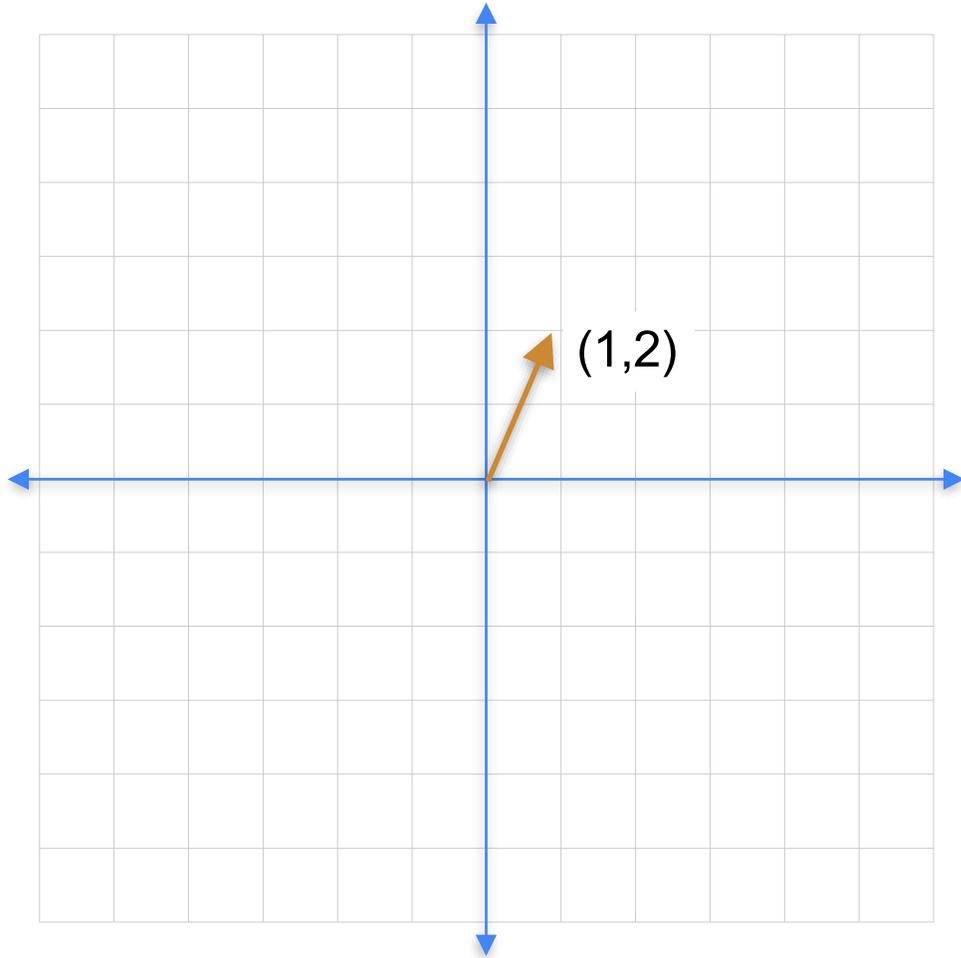
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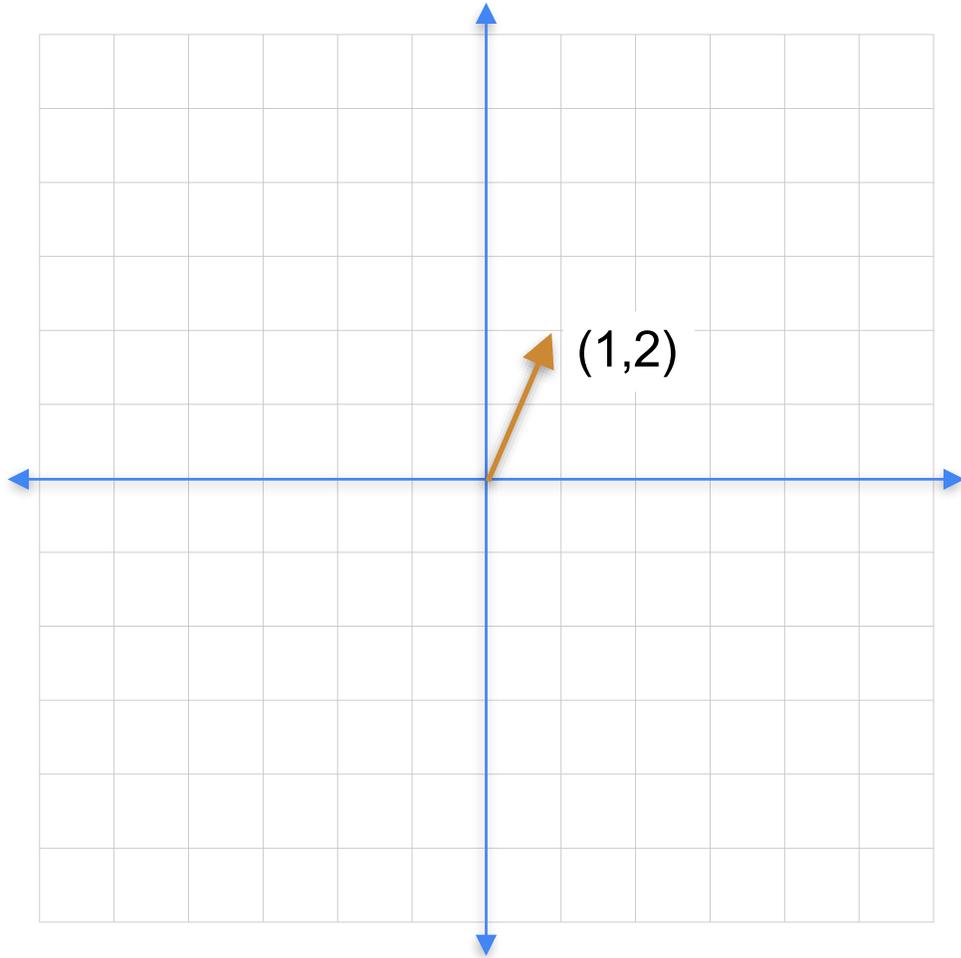


$$u = (1,2)$$

$$\lambda = -2$$

1. 向量及其属性

如果系数为负？



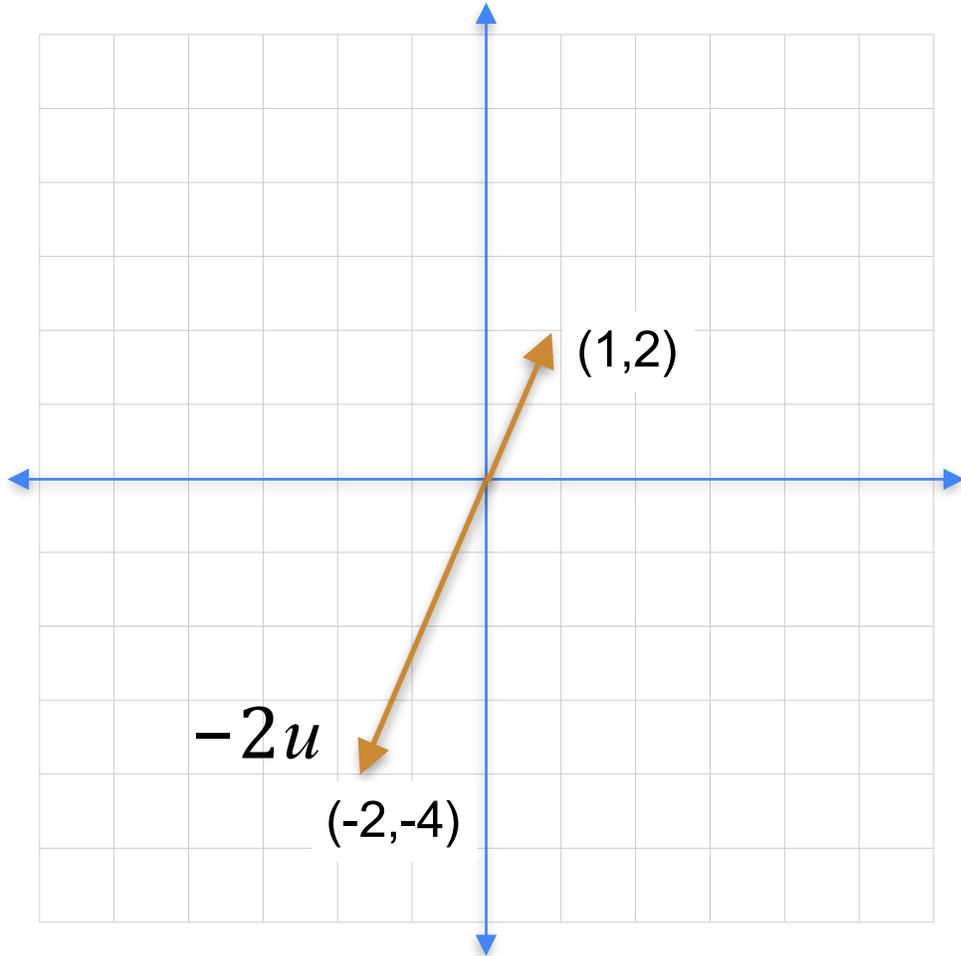
$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

1. 向量及其属性

如果系数为负？



$$u = (1,2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

1. 向量及其属性

线性操作如何简单表达？

1. 向量及其属性

线性操作如何简单表达？

数量

2个苹果

4个香蕉

1个樱桃

1. 向量及其属性

线性操作如何简单表达？

数量

2个苹果

4个香蕉

1个樱桃

价格

苹果：3元一个

香蕉：5元一个

樱桃：2元一个

1. 向量及其属性

线性操作如何简单表达？

数量	价格	总价格
2个苹果	苹果：3元一个	
4个香蕉	香蕉：5元一个	
1个樱桃	樱桃：2元一个	

1. 向量及其属性

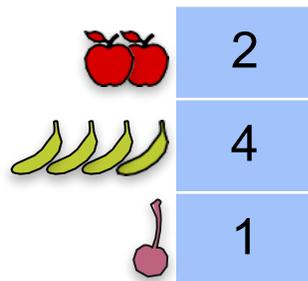
线性操作如何简单表达？

数量

2个苹果

4个香蕉

1个樱桃



价格

苹果：3元一个

香蕉：5元一个

樱桃：2元一个

总价格

1. 向量及其属性

线性操作如何简单表达？

数量

2个苹果

4个香蕉

1个樱桃

	2
	4
	1

价格

苹果：3元一个

香蕉：5元一个

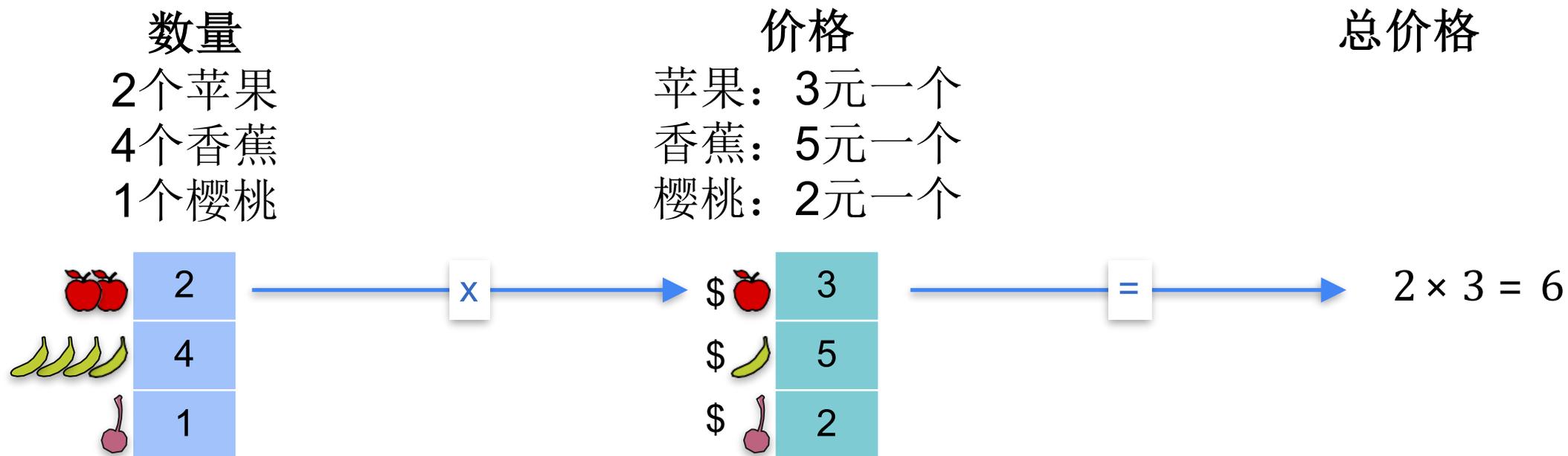
樱桃：2元一个

\$ 	3
\$ 	5
\$ 	2

总价格

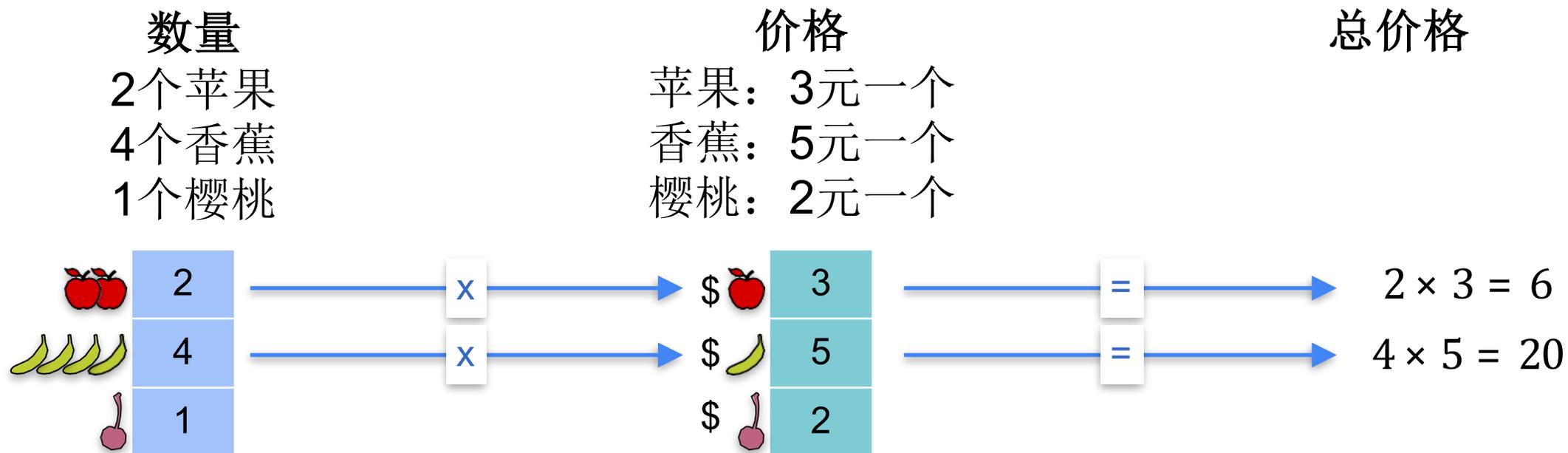
1. 向量及其属性

线性操作如何简单表达？



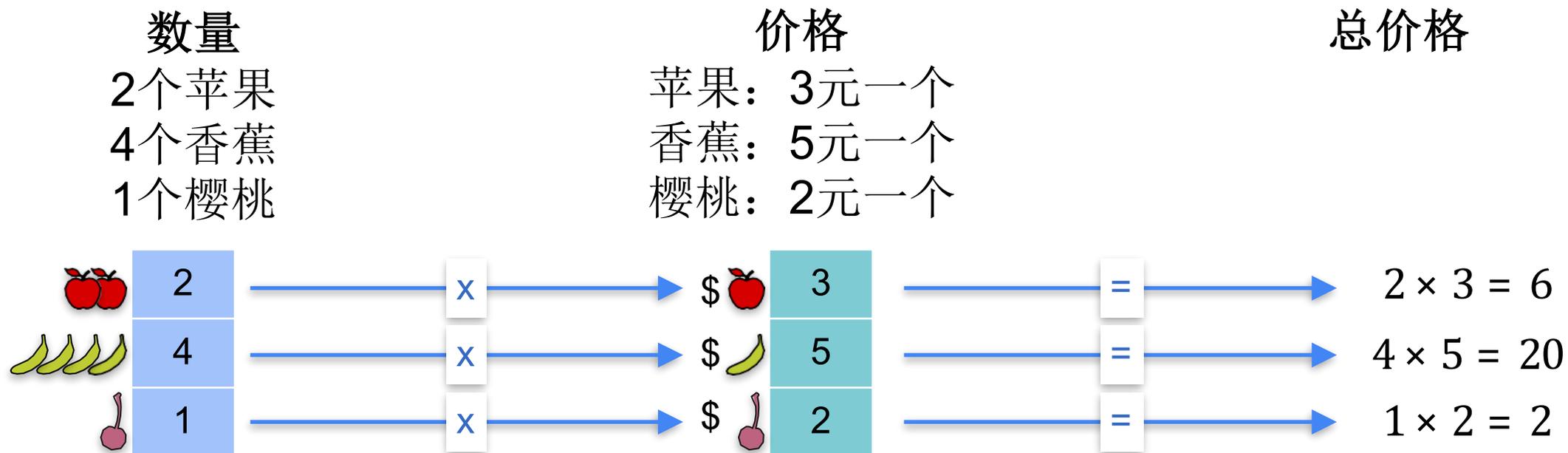
1. 向量及其属性

线性操作如何简单表达？



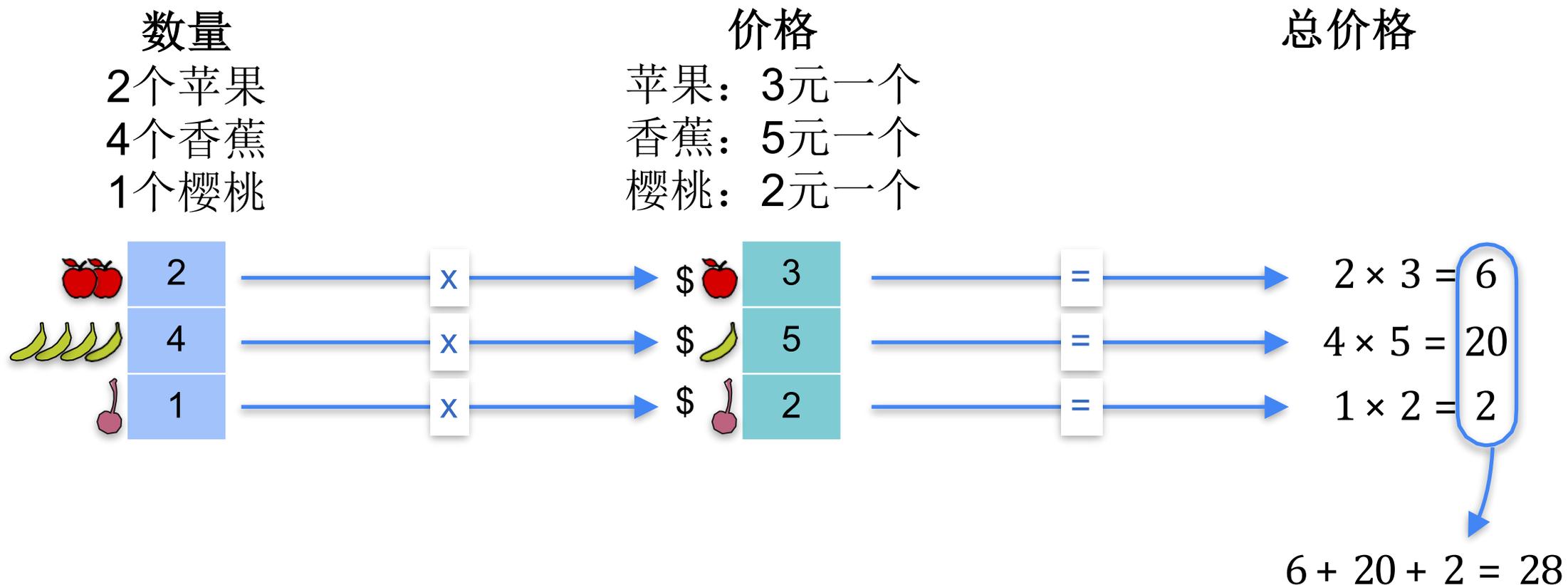
1. 向量及其属性

线性操作如何简单表达？



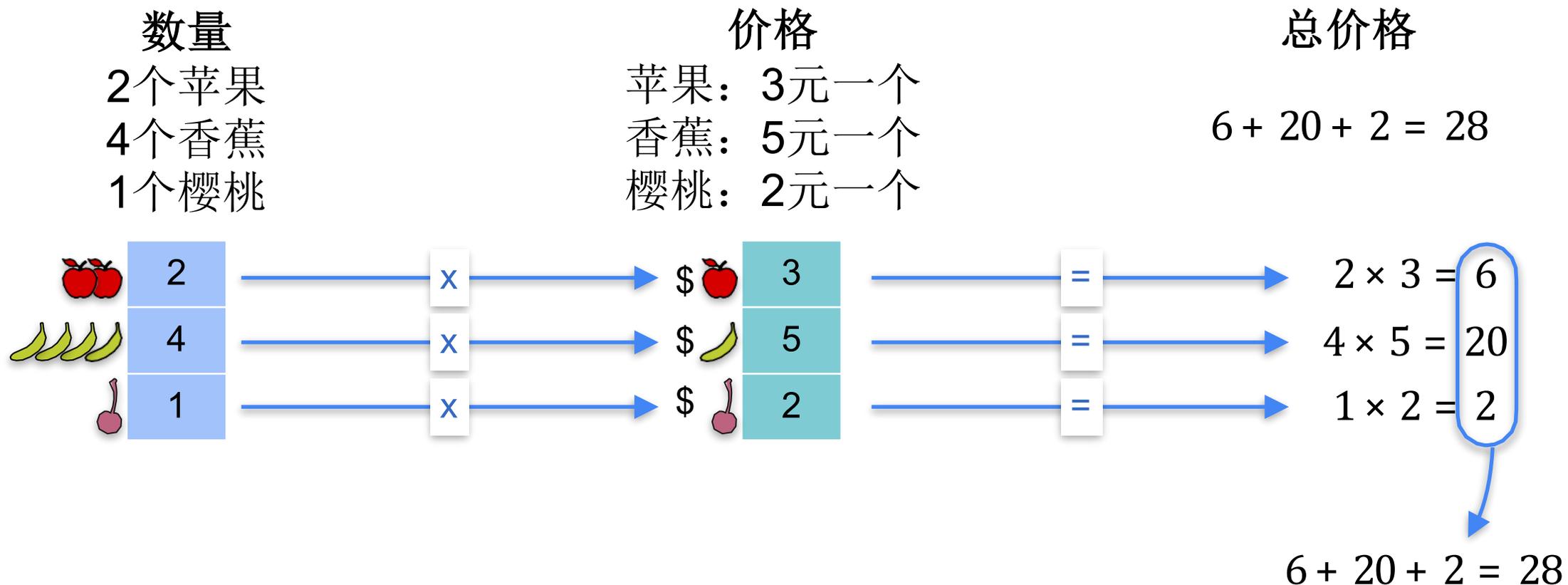
1. 向量及其属性

线性操作如何简单表达？



1. 向量及其属性

线性操作如何简单表达？



1. 向量及其属性

点积 (dot product)

The diagram shows two vectors being multiplied to find a total cost. The first vector, represented by a blue column, contains the quantities of fruit: 2 apples, 4 bananas, and 1 cherry. The second vector, represented by a teal column, contains the prices per unit: \$3 for an apple, \$5 for a banana, and \$2 for a cherry. The dot product of these two vectors is calculated as $2 \times 3 + 4 \times 5 + 1 \times 2 = 6 + 20 + 2 = 28$, resulting in a total cost of \$28.

	2
	4
	1

 ·

\$ 	3
\$ 	5
\$ 	2

 = \$28

1. 向量及其属性

点积 (dot product)

The diagram shows two vectors being multiplied. The first vector is represented by a blue column with three rows: the top row has two red apples and the number 2; the middle row has four yellow bananas and the number 4; the bottom row has one pink cherry and the number 1. The second vector is represented by a teal column with three rows: the top row has a dollar sign, a red apple, and the number 3; the middle row has a dollar sign, a yellow banana, and the number 5; the bottom row has a dollar sign, a pink cherry, and the number 2. A dot product symbol (·) is between the two columns, followed by an equals sign and the result \$28.

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

1. 向量及其属性

点积 (dot product)

The diagram shows two vectors being multiplied. The first vector is represented by three blue boxes containing the numbers 2, 4, and 1. Above these boxes are icons: two red apples above the 2, four yellow bananas above the 4, and one pink cherry above the 1. The second vector is represented by three teal boxes containing the numbers 3, 5, and 2. To the left of these boxes are price tags: '\$' followed by an apple icon above the 3, '\$' followed by a banana icon above the 5, and '\$' followed by a cherry icon above the 2. A dot product symbol (·) is placed between the two vectors, followed by an equals sign and the result '\$28'.

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

1. 向量及其属性

点积 (dot product)

$$\begin{array}{|c|c|c|} \hline 2 & 4 & 1 \\ \hline \end{array} \cdot \begin{array}{|c|} \hline 3 \\ \hline 5 \\ \hline 2 \\ \hline \end{array} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

1. 向量及其属性

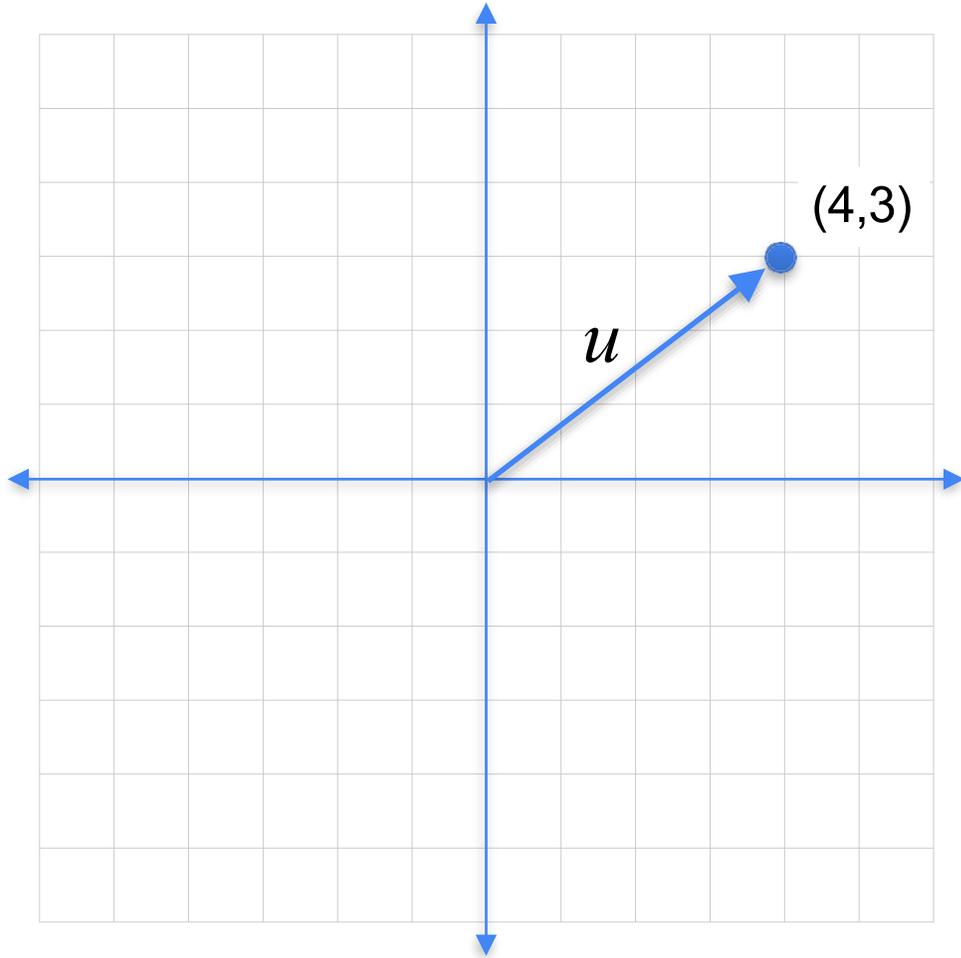
点积 (dot product)

向量 $\vec{a} = [a_1, a_2, \dots, a_n]$ 和 $\vec{b} = [b_1, b_2, \dots, b_n]$ 的点积定义为:

$$\vec{a} \cdot \vec{b} = \sum_{i=1}^n a_i b_i = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$$

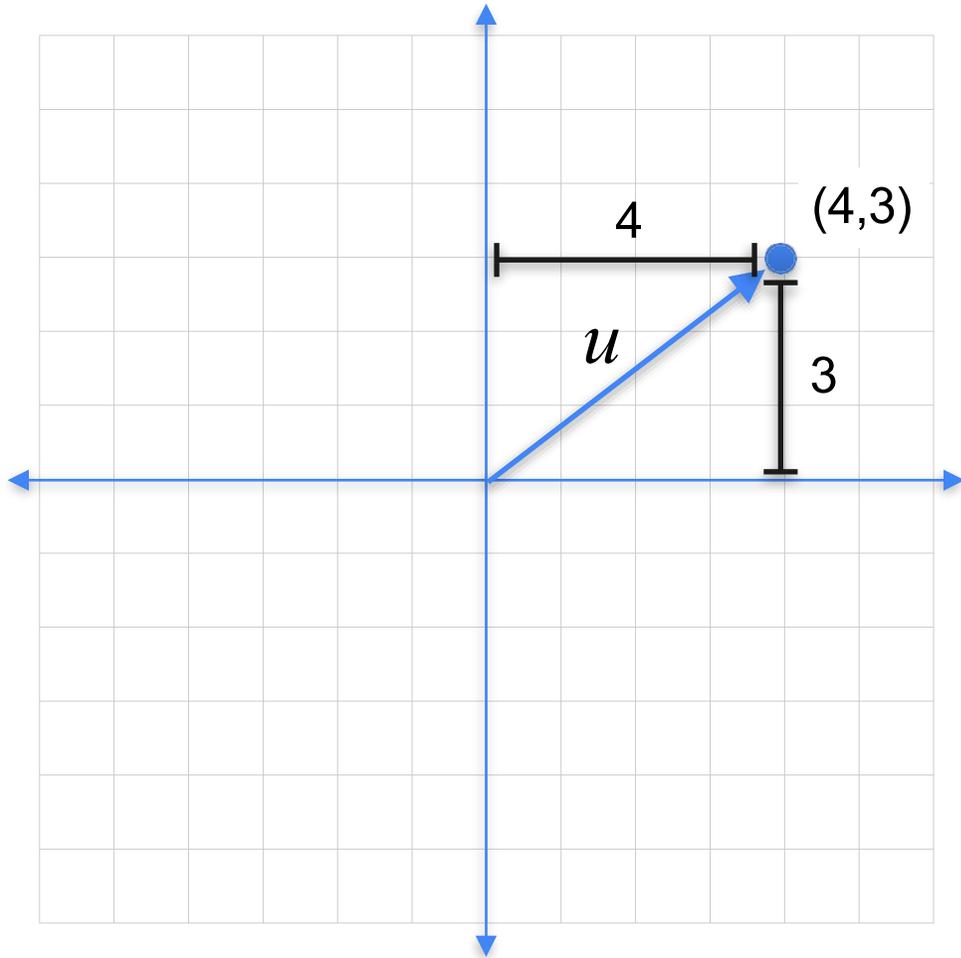
1. 向量及其属性

用点积计算范数



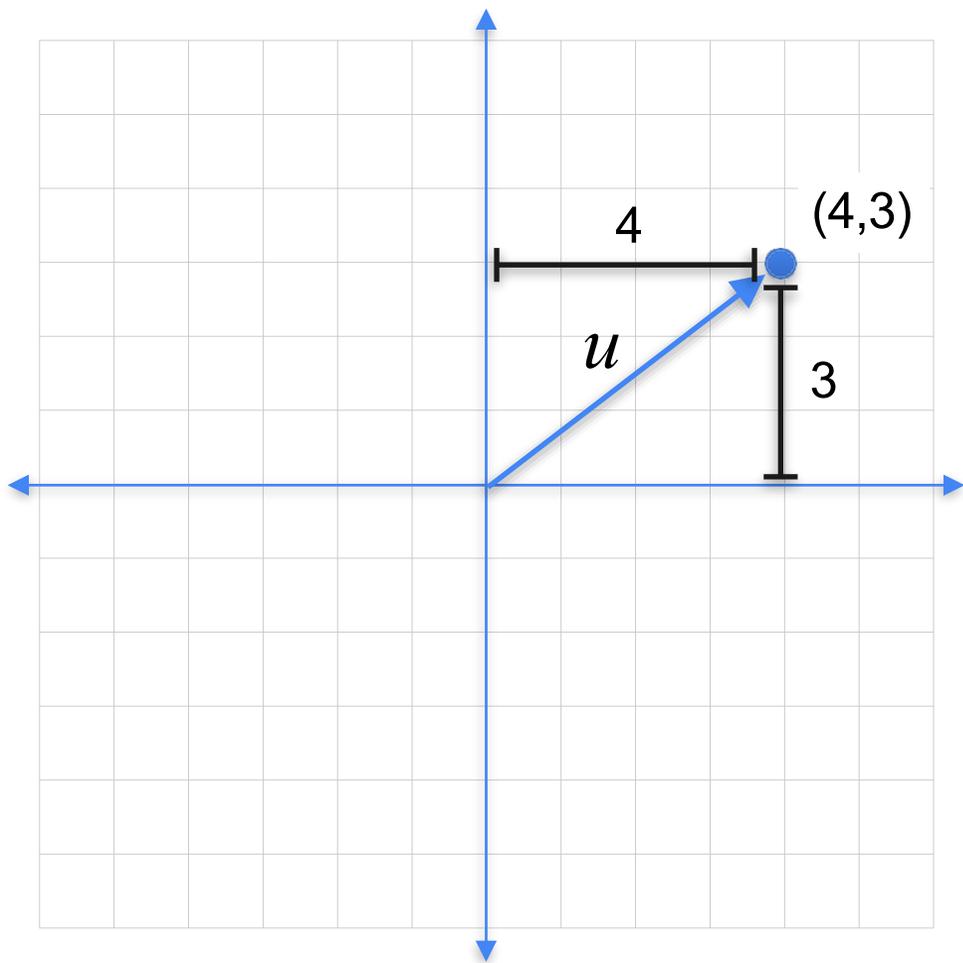
1. 向量及其属性

用点积计算范数



1. 向量及其属性

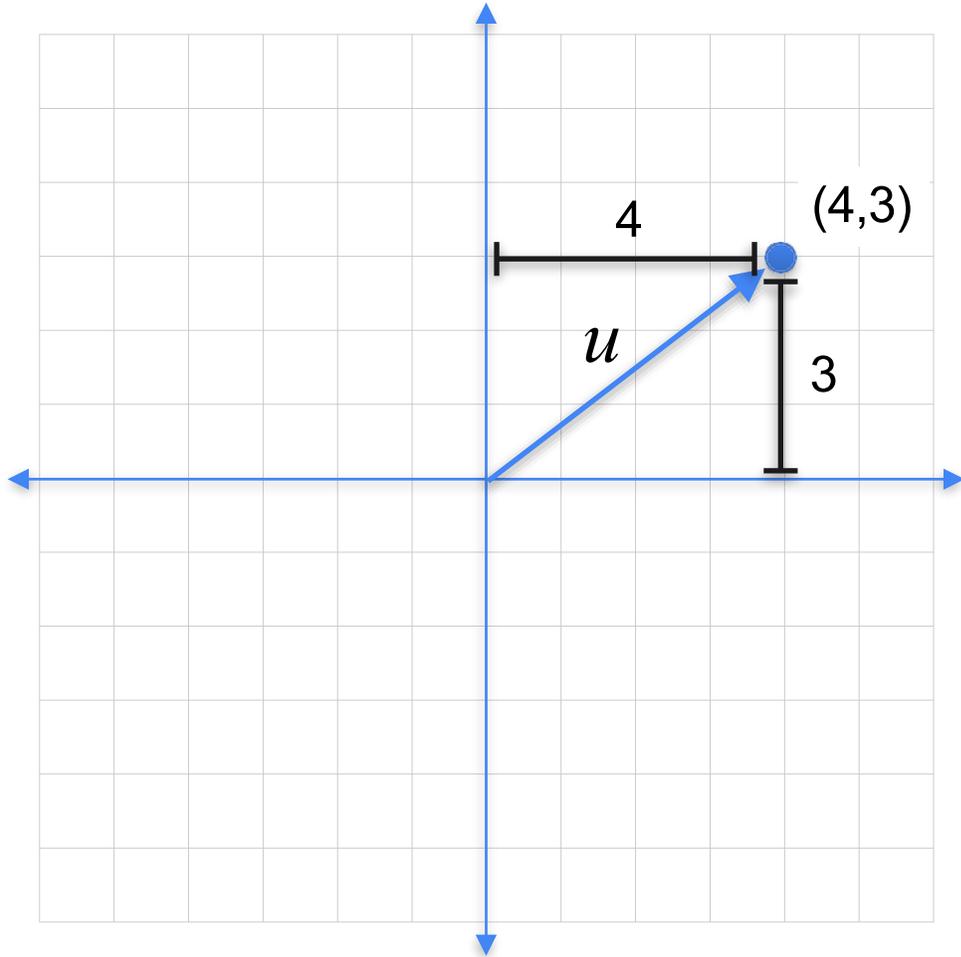
用点积计算范数



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

1. 向量及其属性

用点积计算范数

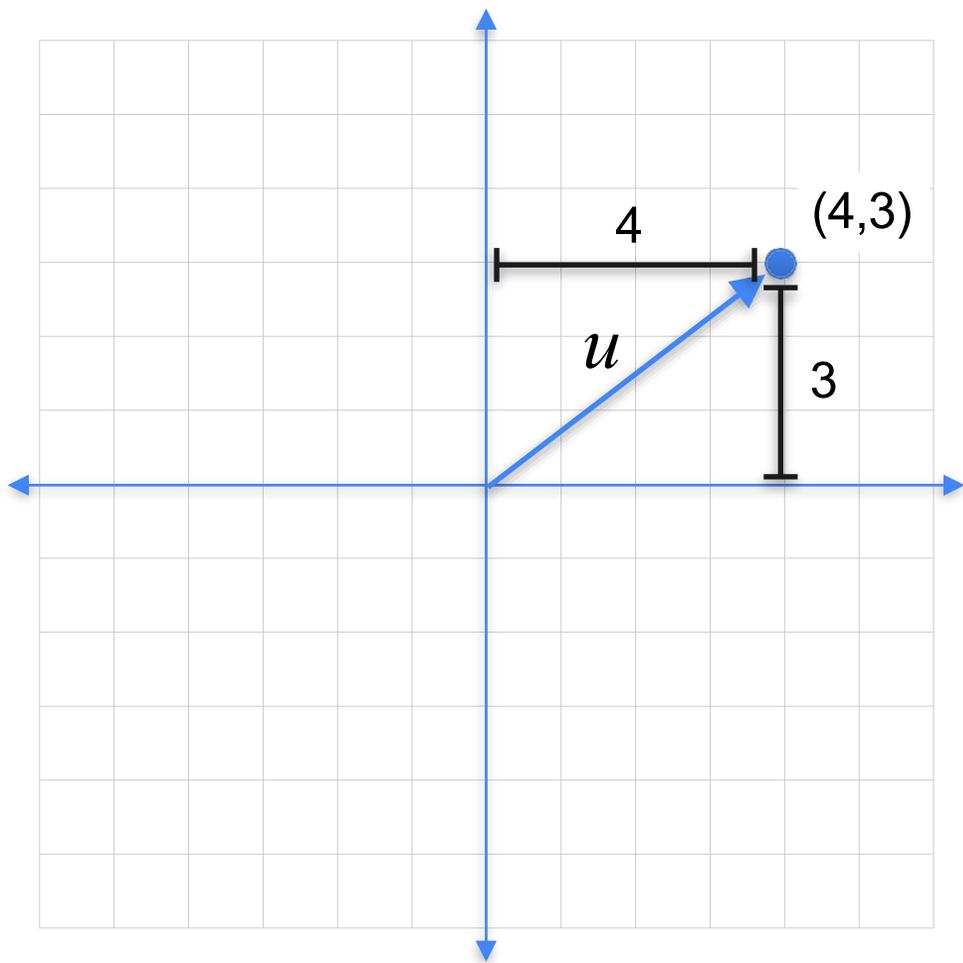


$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|} \hline 4 \\ \hline \end{array} \begin{array}{|c|} \hline 3 \\ \hline \end{array} \begin{array}{|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 25 \\ \hline \end{array}$$

1. 向量及其属性

用点积计算范数



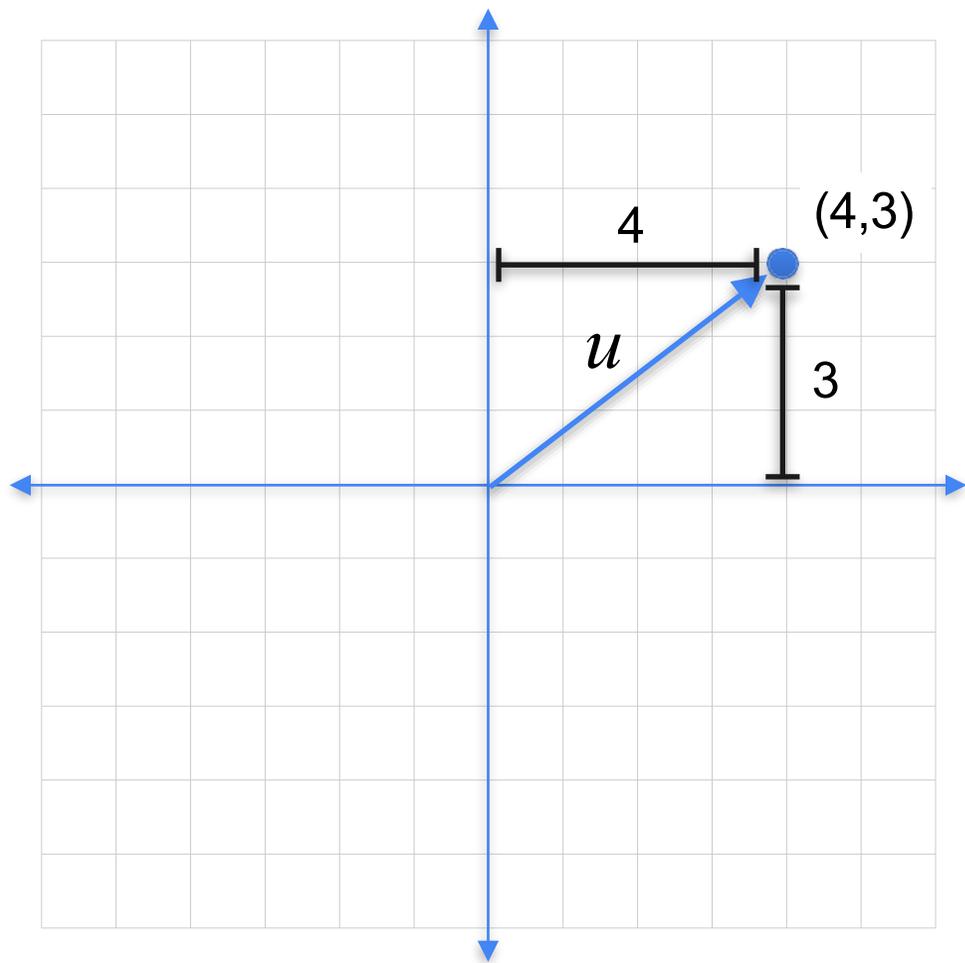
$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

1. 向量及其属性

用点积计算范数



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

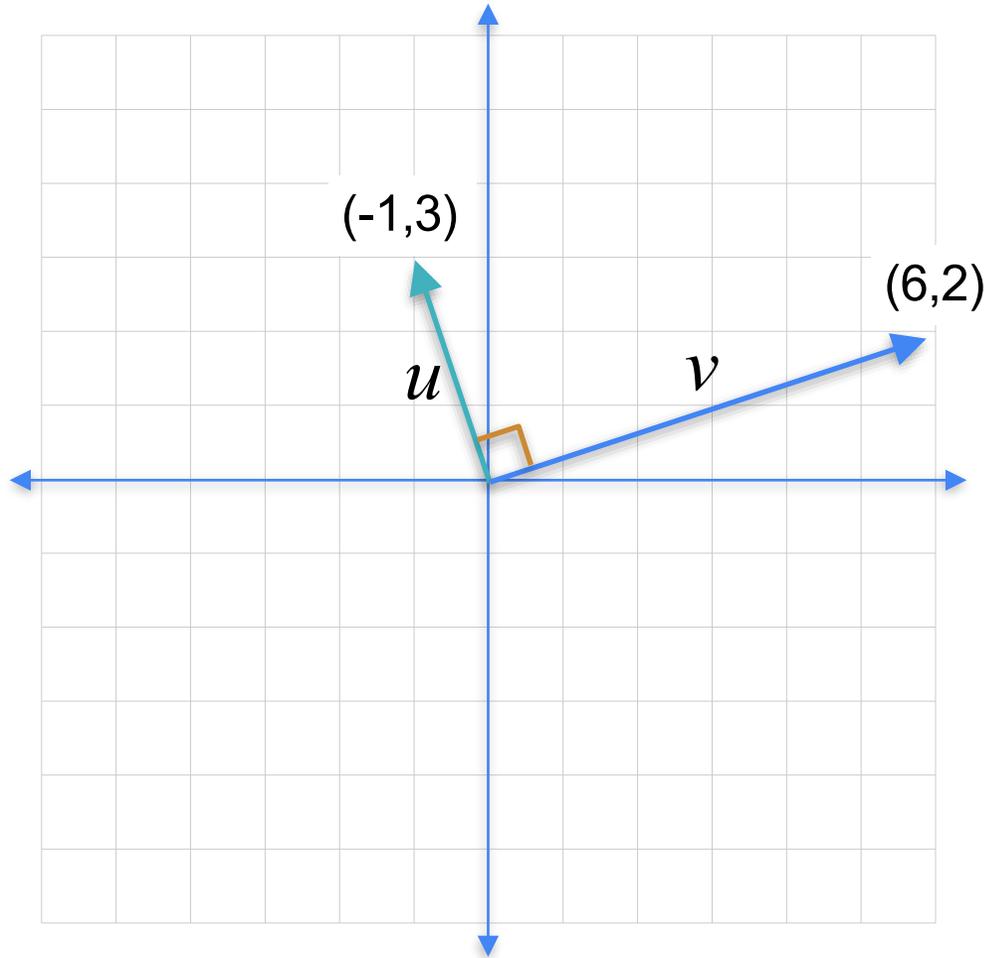
$$\begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 25$$

$$L2 - norm = \sqrt{\text{dot product}(u, u)}$$

$$\|u\|_2 = \sqrt{\langle u, u \rangle}$$

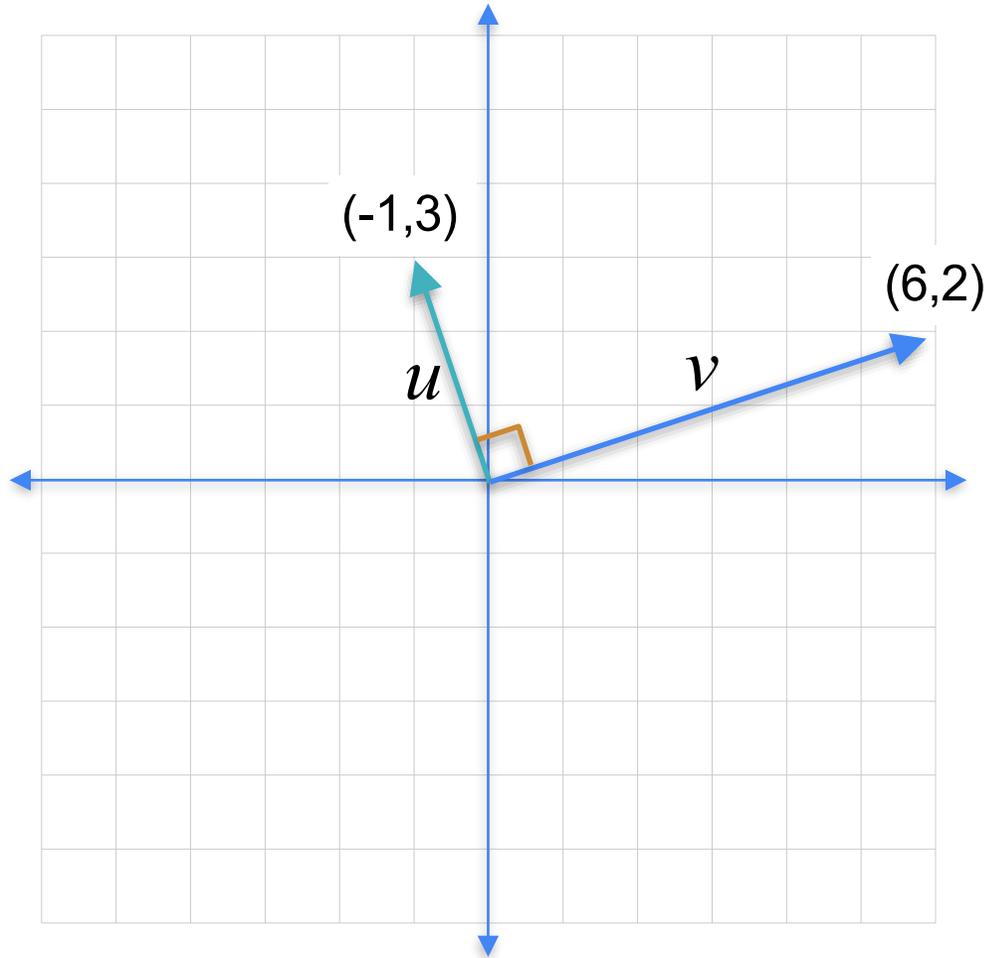
1. 向量及其属性

正交向量点积为0



1. 向量及其属性

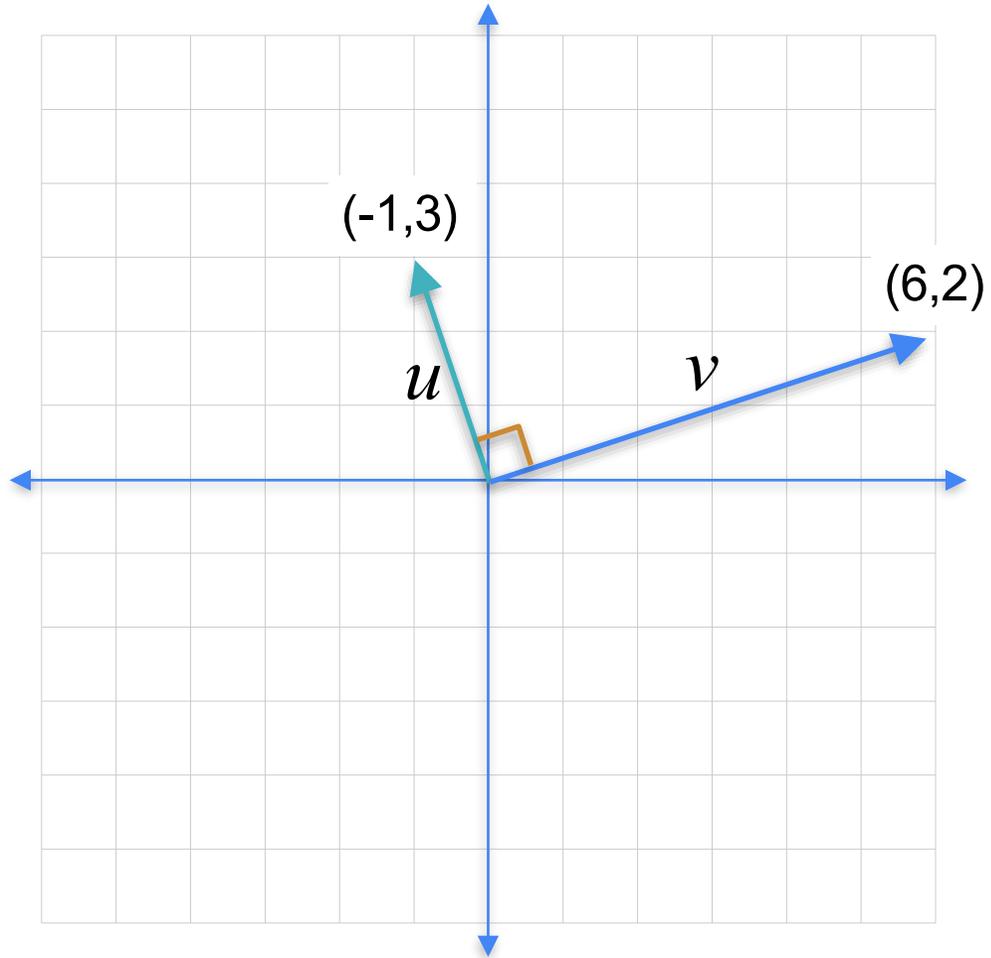
正交向量点积为0



6 2

1. 向量及其属性

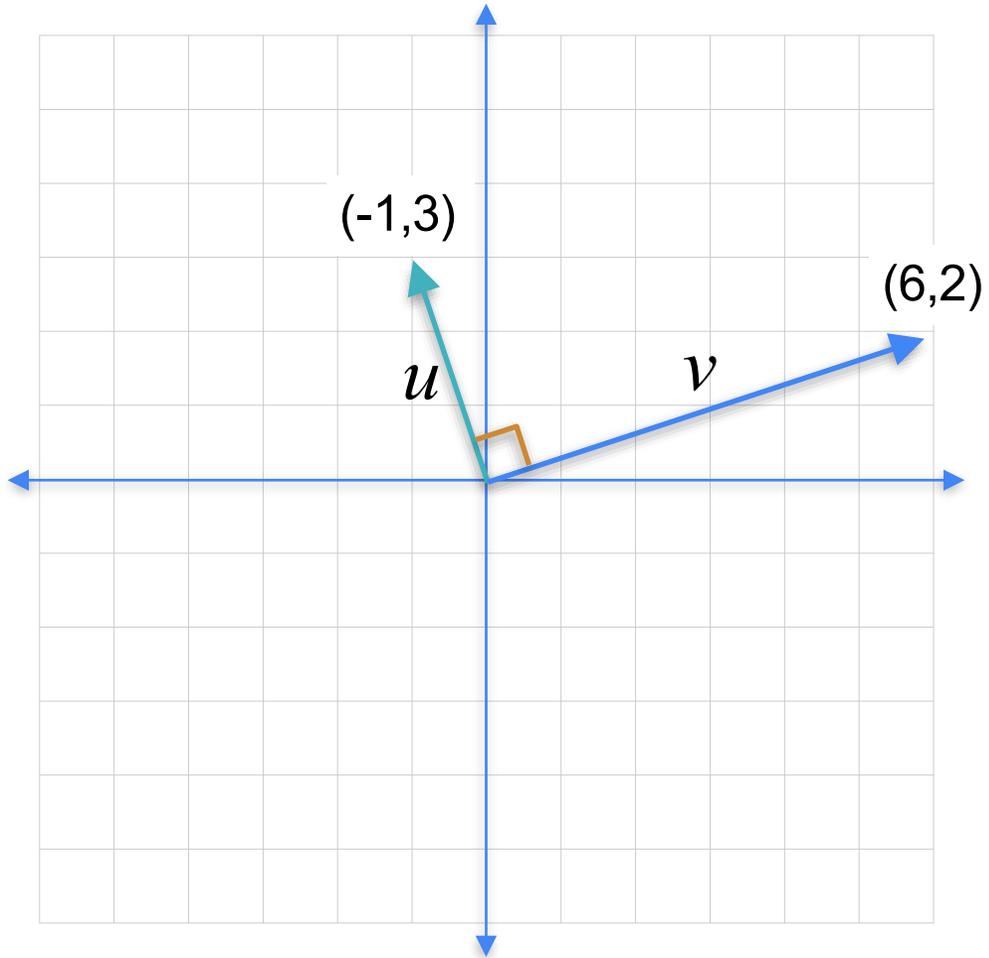
正交向量点积为0



6	2	-1
		3

1. 向量及其属性

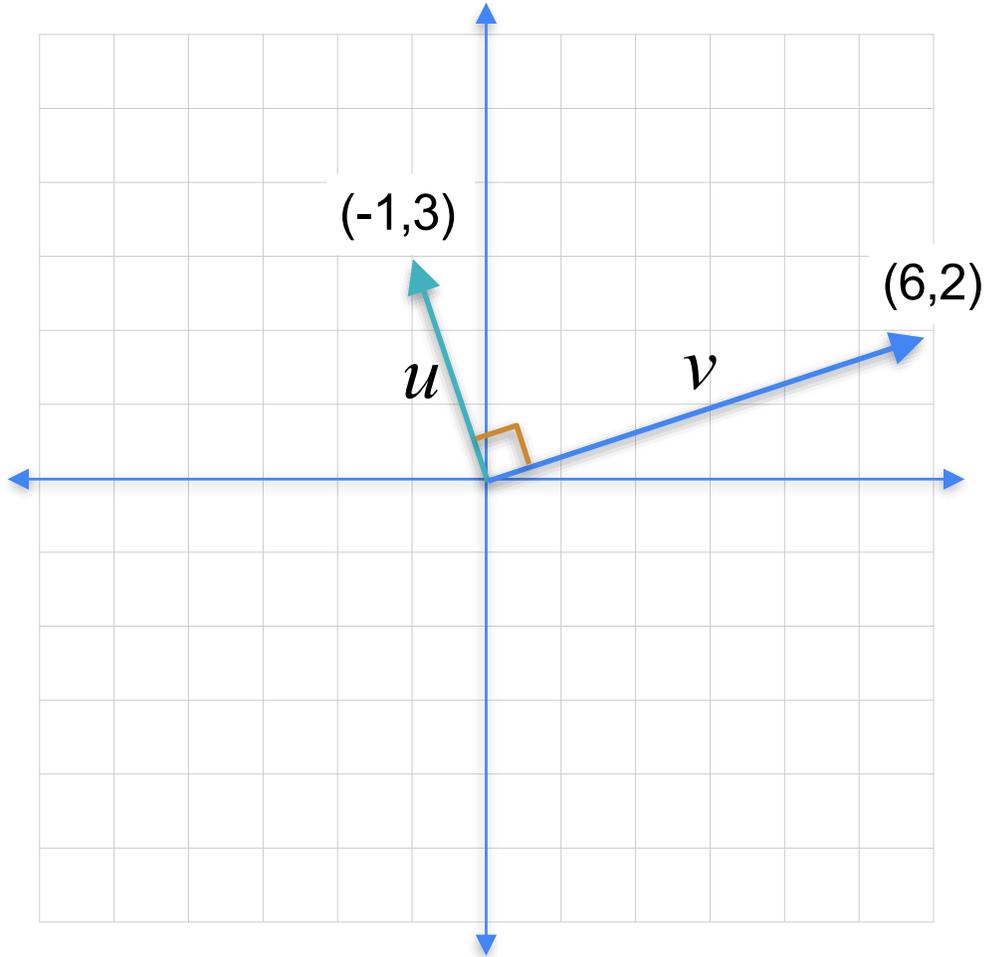
正交向量点积为0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

1. 向量及其属性

正交向量点积为0



$$\begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 0$$

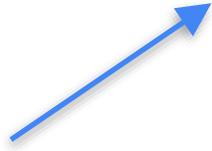
$$\langle u, v \rangle = 0$$

1. 向量及其属性

点积

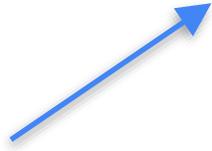
1. 向量及其属性

点积



1. 向量及其属性

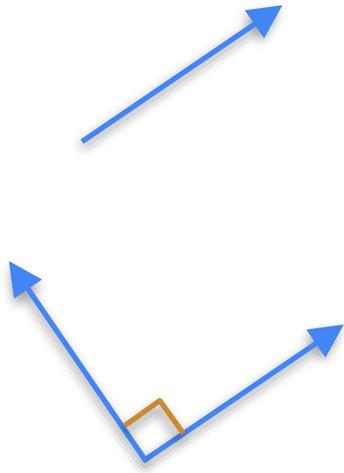
点积



$$\langle u, u \rangle = |u|^2$$

1. 向量及其属性

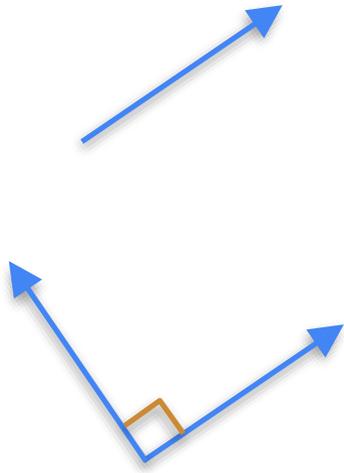
点积



$$\langle u, u \rangle = |u|^2$$

1. 向量及其属性

点积

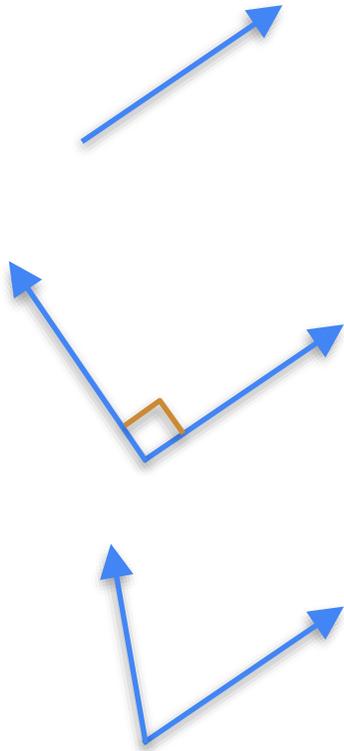


$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

1. 向量及其属性

点积

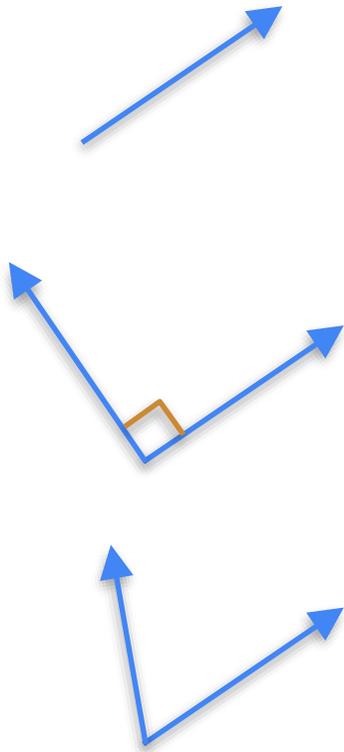


$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

1. 向量及其属性

点积



$$\langle u, u \rangle = |u|^2$$

$$\langle u, v \rangle = 0$$

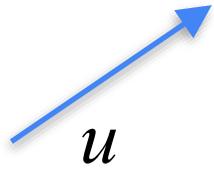
$$\langle u, v \rangle = ?$$

1. 向量及其属性

点积

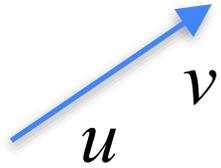
1. 向量及其属性

点积



1. 向量及其属性

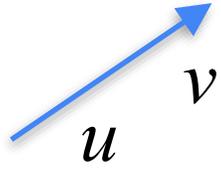
点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

1. 向量及其属性

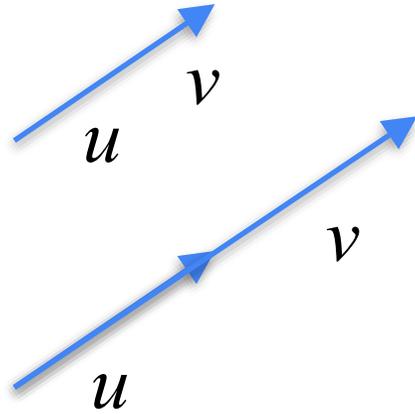
点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

1. 向量及其属性

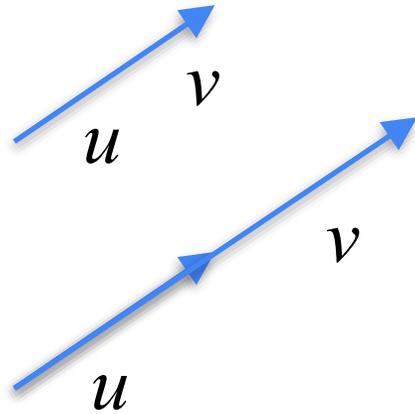
点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

1. 向量及其属性

点积

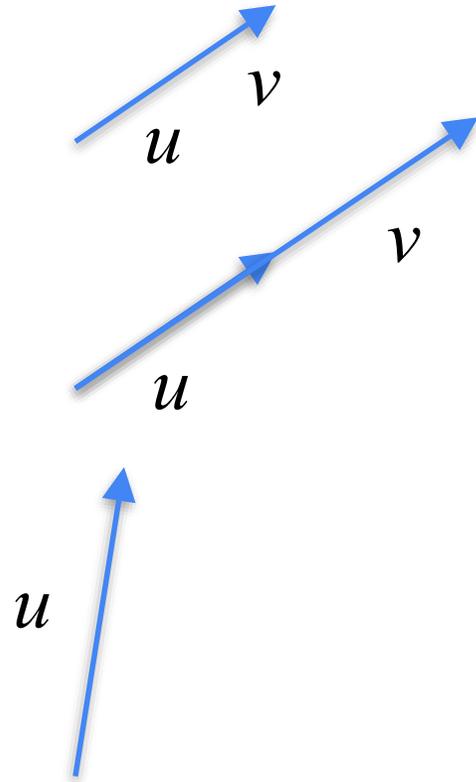


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

1. 向量及其属性

点积

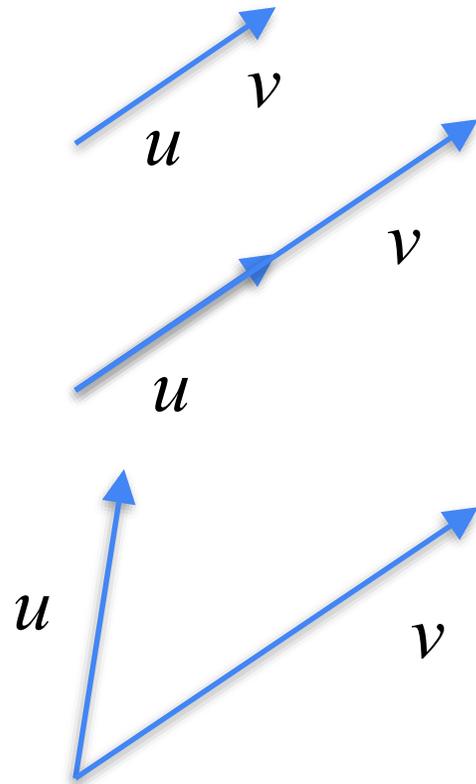


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

$$\langle u, v \rangle = |u| \cdot |v|$$

1. 向量及其属性

点积

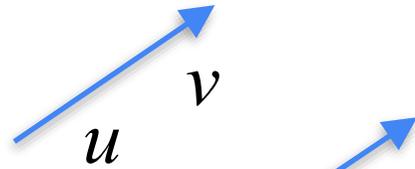


$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

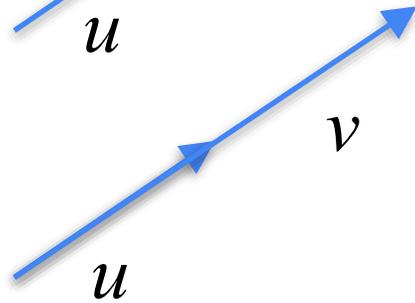
$$\langle u, v \rangle = |u| \cdot |v|$$

1. 向量及其属性

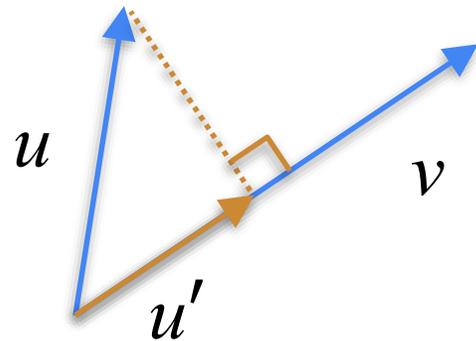
点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

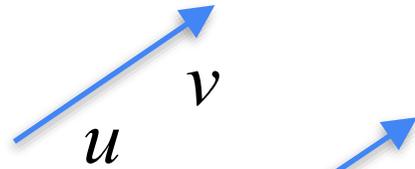


$$\langle u, v \rangle = |u| \cdot |v|$$

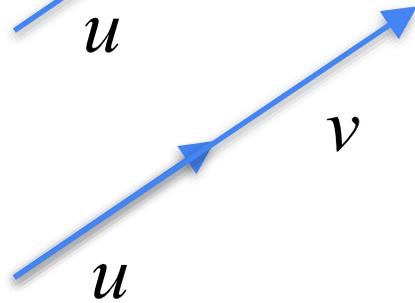


1. 向量及其属性

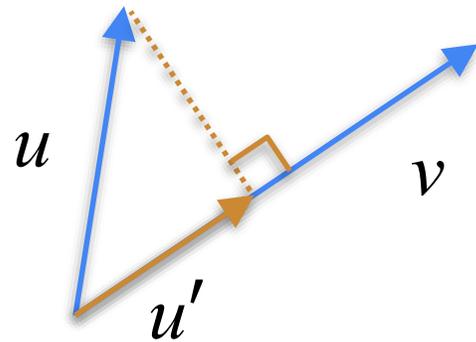
点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



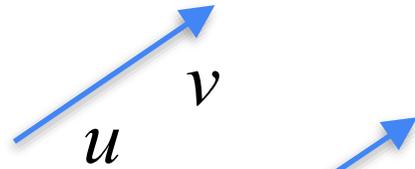
$$\langle u, v \rangle = |u| \cdot |v|$$



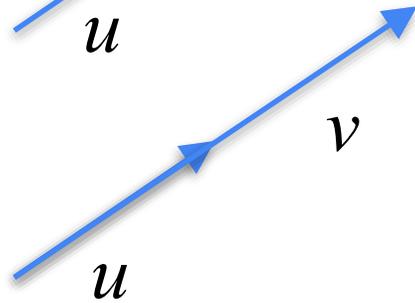
$$\langle u, v \rangle = |u'| \cdot |v|$$

1. 向量及其属性

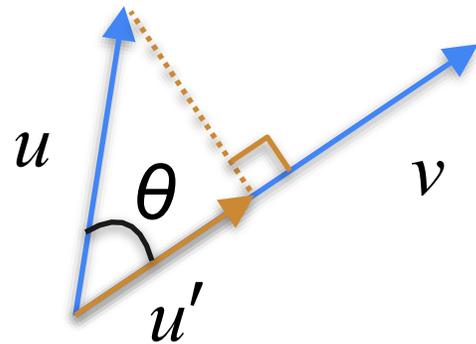
点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$



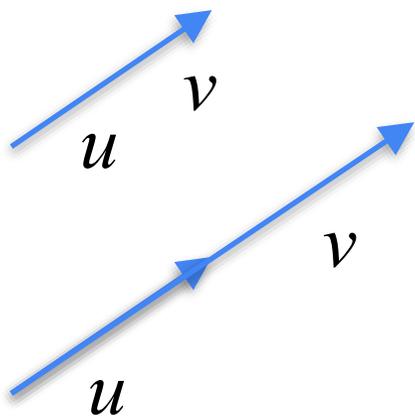
$$\langle u, v \rangle = |u| \cdot |v|$$



$$\langle u, v \rangle = |u'| \cdot |v|$$

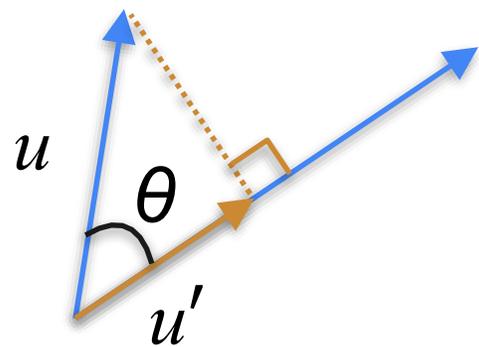
1. 向量及其属性

点积



$$\langle u, u \rangle = |u|^2 = |u| \cdot |u|$$

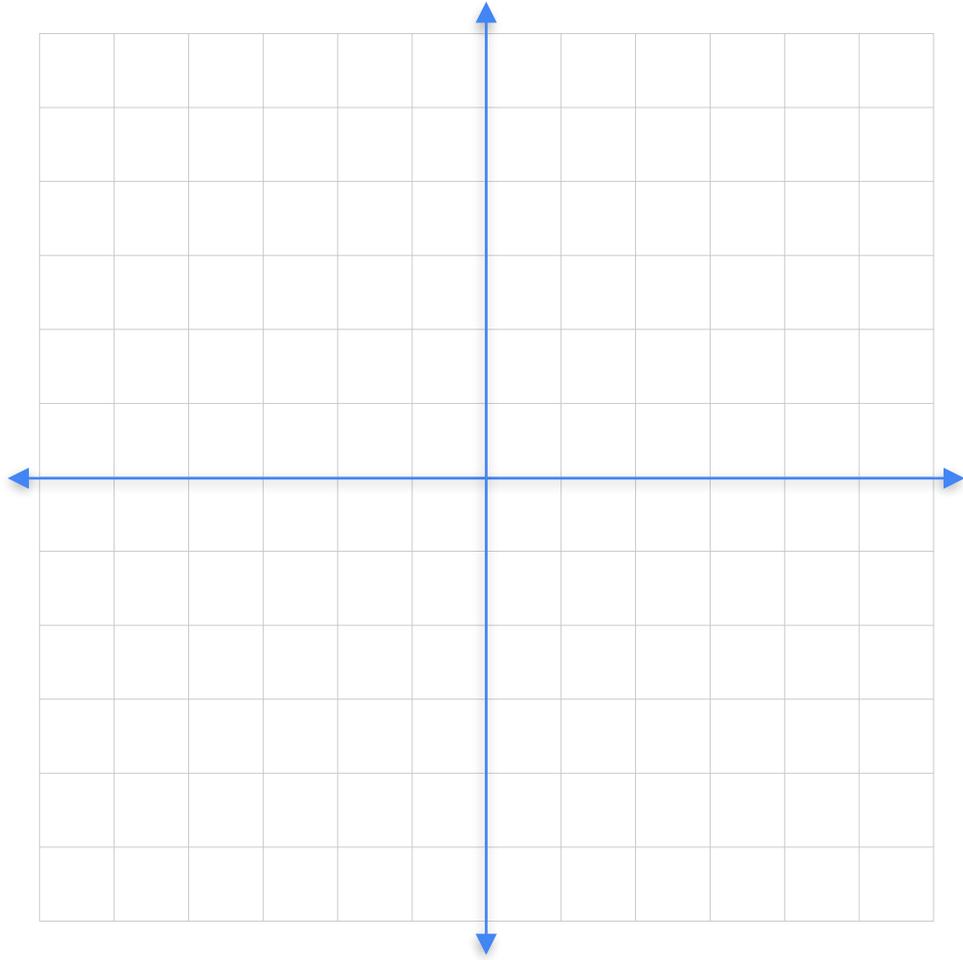
$$\langle u, v \rangle = |u| \cdot |v|$$



$$\begin{aligned} \langle u, v \rangle &= |u'| \cdot |v| \\ &= |u| |v| \cos(\theta) \end{aligned}$$

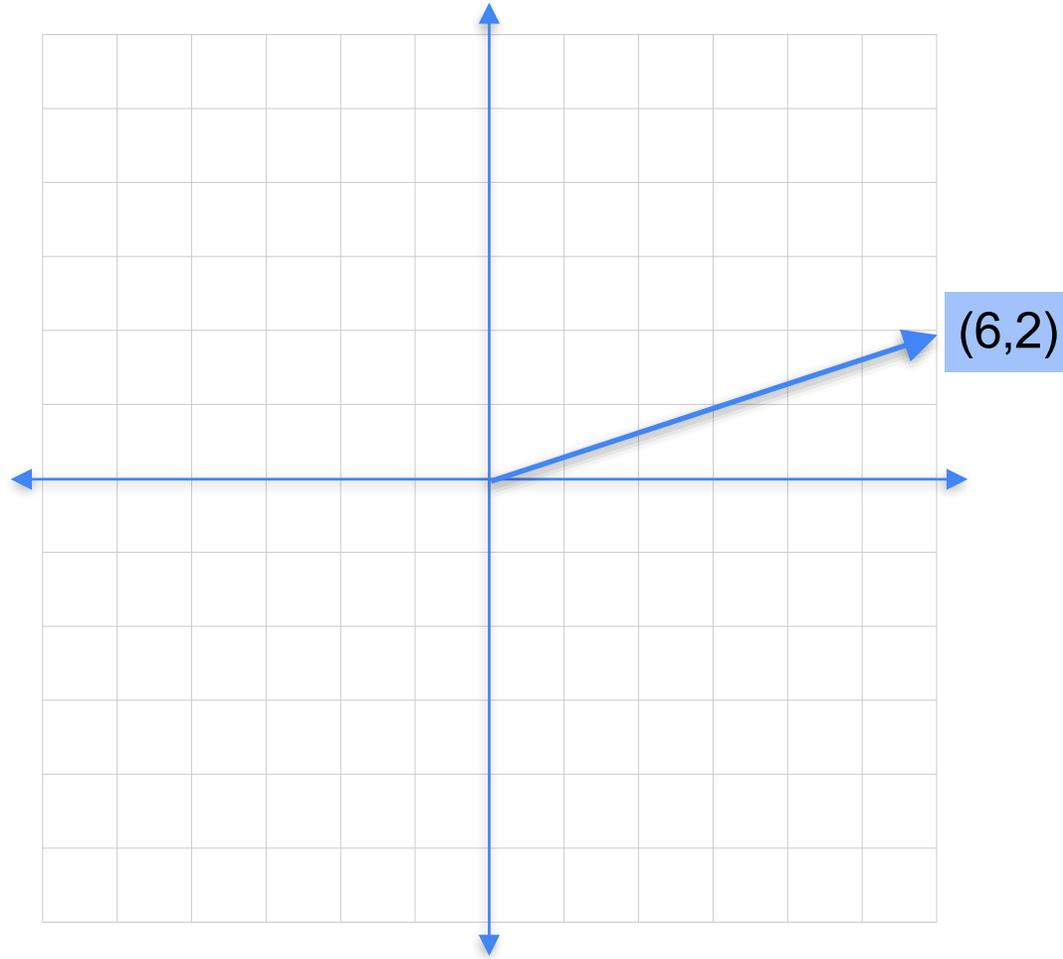
1. 向量及其属性

点积



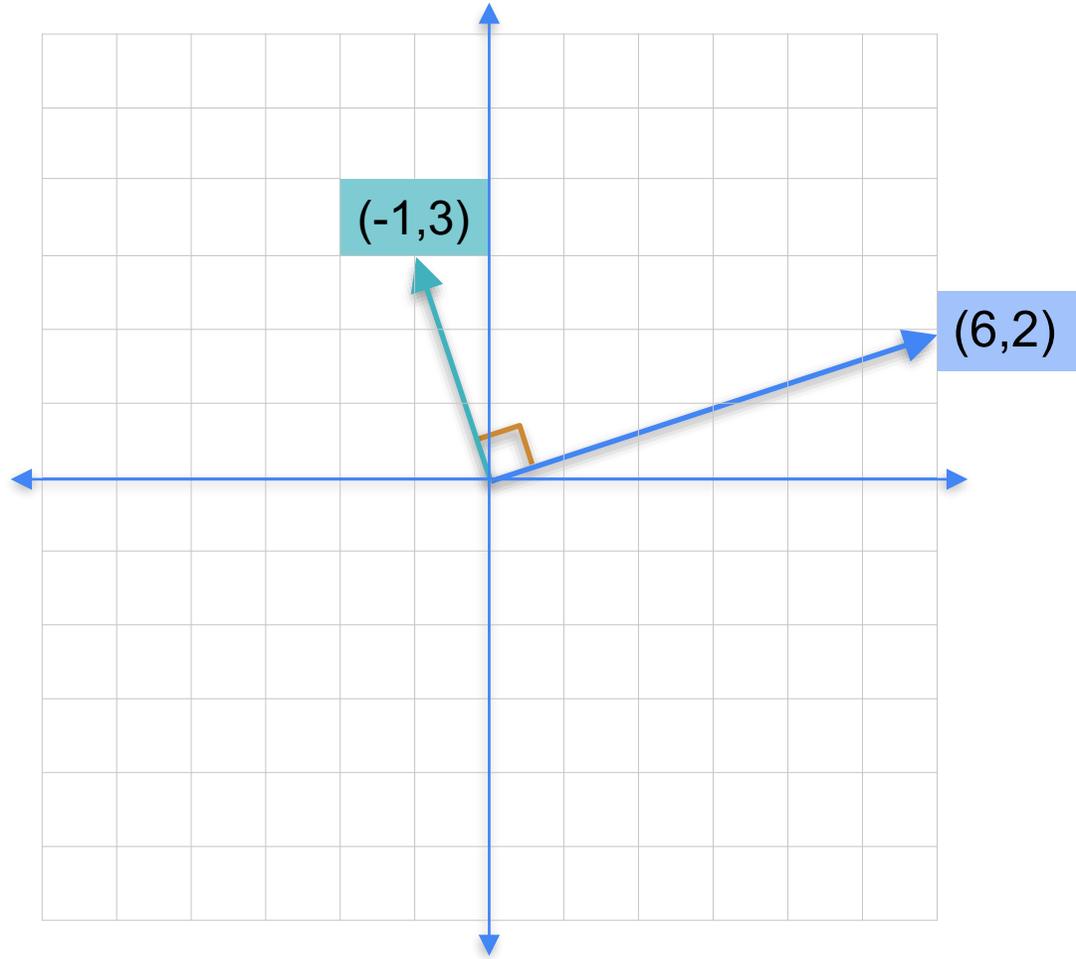
1. 向量及其属性

点积



1. 向量及其属性

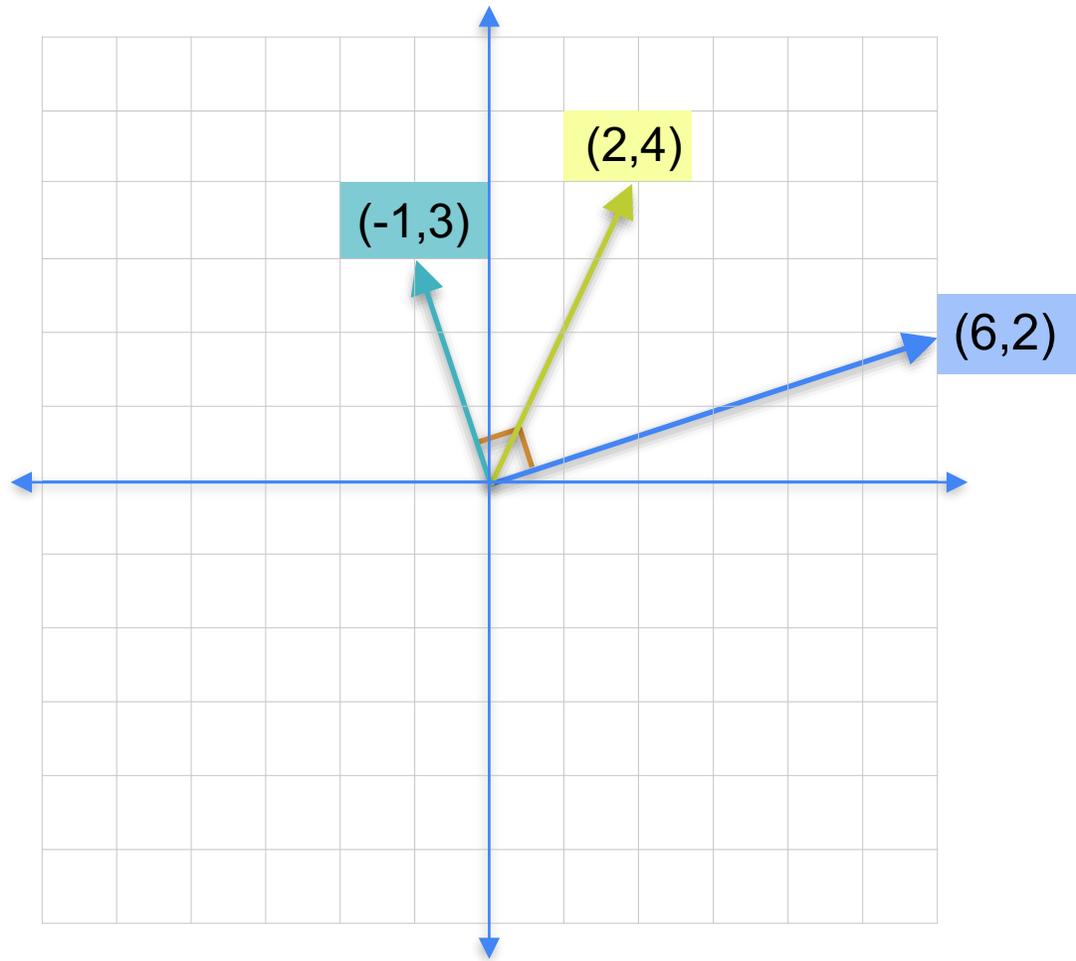
点积



$$\begin{matrix} 6 & 2 \end{matrix} \cdot \begin{matrix} -1 \\ 3 \end{matrix} = 0$$

1. 向量及其属性

点积

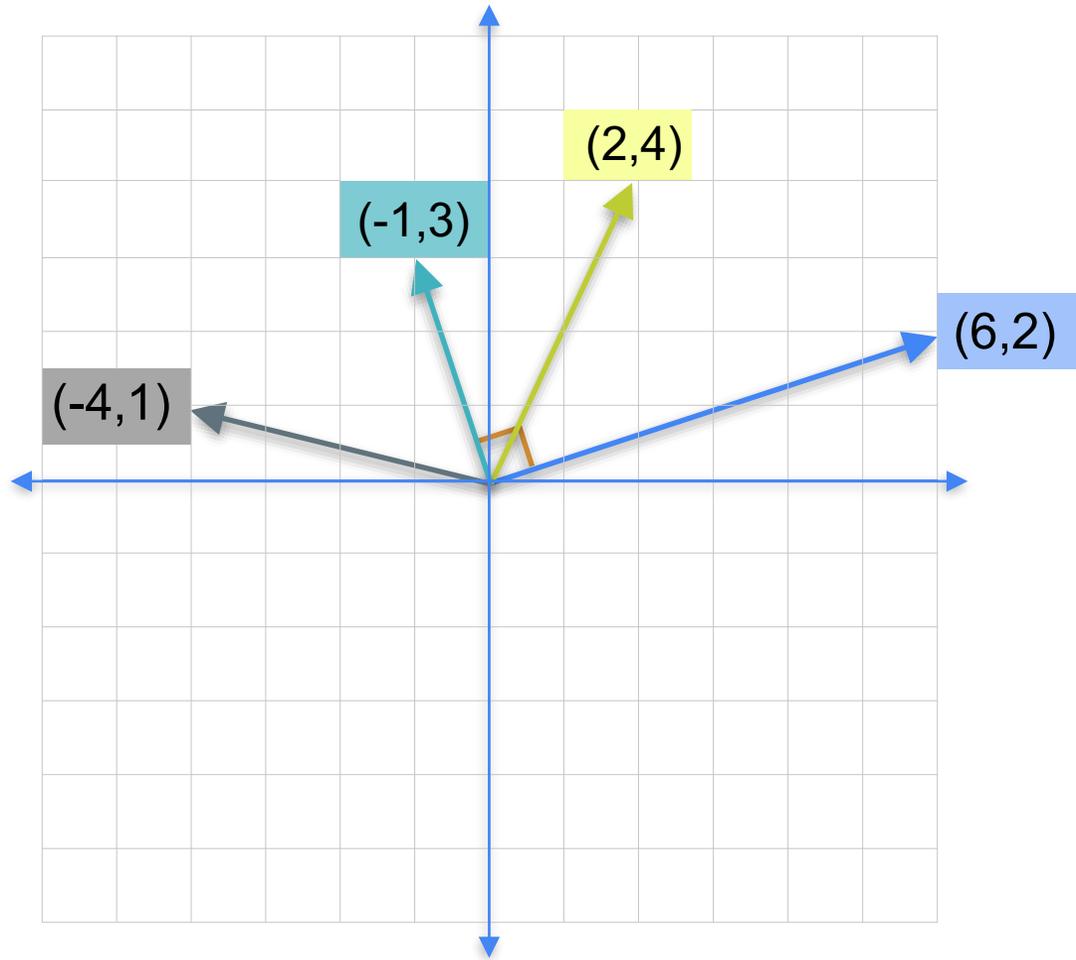


$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

1. 向量及其属性

点积



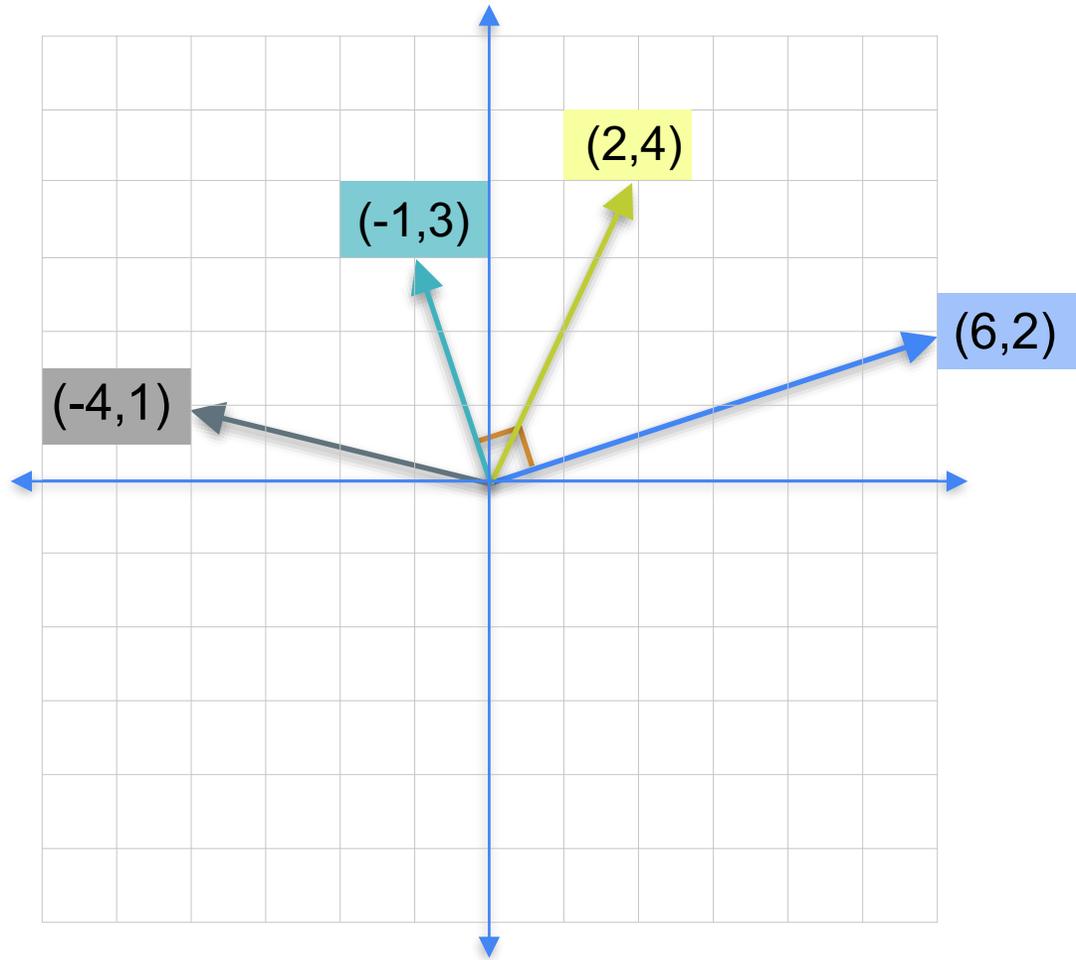
$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22$$

1. 向量及其属性

点积



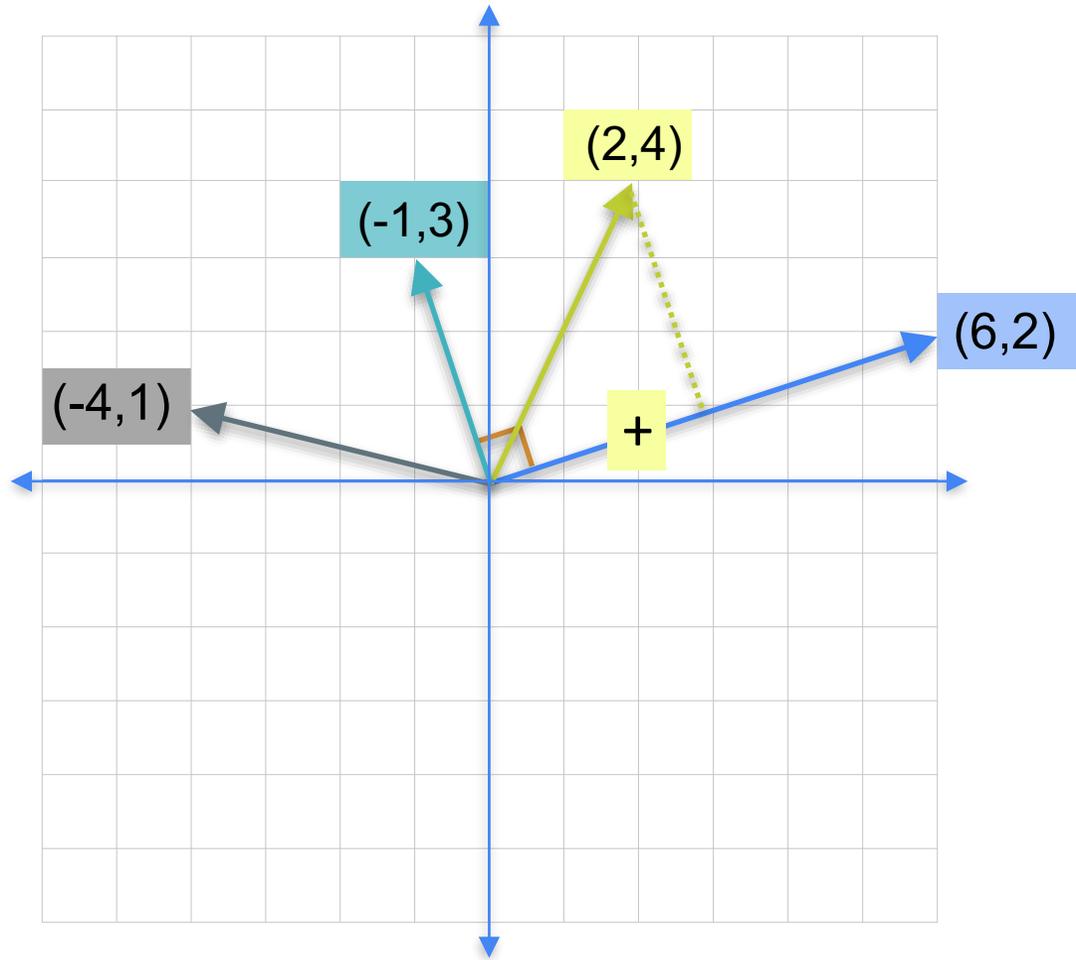
$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22$$

1. 向量及其属性

点积



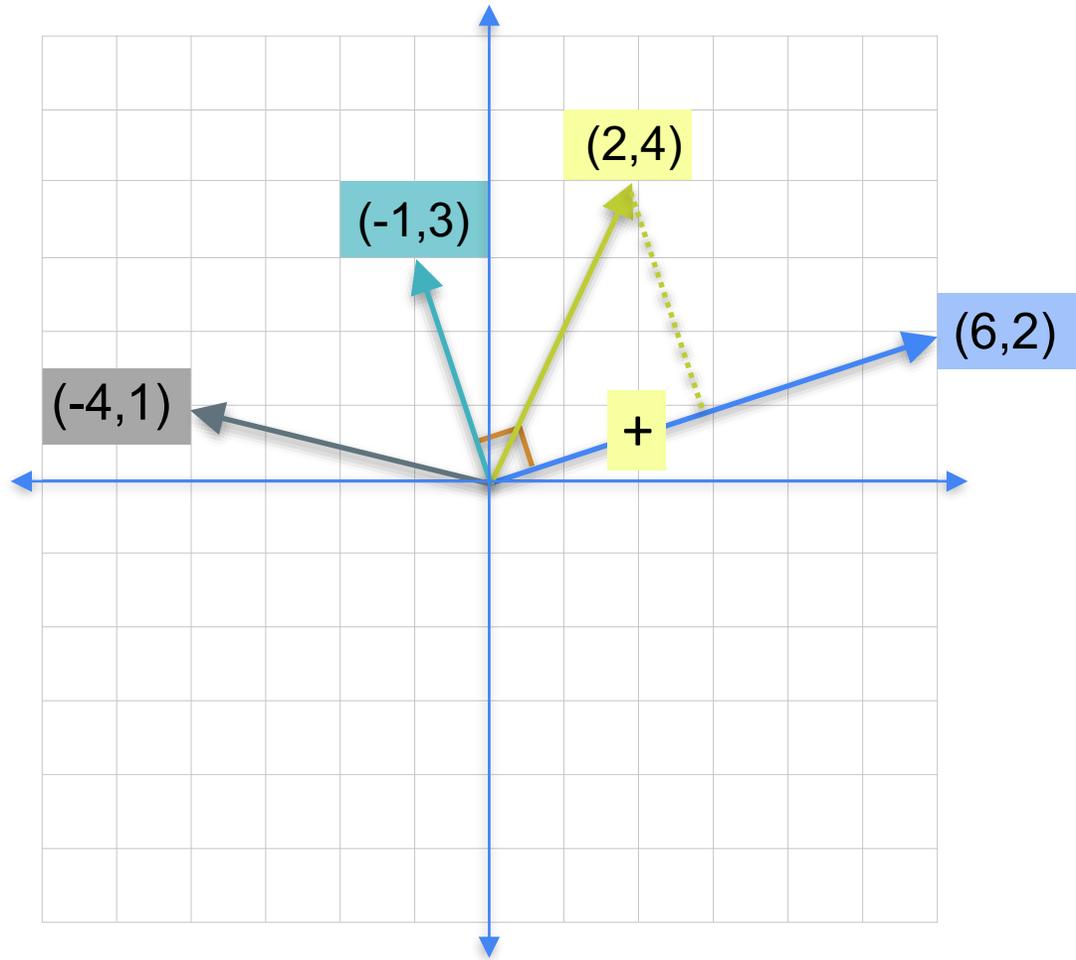
$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22$$

1. 向量及其属性

点积



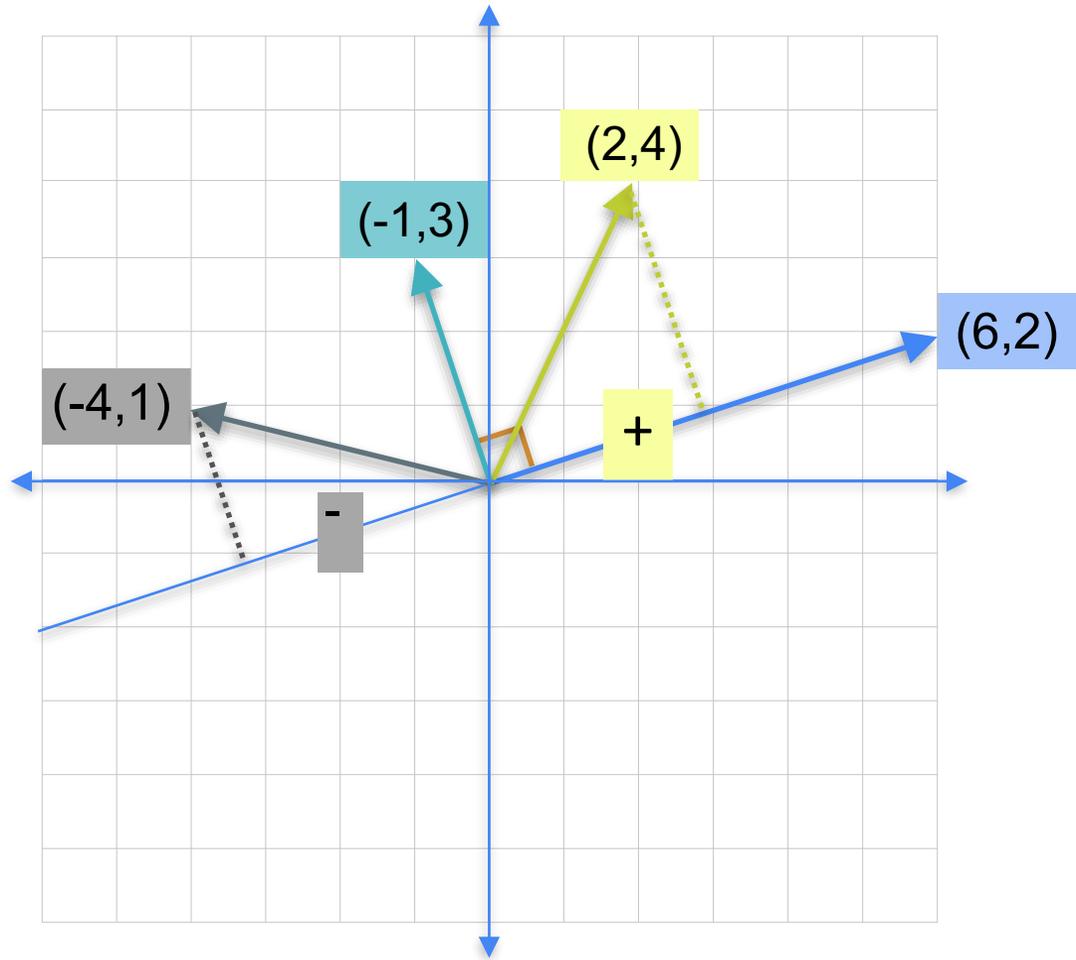
$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22 \quad \text{Negative}$$

1. 向量及其属性

点积



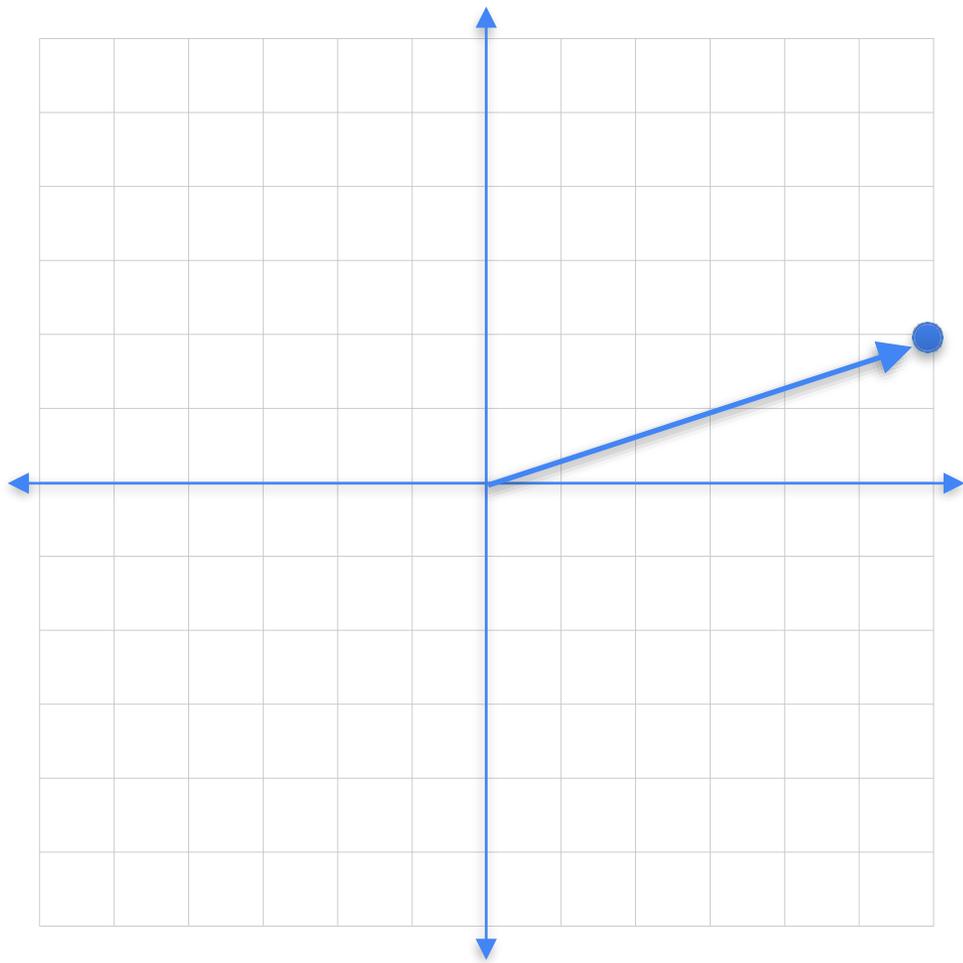
$$\begin{matrix} 6 & 2 \\ 2 & 4 \end{matrix} = 20 \quad \text{Positive}$$

$$\begin{matrix} 6 & 2 \\ -1 & 3 \end{matrix} = 0$$

$$\begin{matrix} 6 & 2 \\ -4 & 1 \end{matrix} = -22 \quad \text{Negative}$$

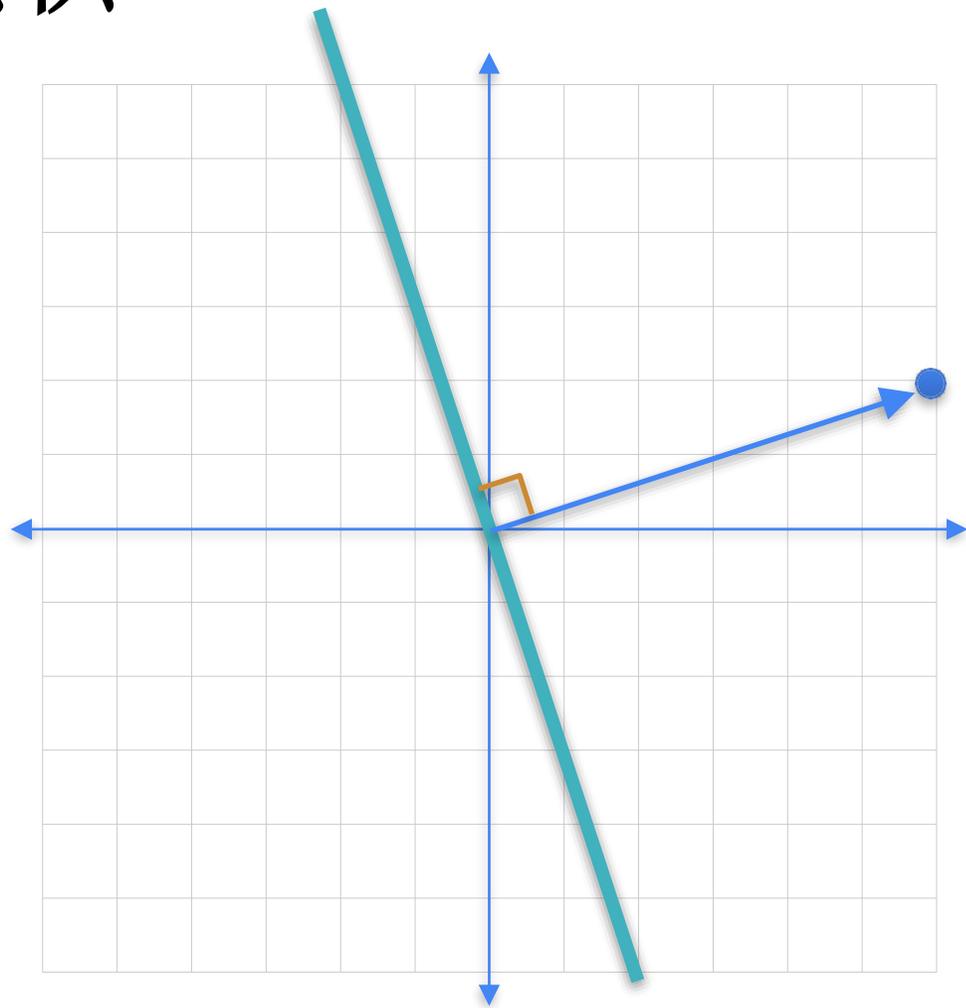
1. 向量及其属性

点积



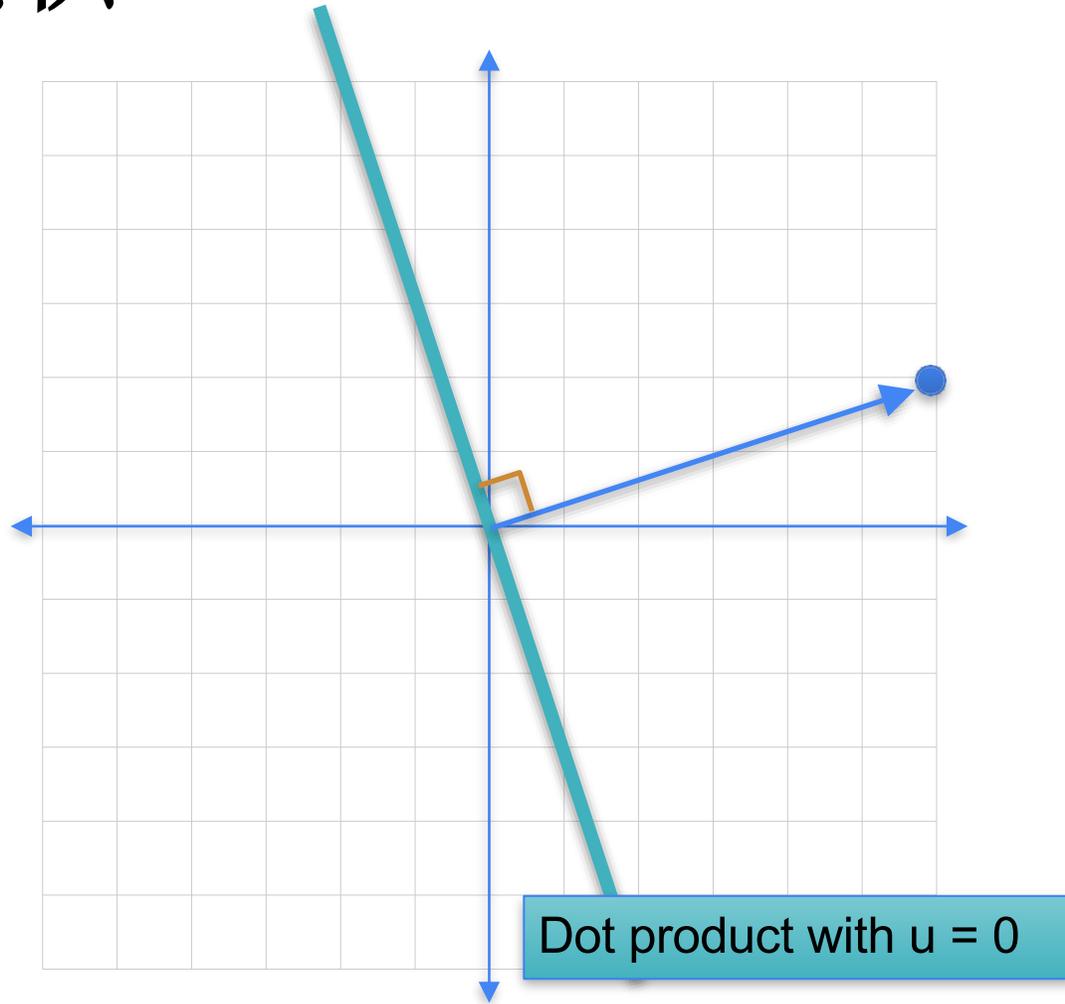
1. 向量及其属性

点积



1. 向量及其属性

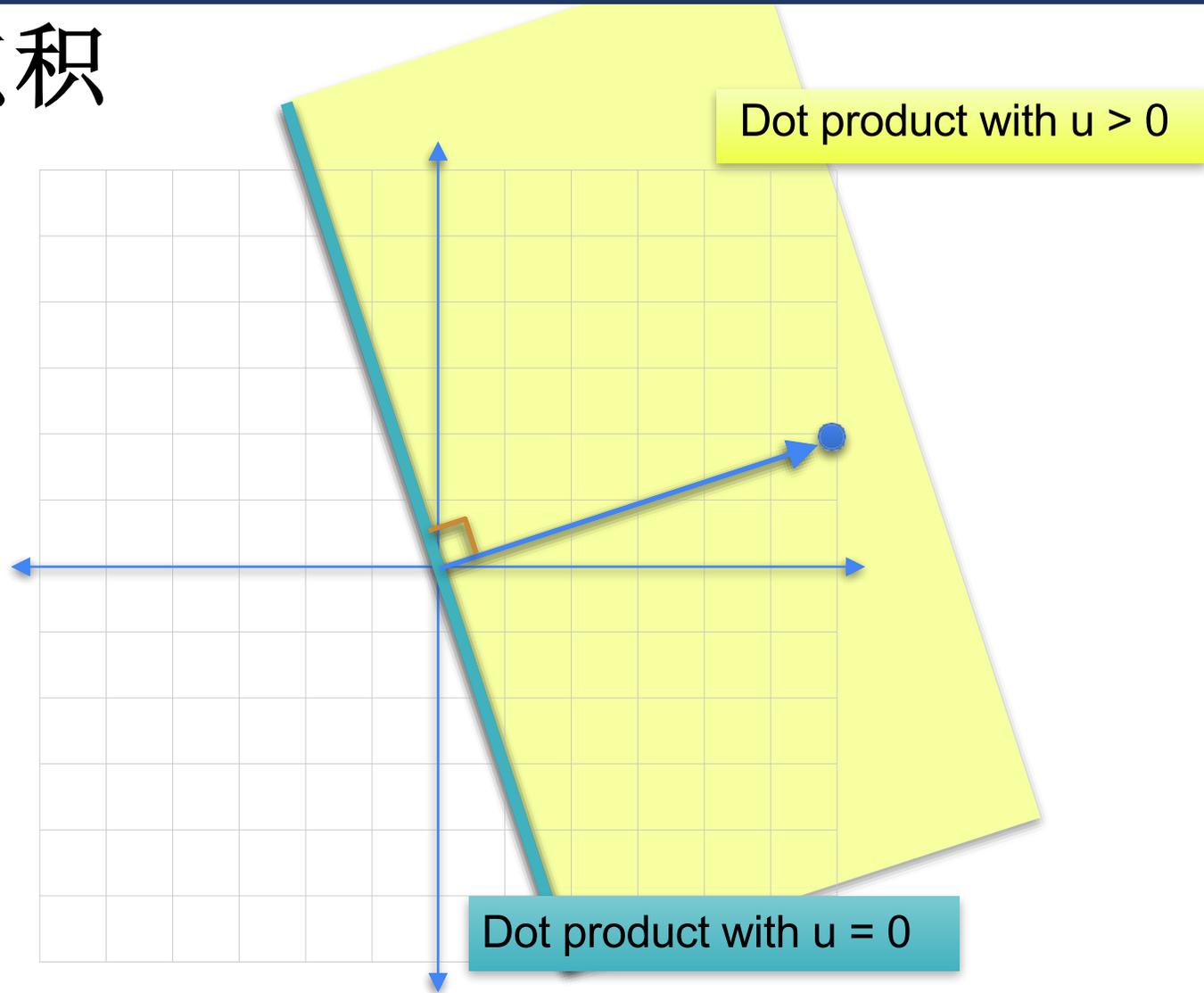
点积



$$\langle u, v \rangle = 0$$

1. 向量及其属性

点积

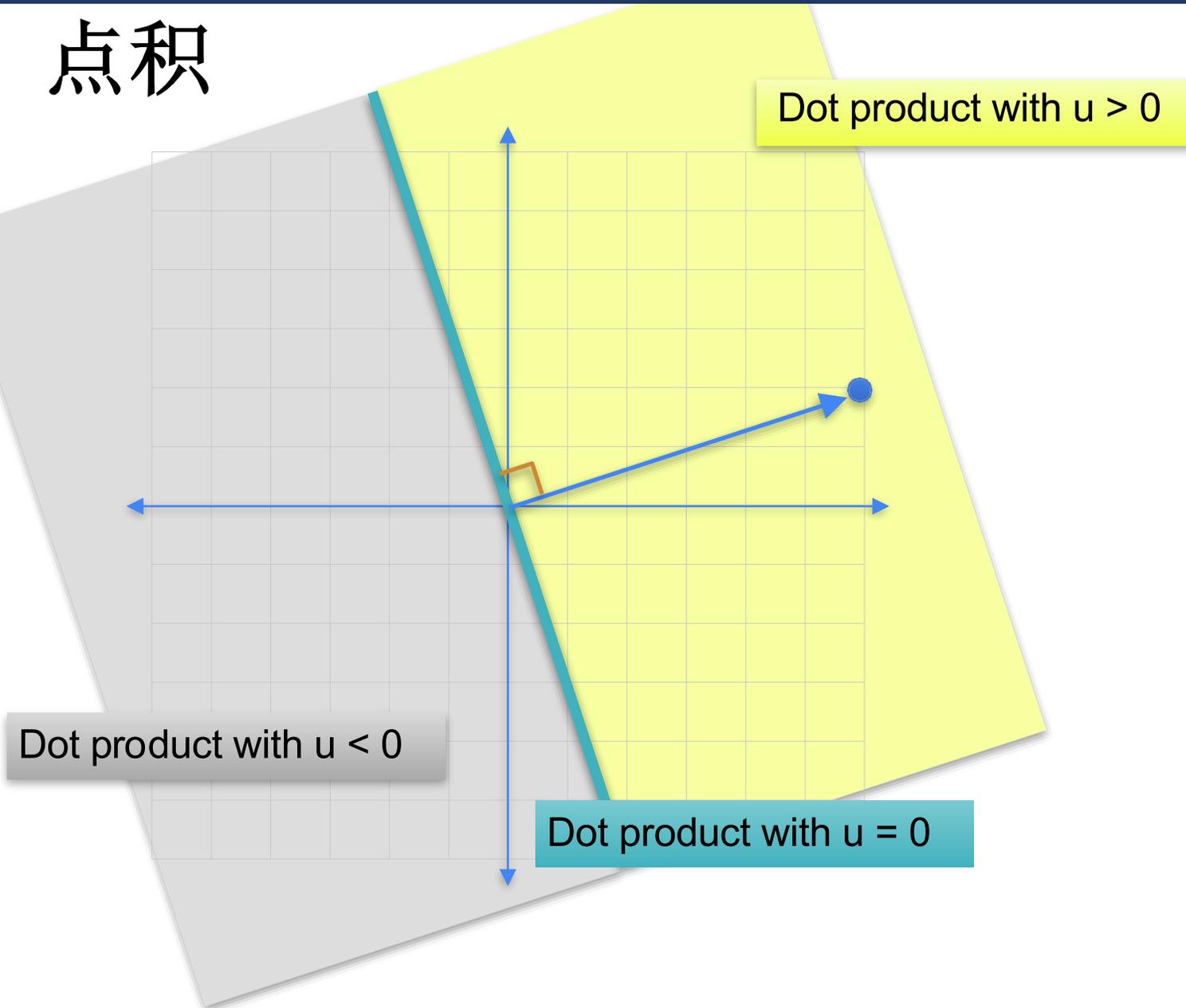


$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

1. 向量及其属性

点积



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$

01 向量及其属性

02 矩阵及其属性

03 线性变换和矩阵乘法

04 逆矩阵

05 神经网络与矩阵

目录

2. 矩阵及其属性

用点积表示线性方程组

$$2a + 4b + c = 28$$

												
2	4	1	·	<table border="1"><tr><td>\$ </td><td>a</td></tr><tr><td>\$ </td><td>b</td></tr><tr><td>\$ </td><td>c</td></tr></table>	\$ 	a	\$ 	b	\$ 	c	=	\$ 28
\$ 	a											
\$ 	b											
\$ 	c											

用点积表示线性方程组

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

2.矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$



1	1	1
---	---	---

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

2.矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

			\$		a	
1	1	1	·	\$		b
			\$		c	

2.矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Diagram illustrating the dot product representation of the first equation:

			\$		a	
1	1	1	·	\$		b
				\$		c

= \$ 10

2. 矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

			\$		<table border="1" data-bbox="504 586 614 648"><tr><td>a</td></tr></table>	a	= \$ 10
a							
1	1	1	·	\$		<table border="1" data-bbox="504 672 614 733"><tr><td>b</td></tr></table>	
b							
			\$		<table border="1" data-bbox="504 733 614 795"><tr><td>c</td></tr></table>	c	
c							

$$a + 2b + c = 15$$

		
1	2	1

$$a + b + 2c = 12$$

2.矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 15$

$$a + b + 2c = 12$$

2. 矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

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$$a + b + 2c = 12$$

2. 矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \10

$$a + 2b + c = 15$$

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \15

$$a + b + 2c = 12$$

$\begin{bmatrix} 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

2.矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

			·	\$ 	<table border="1"><tr><td>a</td></tr><tr><td>b</td></tr><tr><td>c</td></tr></table>	a	b	c	=	\$ 10
a										
b										
c										
1	1	1								

$$a + 2b + c = 15$$

			·	\$ 	<table border="1"><tr><td>a</td></tr><tr><td>b</td></tr><tr><td>c</td></tr></table>	a	b	c	=	\$ 15
a										
b										
c										
1	2	1								

$$a + b + 2c = 12$$

			·	\$ 	<table border="1"><tr><td>a</td></tr><tr><td>b</td></tr><tr><td>c</td></tr></table>	a	b	c	=	\$ 12
a										
b										
c										
1	1	2								

2.矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \$ 10$

$$a + 2b + c = 15$$

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2.矩阵及其属性

用点积表示线性方程组

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2.矩阵及其属性

用点积表示线性方程组

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

			\$		a	= \$ 10	
1	1	1	·	\$			b
				\$			c

			\$		a	= \$ 15	
1	2	1	·	\$			b
				\$			c

			\$		a	= \$ 12	
1	1	2	·	\$			b
				\$			c

2.矩阵及其属性

用点积表示线性方程组

线性方程组

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

矩阵和向量相乘

						
1	1	1	\$		a	10
1	2	1	· \$		b	15
1	1	2	\$		c	12

2.矩阵及其属性

用点积表示线性方程组

线性方程组

$$a + b + c = 10$$

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矩阵和向量相乘

1	1	1	a	=	10
1	2	1	b		15
1	1	2	c		12

矩阵



2.矩阵及其属性

矩阵

数学上，一个 $m \times n$ 的矩阵 (matrix) 是一个有 m 行 (row) n 列 (column) 元素的矩形阵列。

行和列相同的元素称为矩阵主对角线上的元素，如 a_{11} ， a_{22} ， a_{33} 等。

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \cdots & \cdots & & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

特殊矩阵

行数与列数相同的矩阵称为方块矩阵，简称方阵。

如果一个方阵只有主对角线上的元素不是0，其它都是0，那么称其为对角矩阵（diagonal matrix）。

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

主对角线元素全部为1的对角矩阵称为单位矩阵（identity matrix），一般用 I_n 表示。

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

特殊矩阵

如果主对角线上方的元素都是0，那么称为下三角矩阵 (lower triangular matrix)。

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

如果主对角线下方的元素都是0，那么称为上三角矩阵 (upper triangular matrix)。

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

2.矩阵及其属性

矩阵的基本运算

矩阵的最基本运算包括矩阵加（减）法，数乘和转置运算。

加（减）法：

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 5 \\ 7 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 & 1+5 \\ 1+7 & 0+5 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 8 & 5 & 0 \end{bmatrix}$$

数乘：

$$2 \cdot \begin{bmatrix} 1 & 8 & -3 \\ 4 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 \cdot 1 & 2 \cdot 8 & 2 \cdot (-3) \\ 2 \cdot 4 & 2 \cdot (-2) & 2 \cdot 5 \end{bmatrix} = \begin{bmatrix} 2 & 16 & -6 \\ 8 & -4 & 10 \end{bmatrix}$$

转置：

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -6 & 7 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 2 & -6 \\ 3 & 7 \end{bmatrix}$$

- 01 向量及其属性
- 02 矩阵及其属性
- 03 线性变换和矩阵乘法
- 04 逆矩阵
- 05 机器学习模型实例

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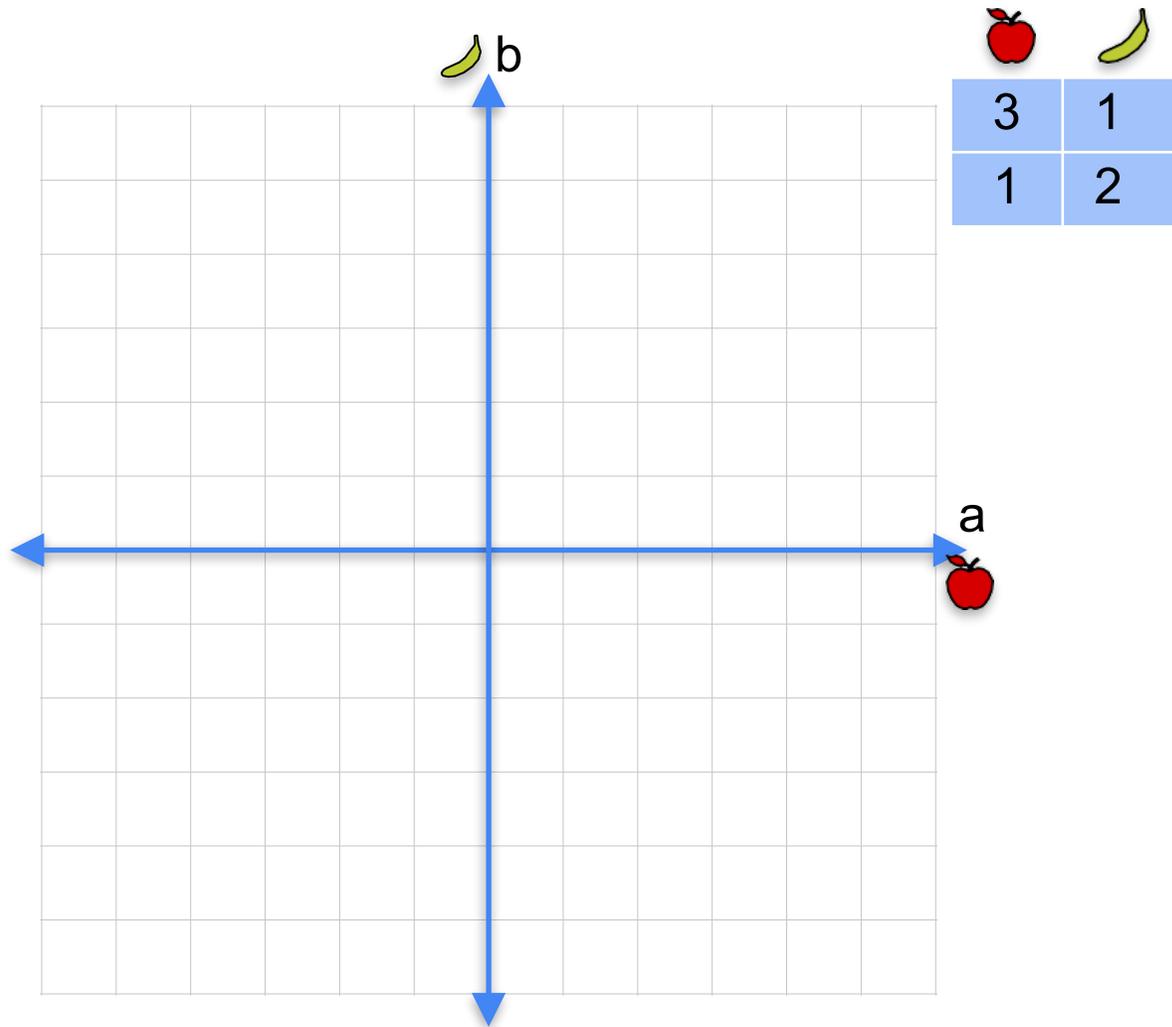
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矩阵是线性变换

	
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1	2

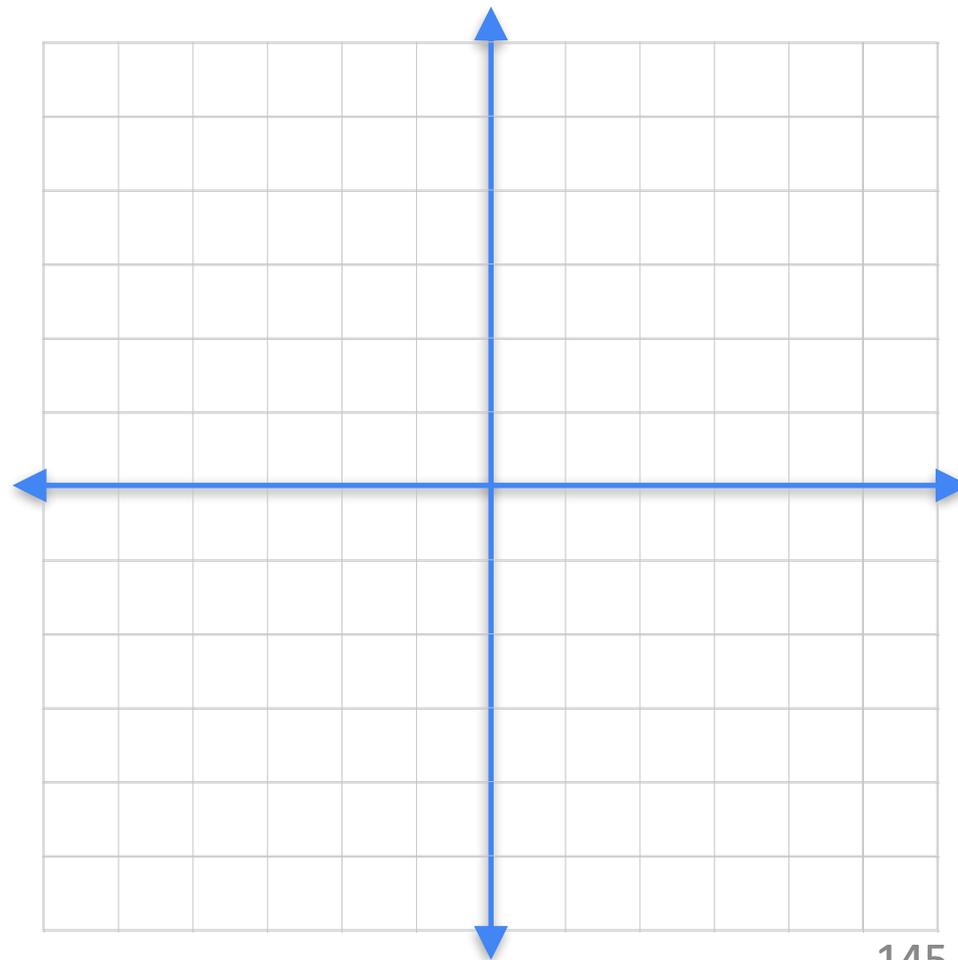
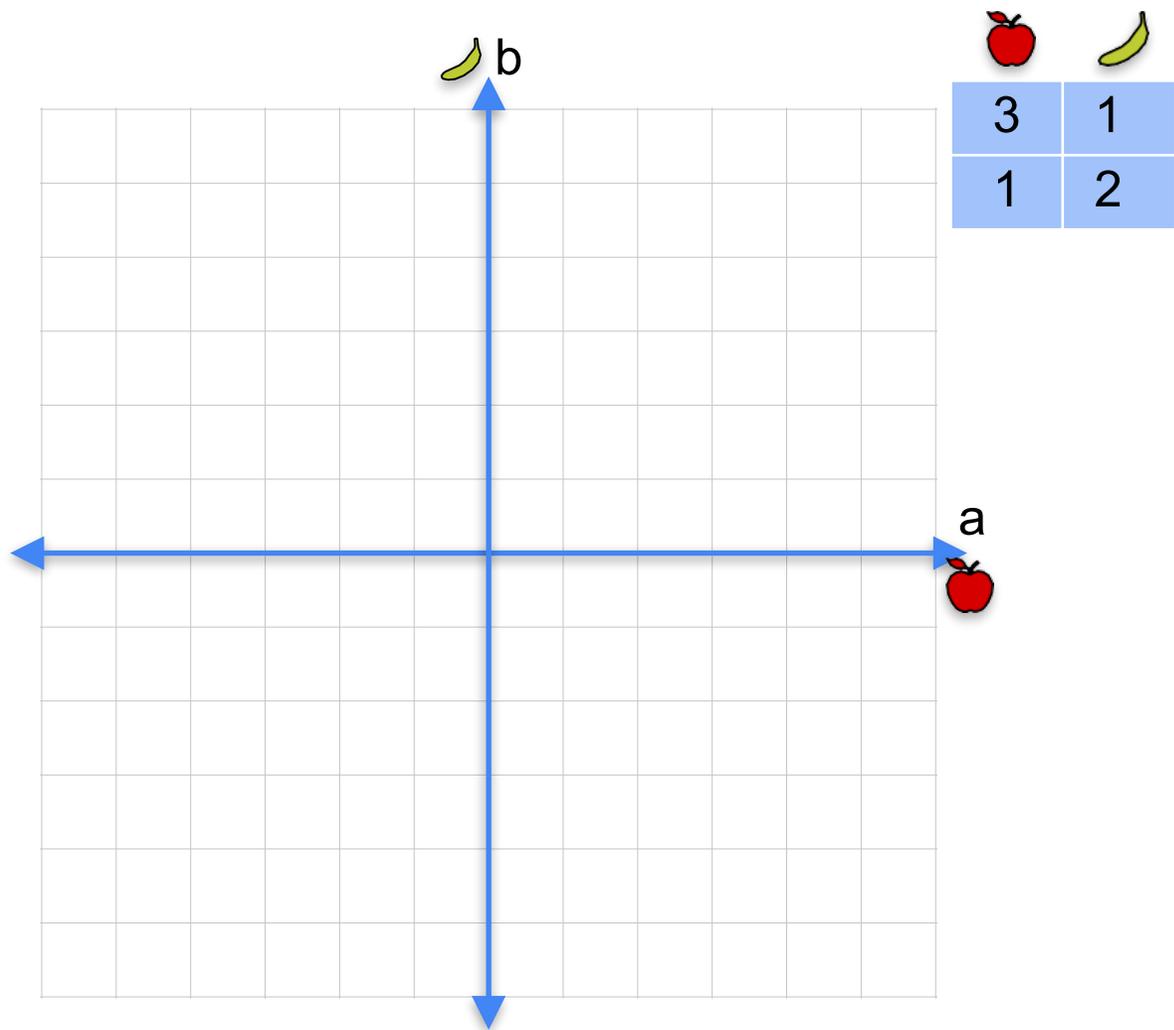
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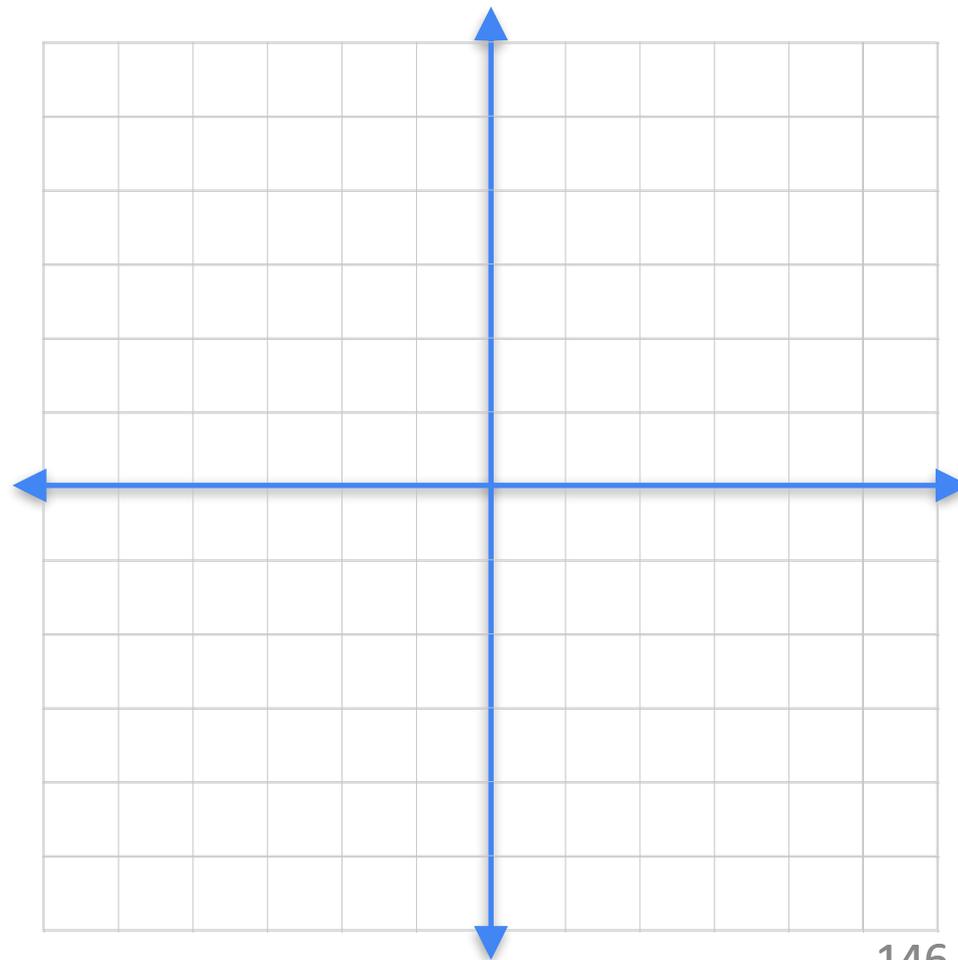
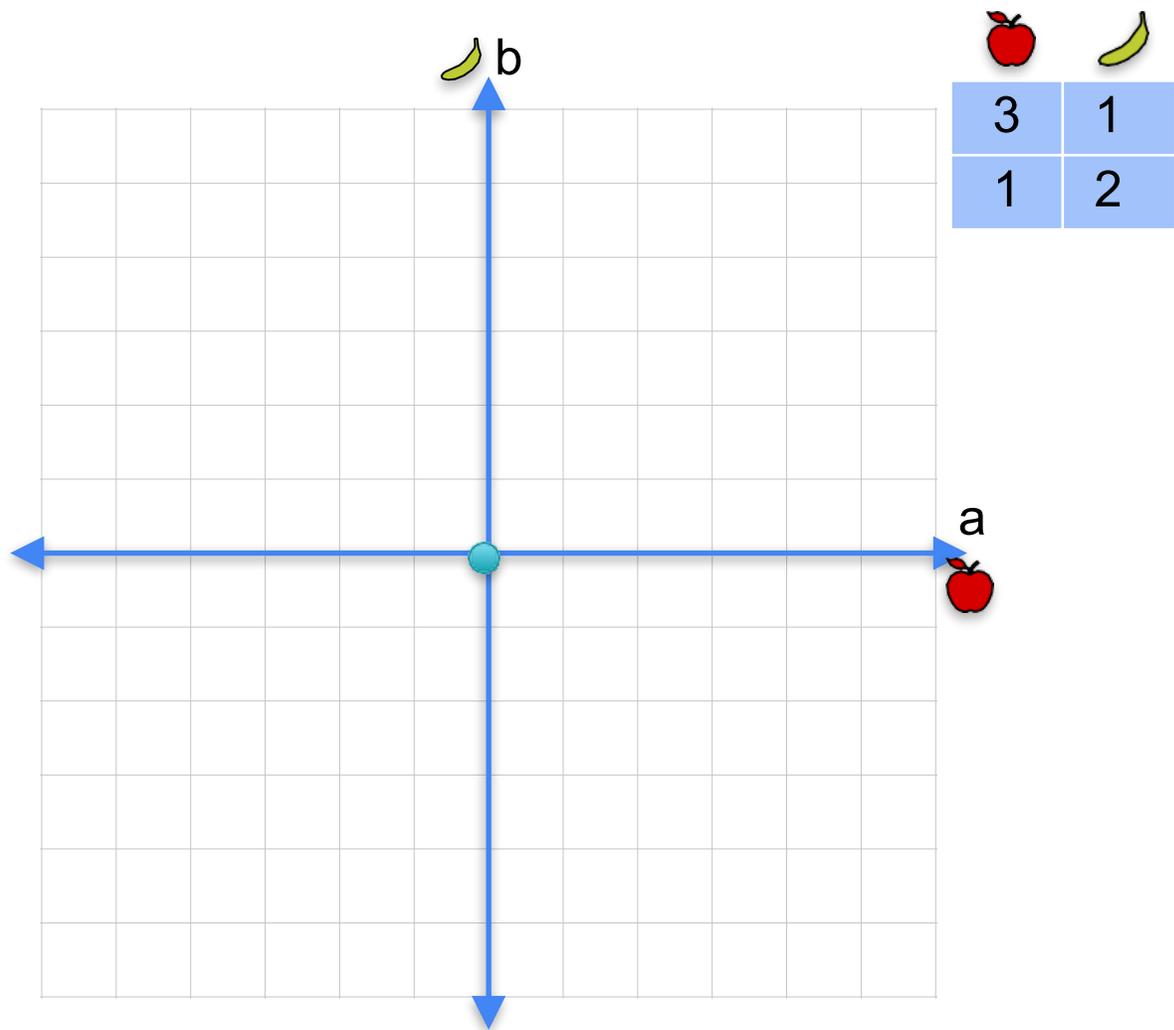
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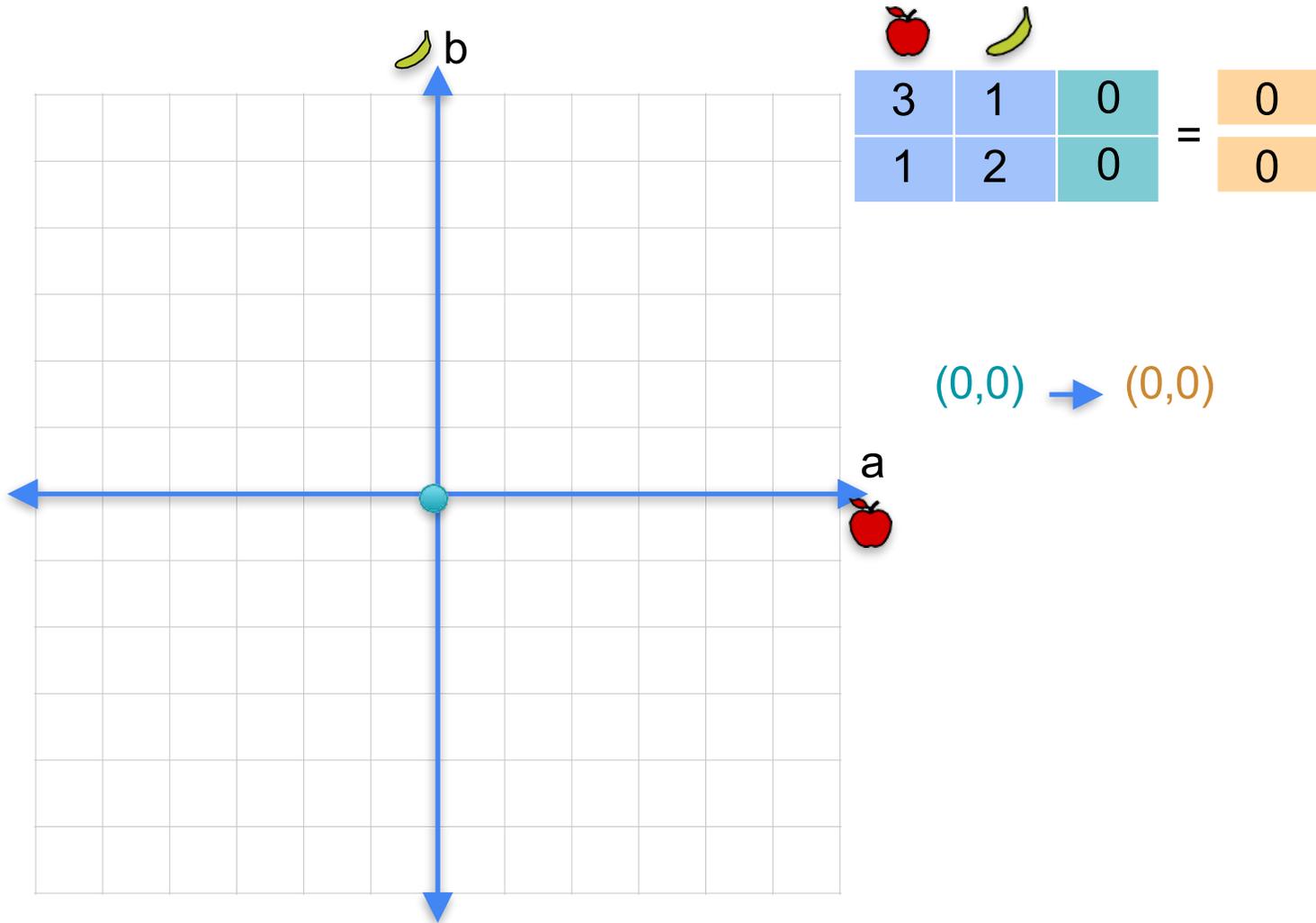
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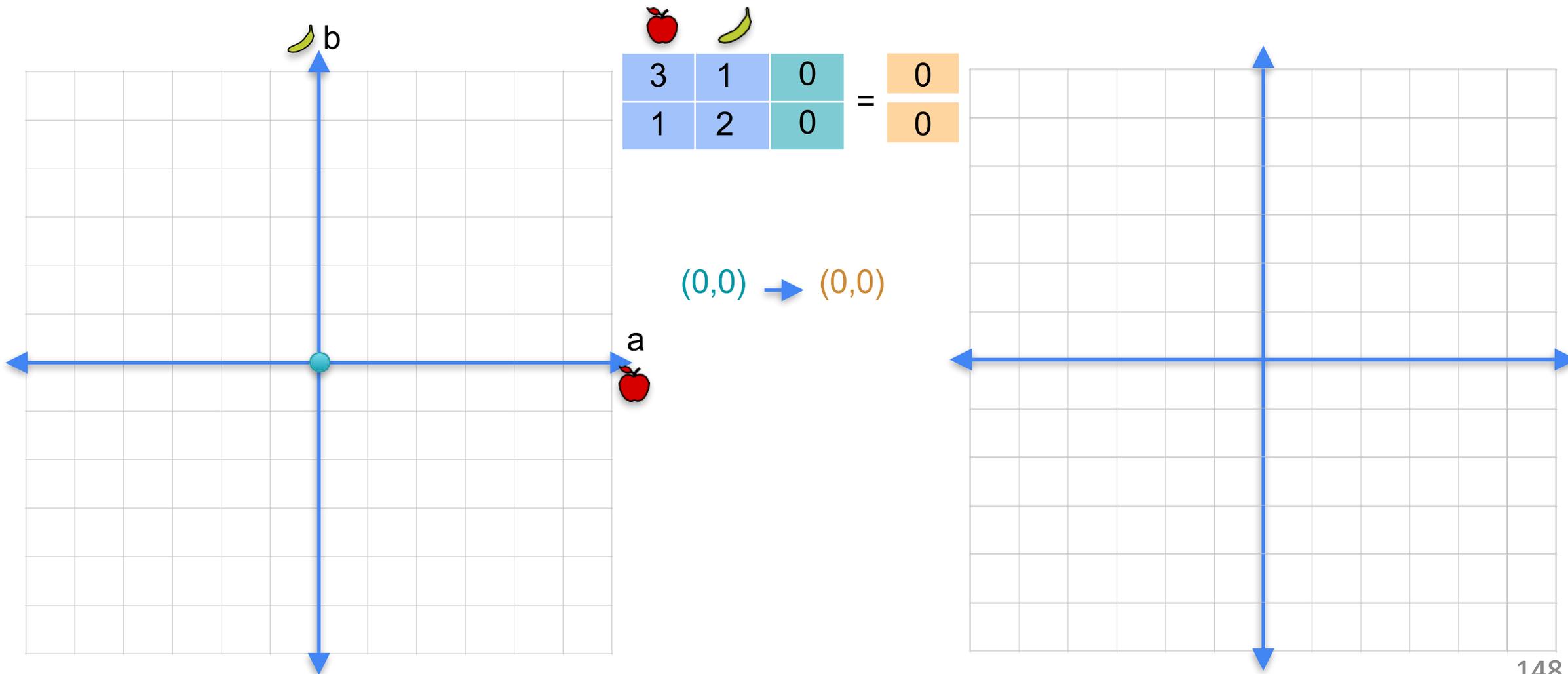
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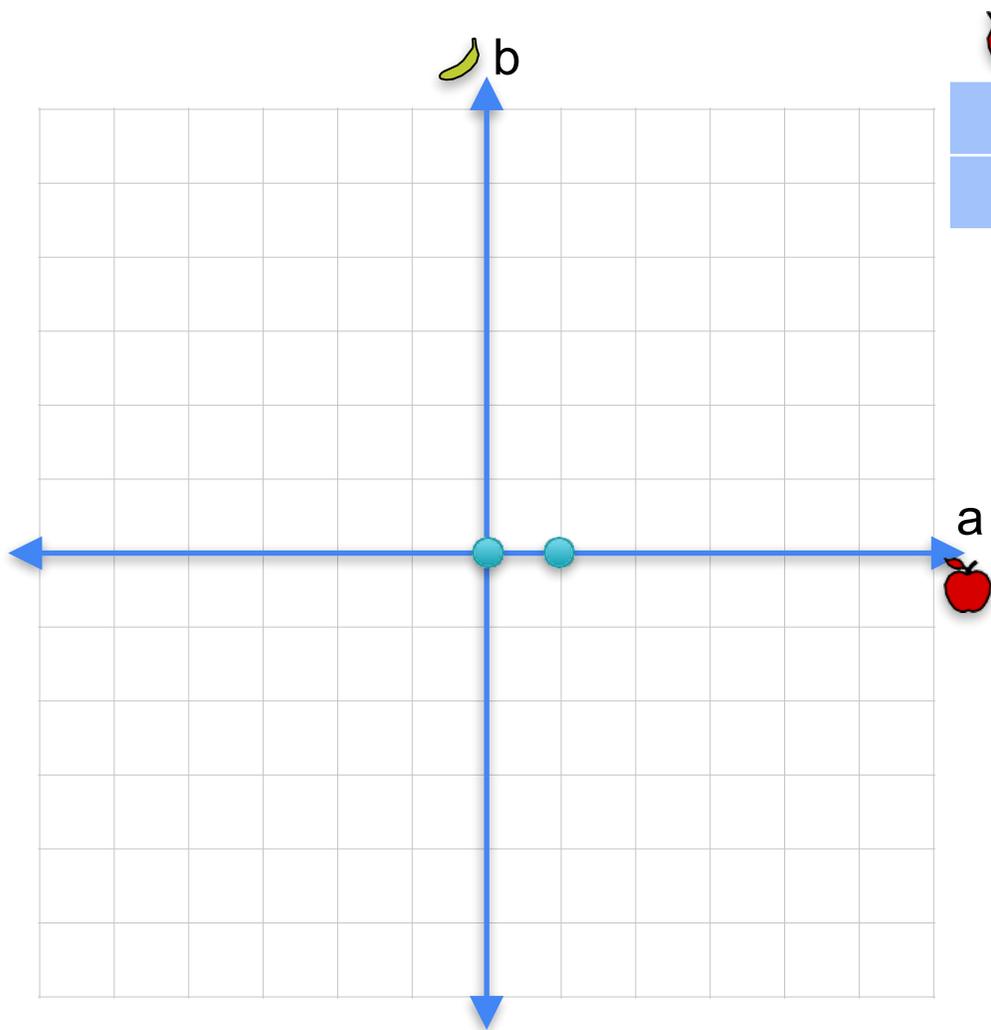
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3. 线性变换和矩阵乘法

矩阵是线性变换

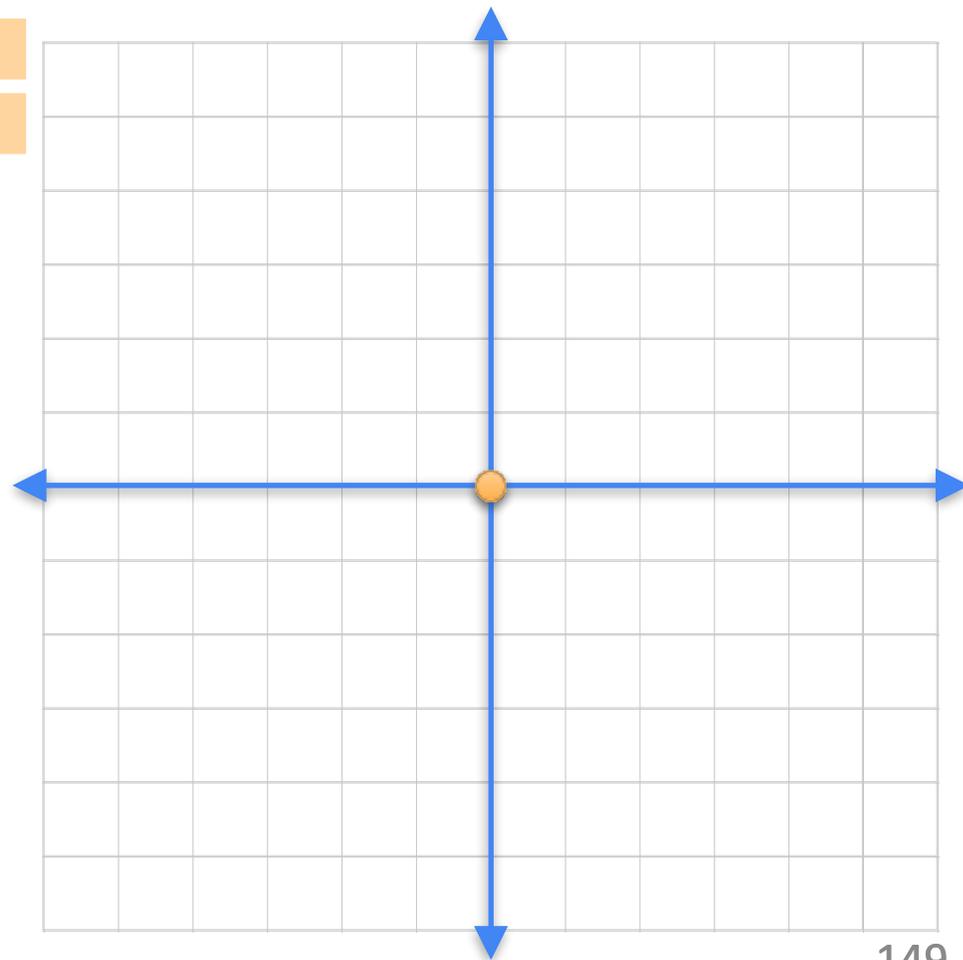


			
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1	2	0	0

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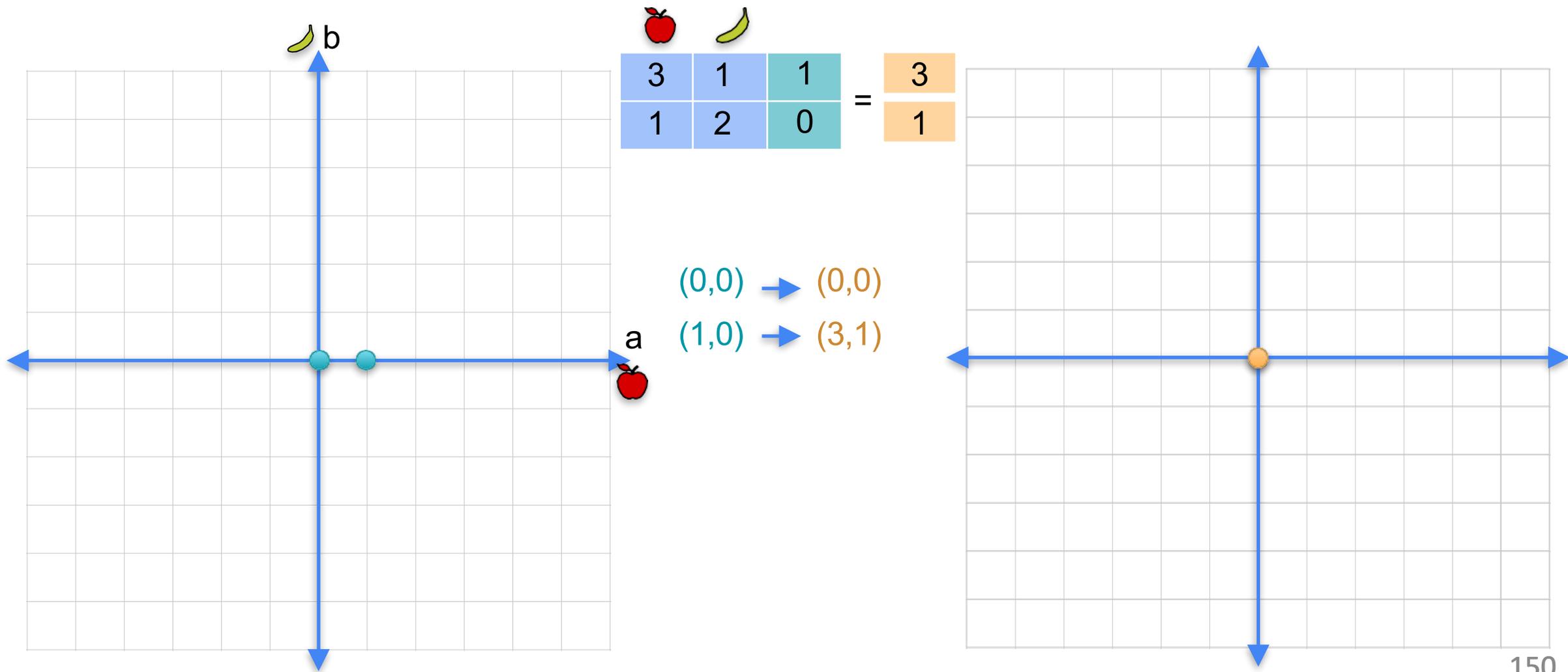
0
0

$(0,0) \rightarrow (0,0)$



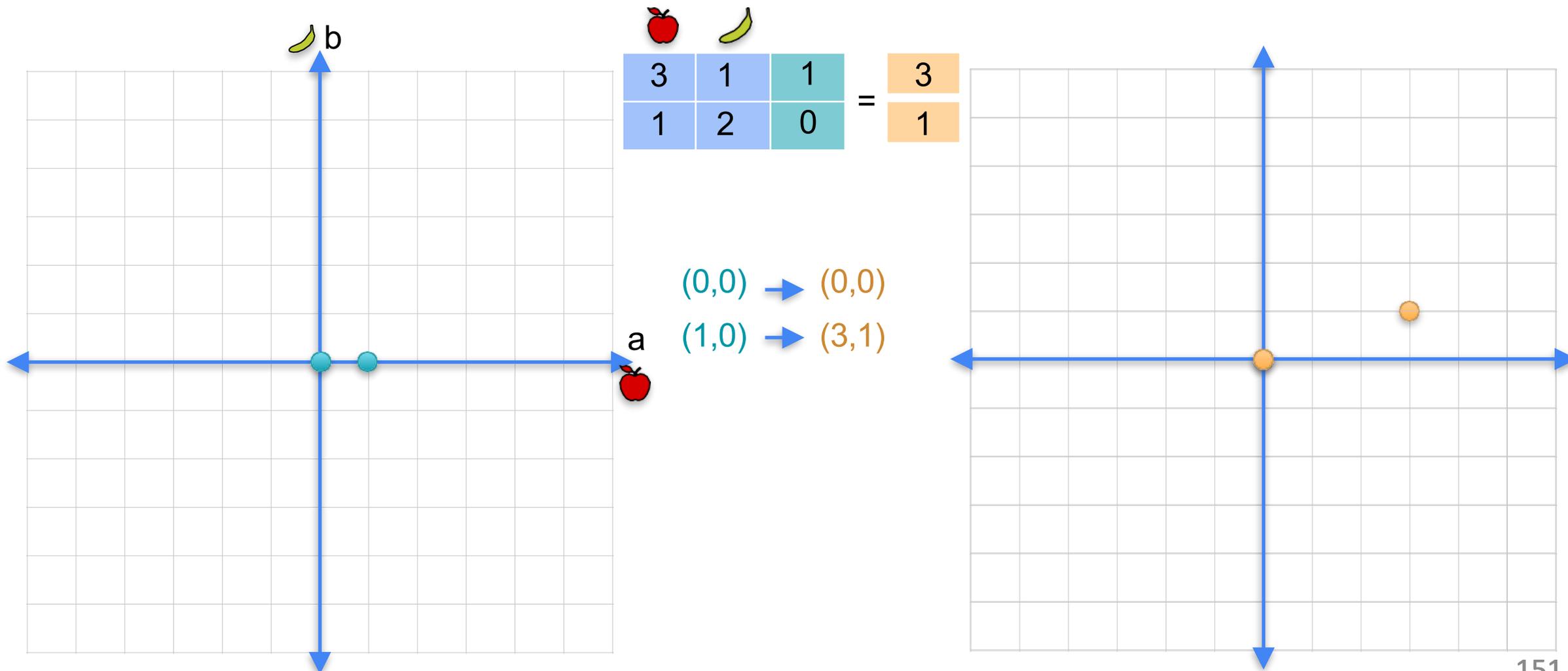
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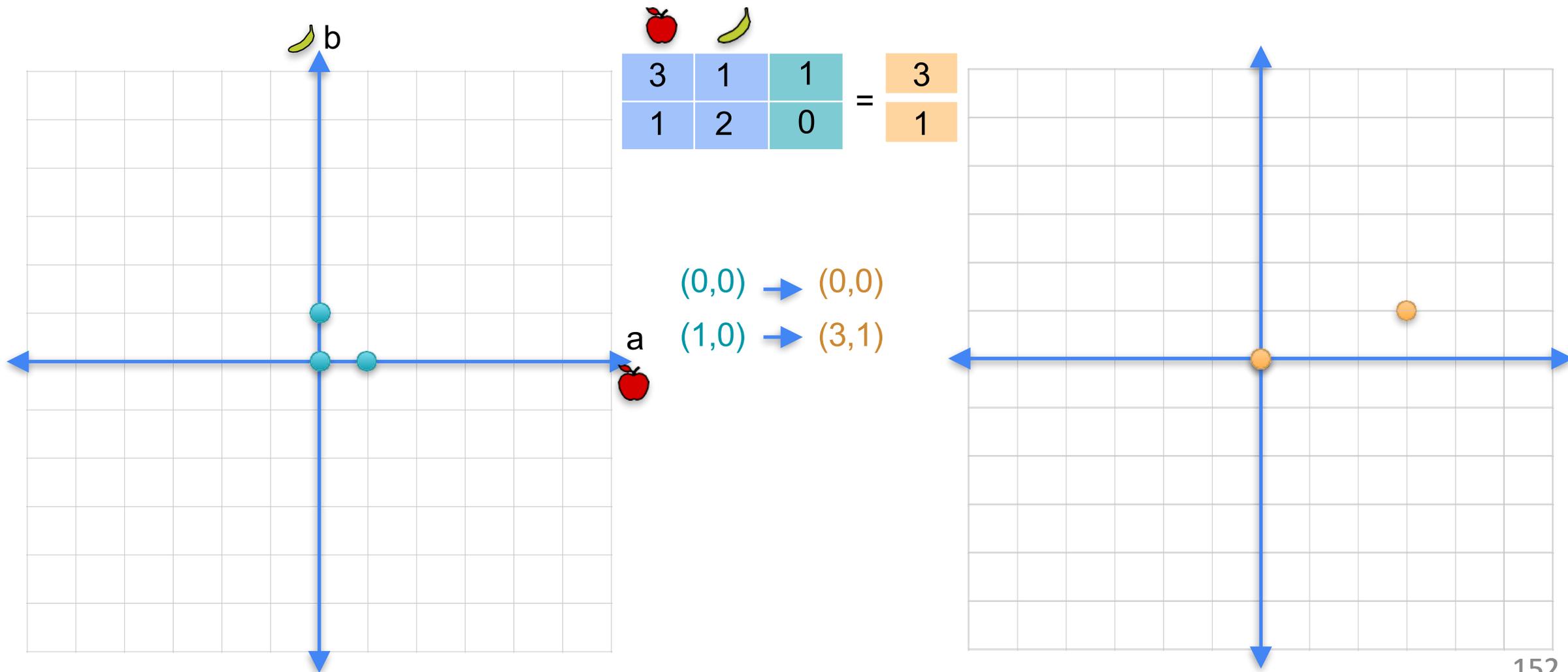
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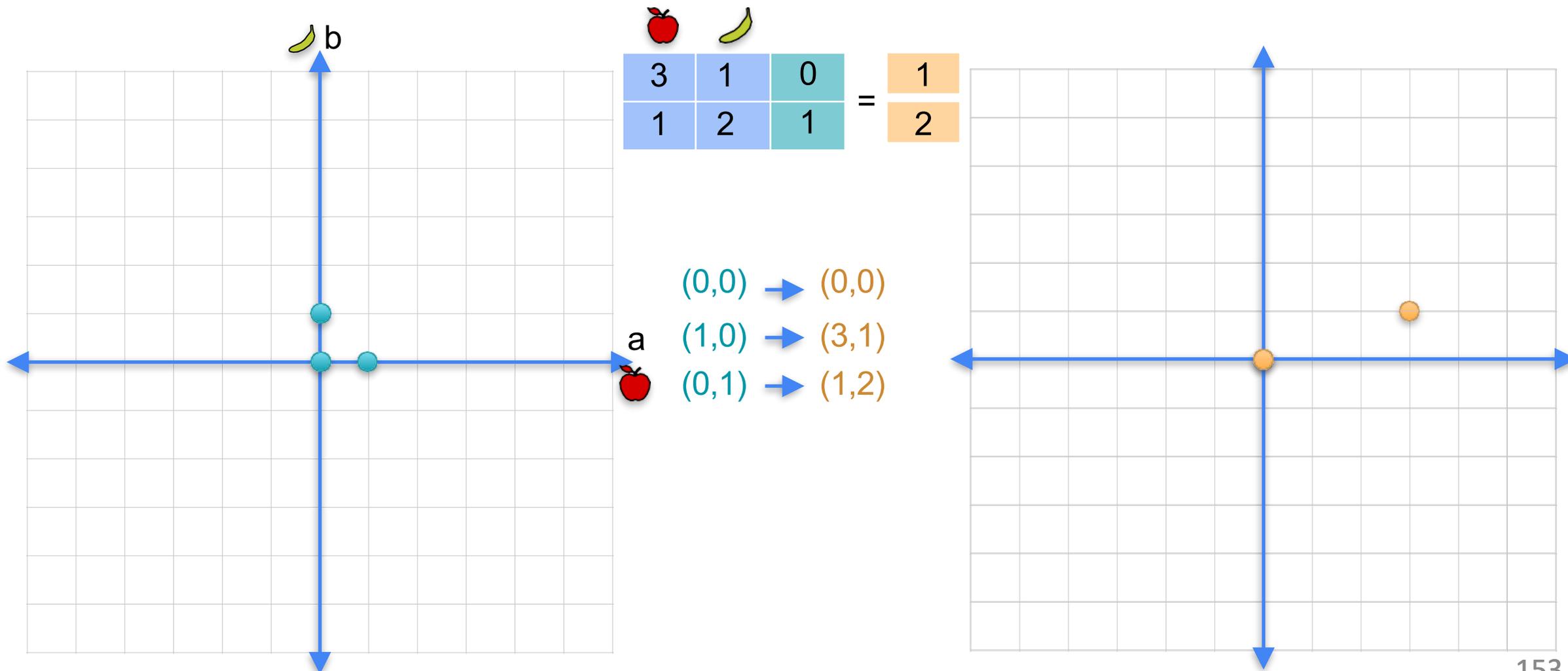
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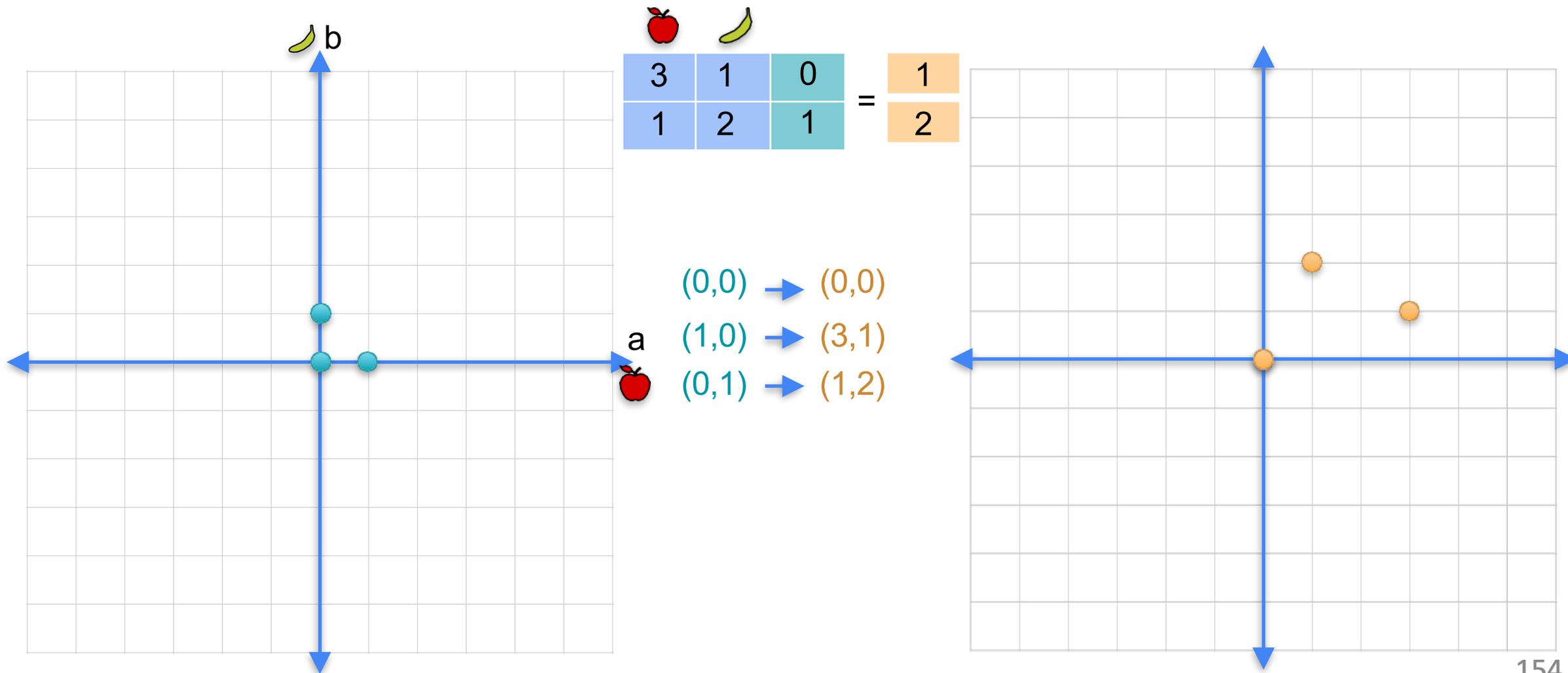
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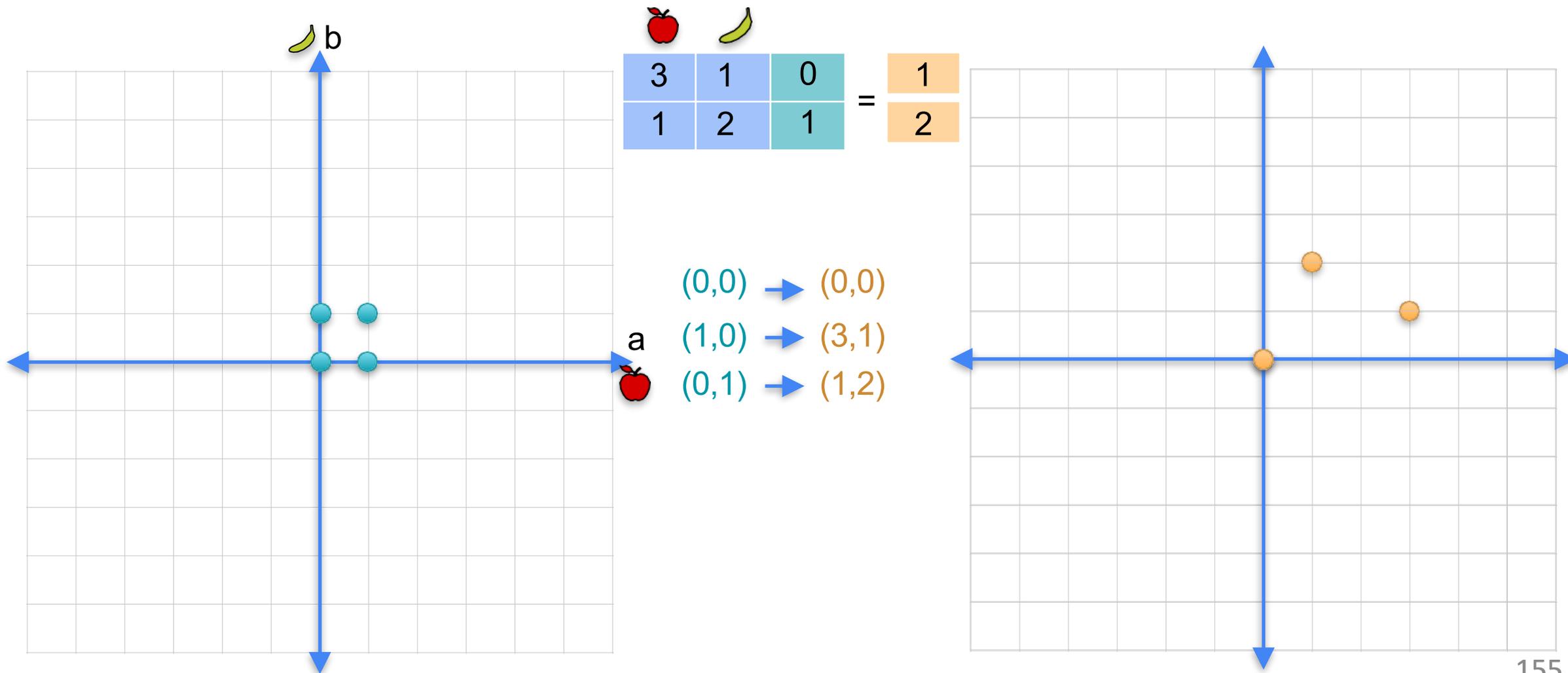
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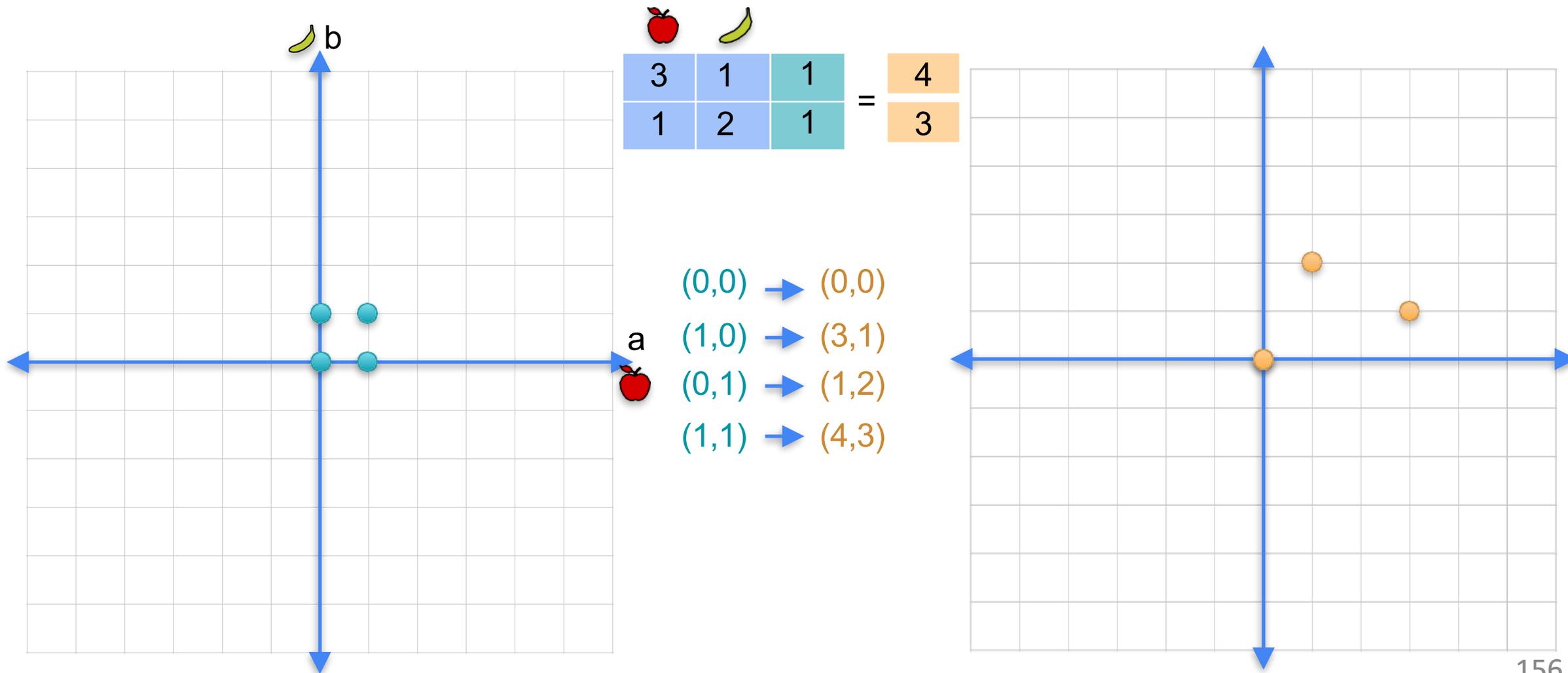
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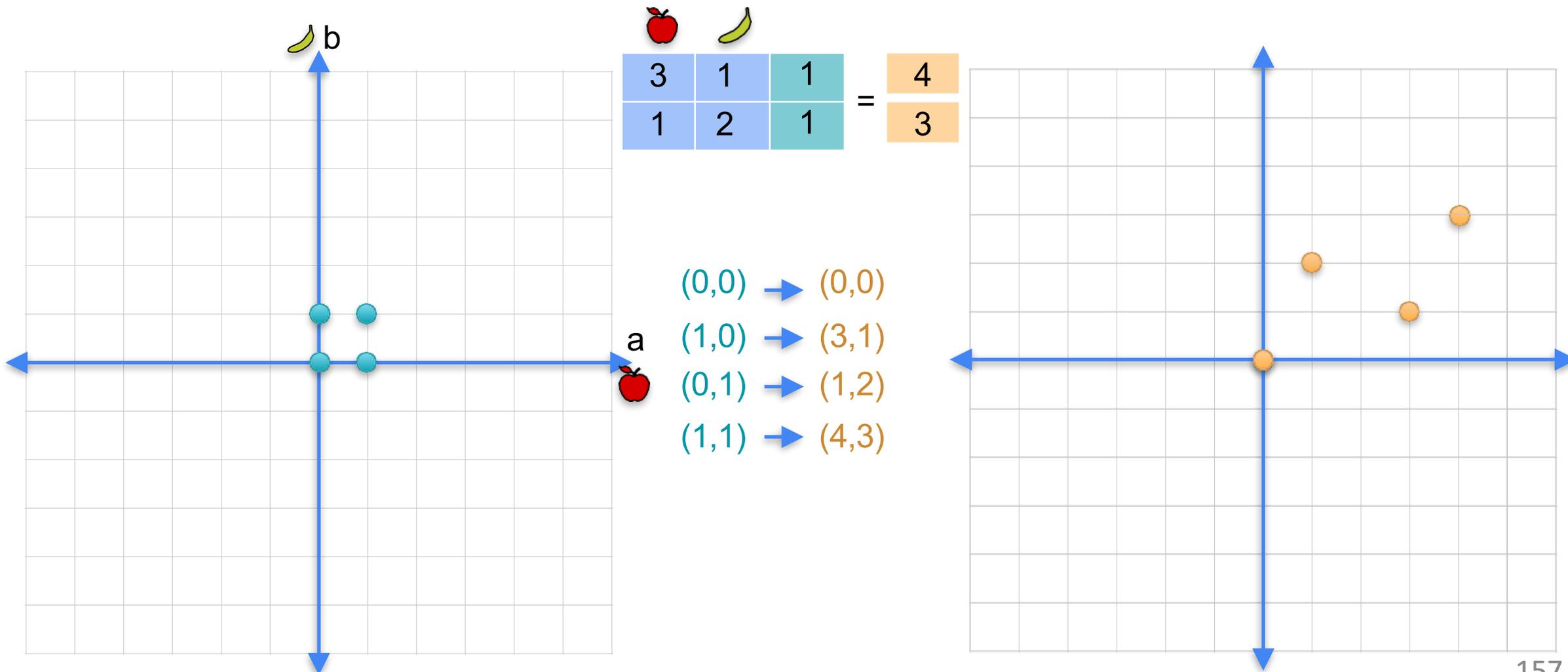
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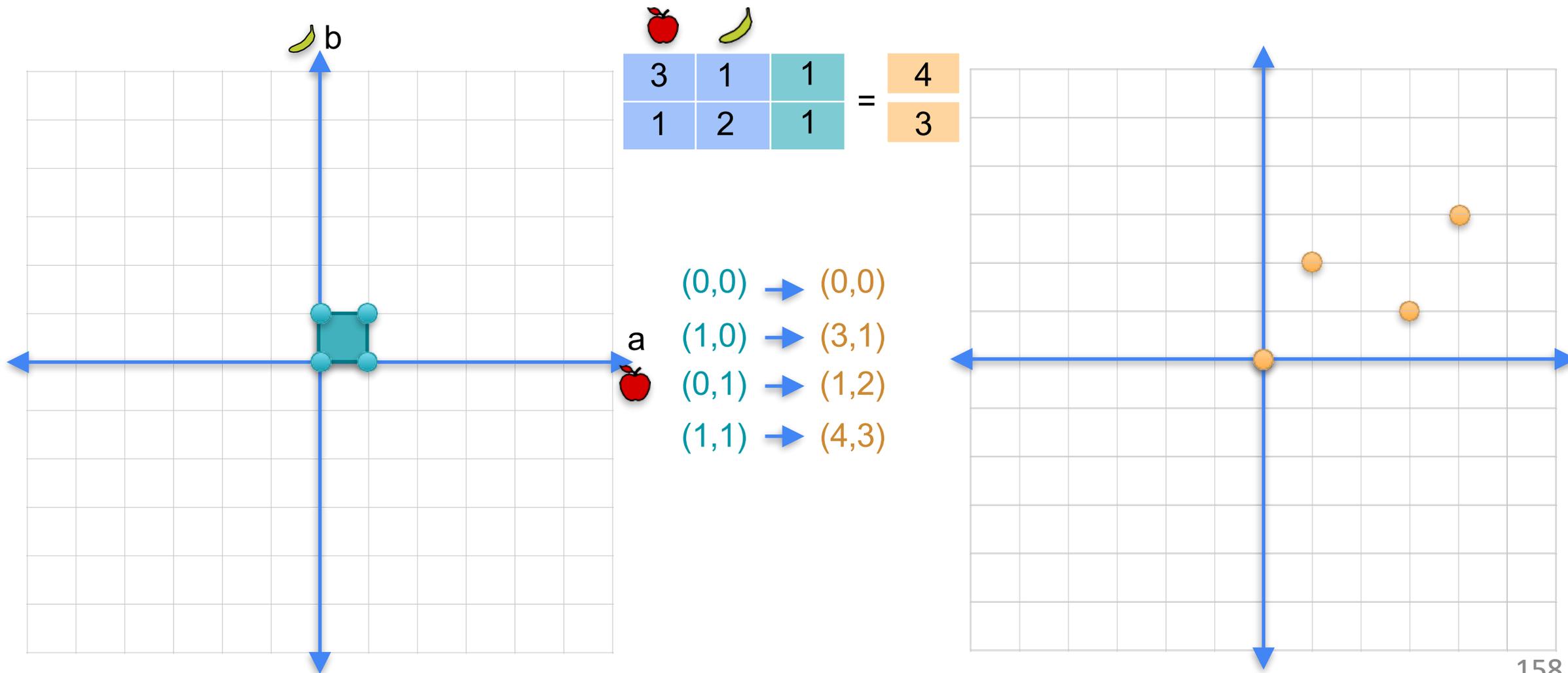
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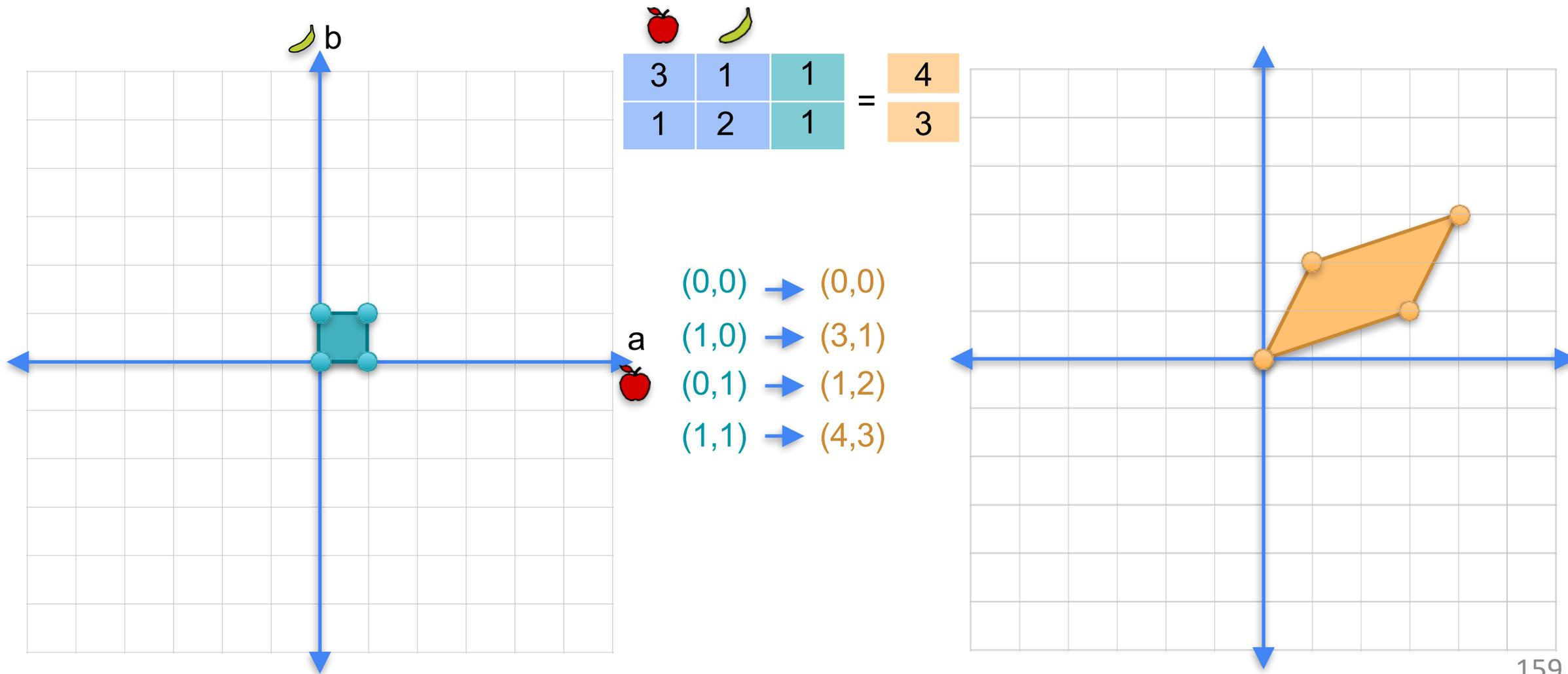
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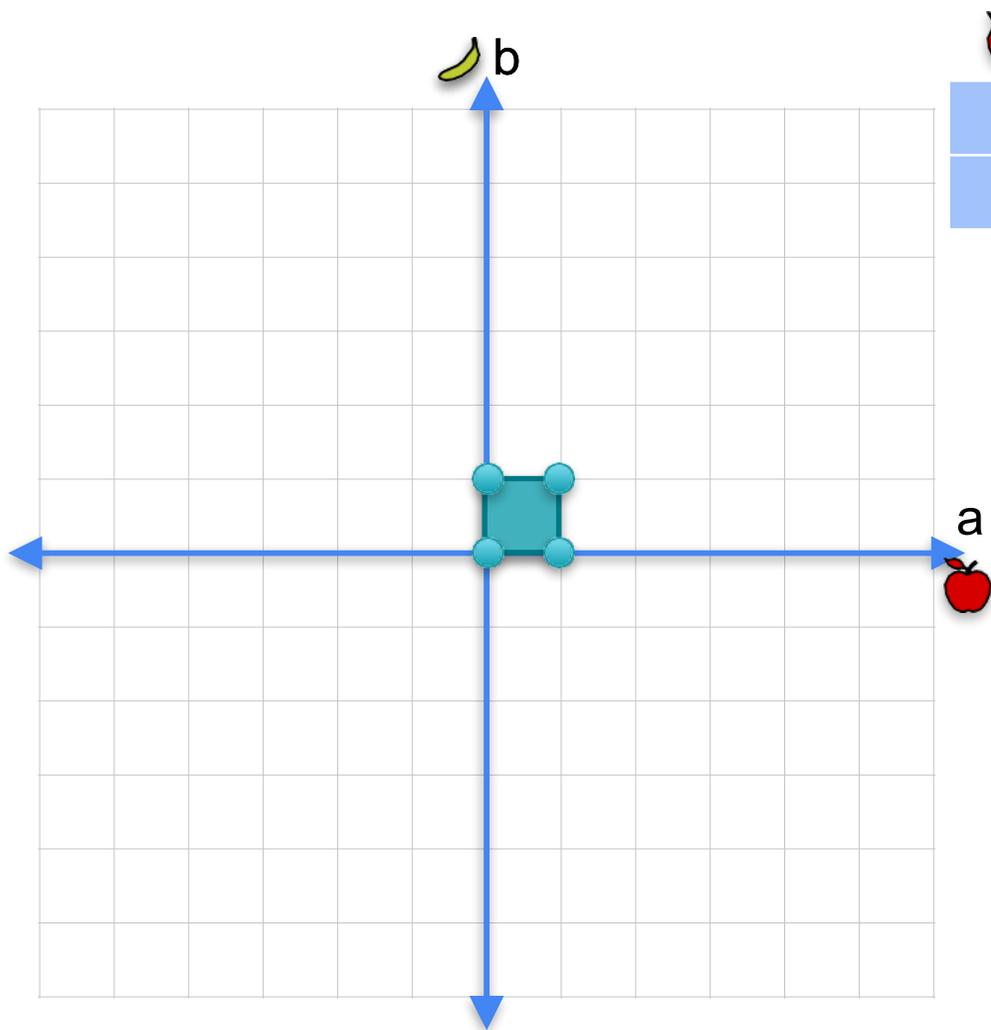
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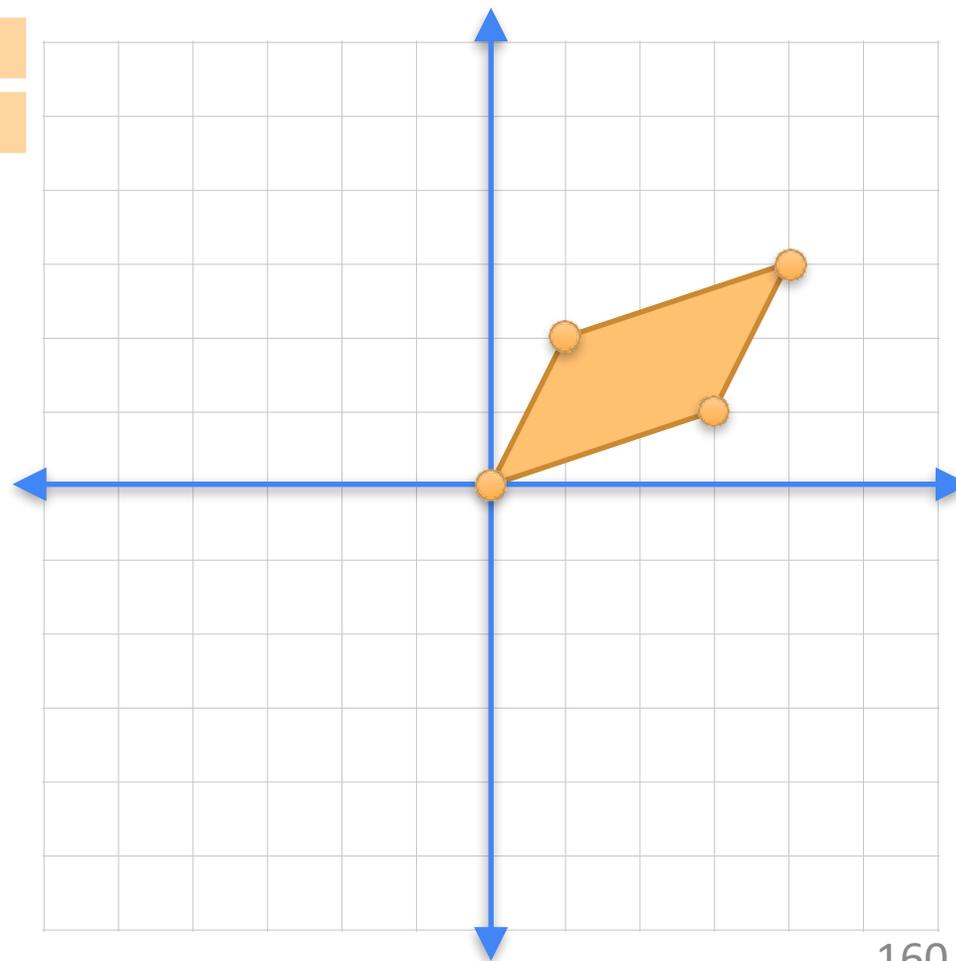
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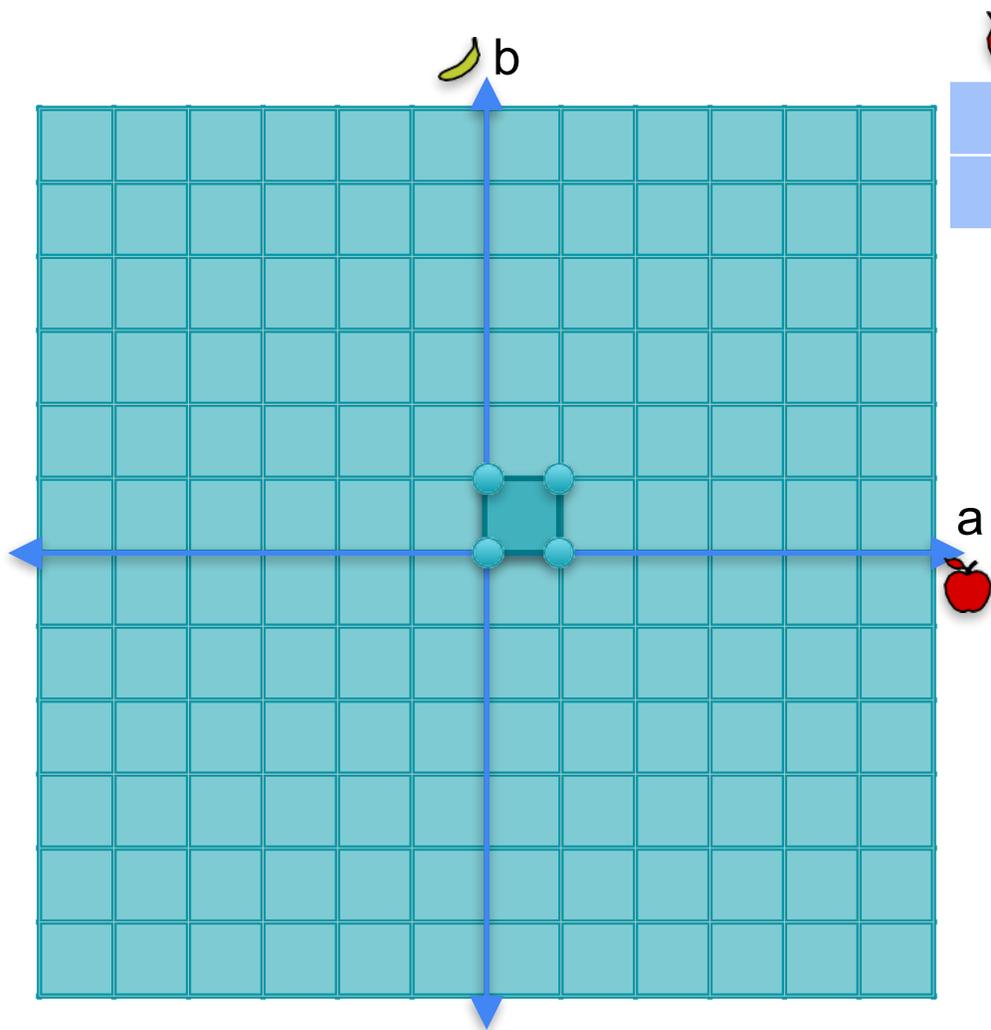
			
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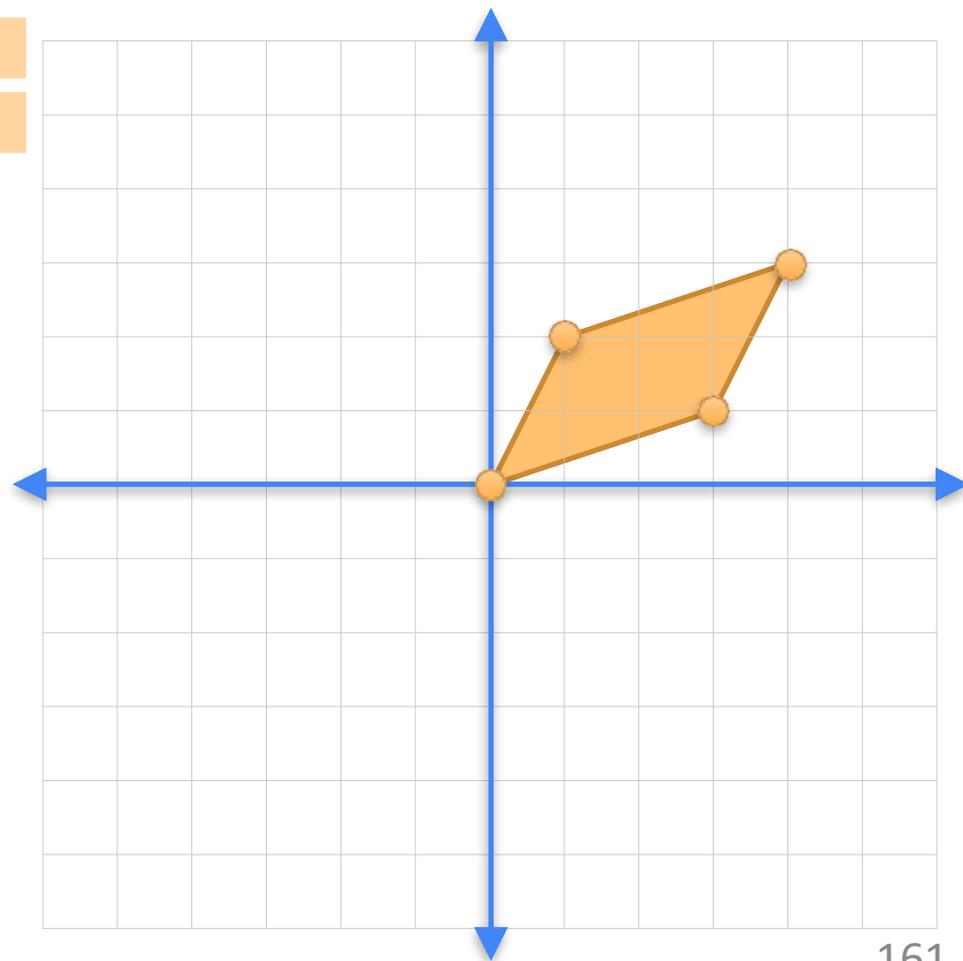
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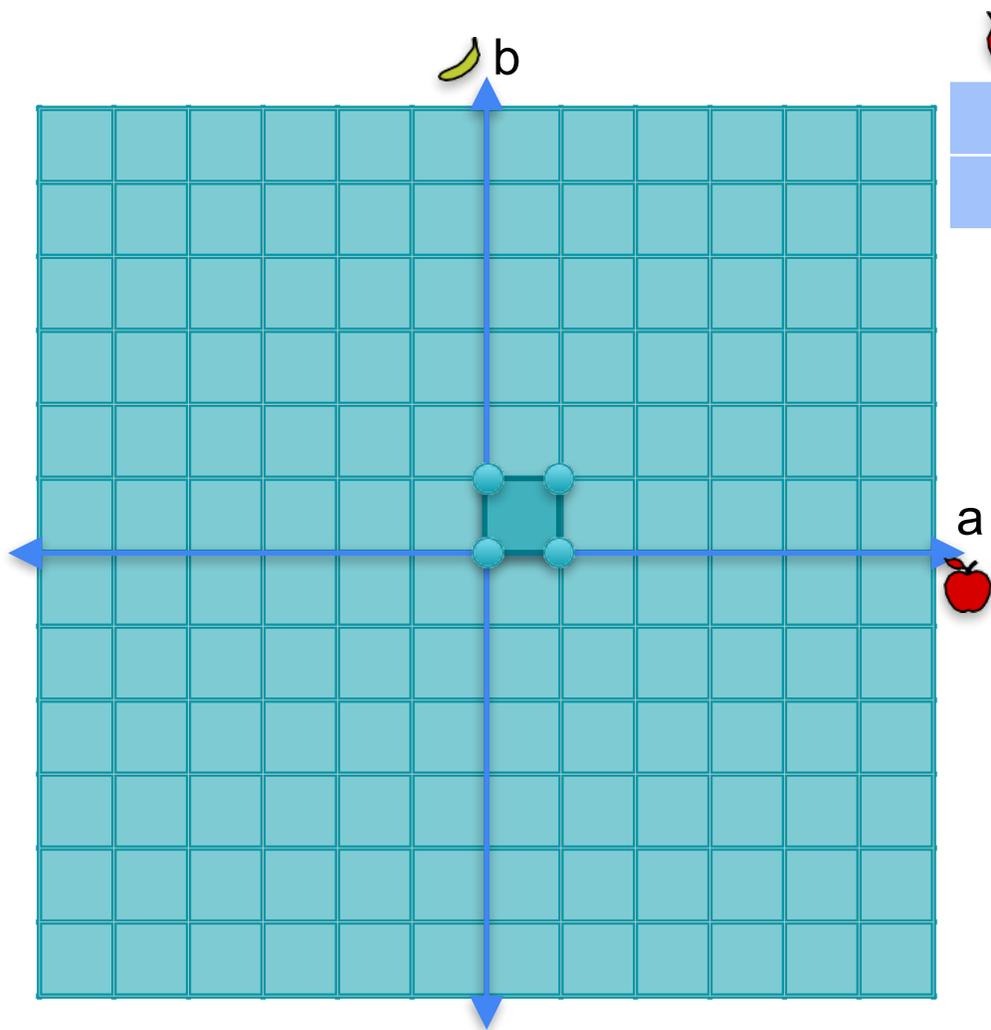
			
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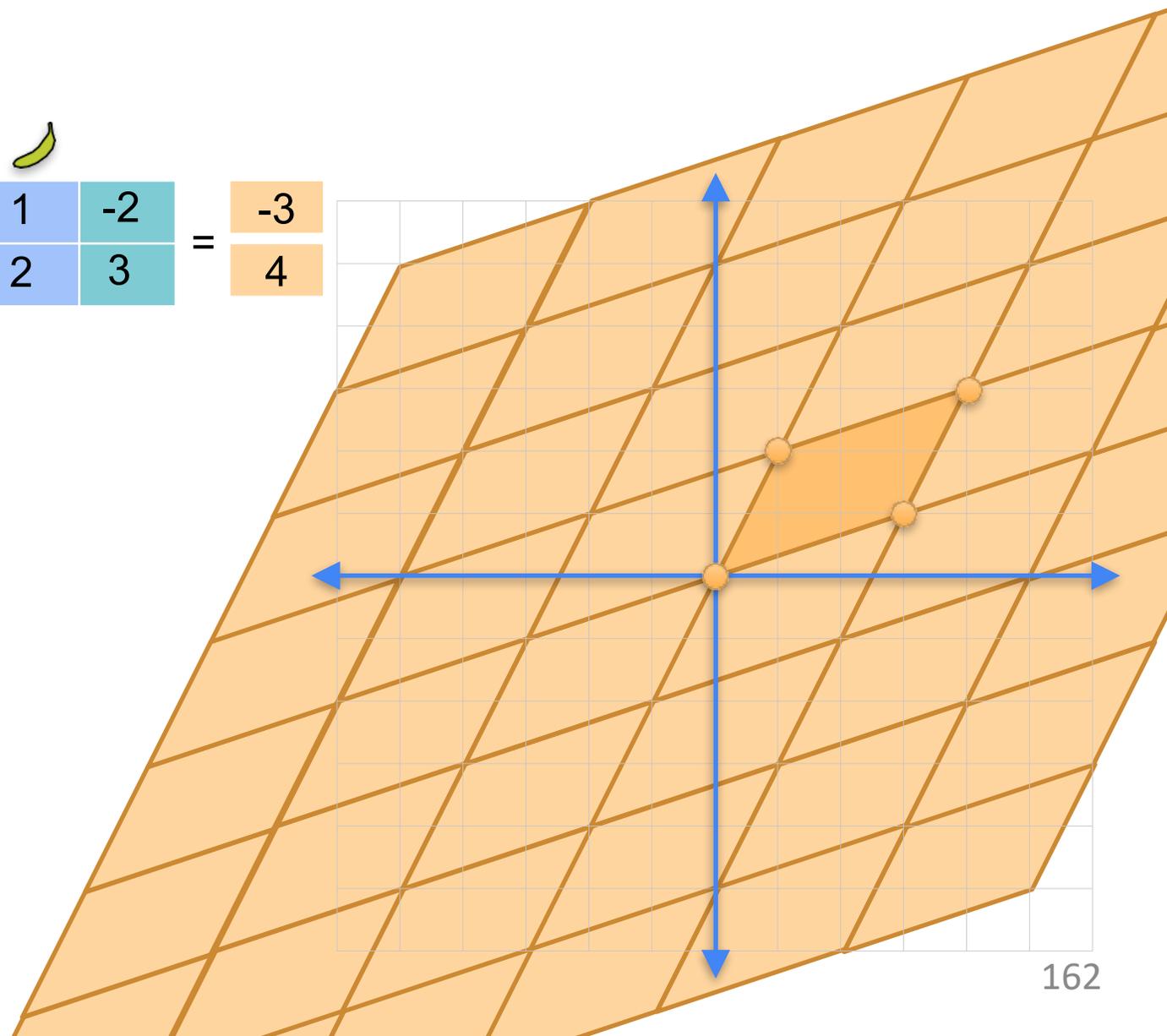
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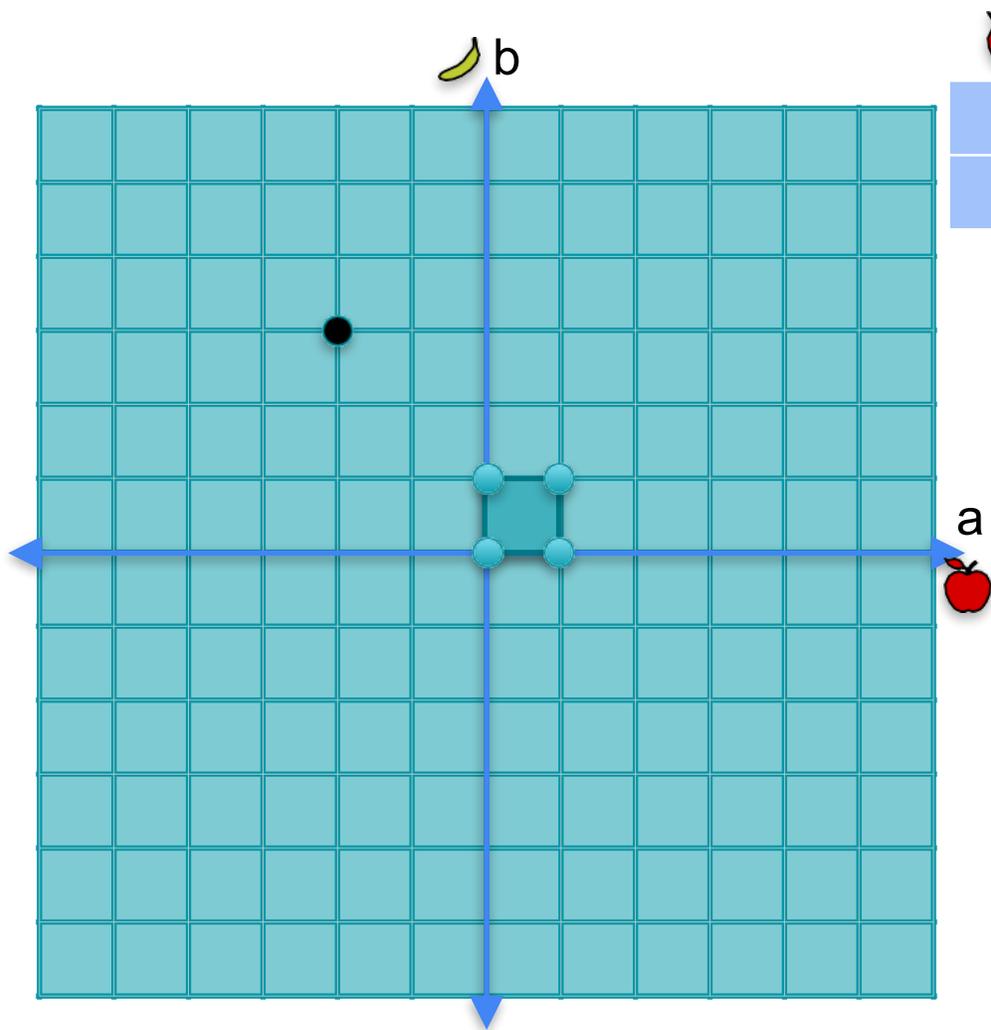
			
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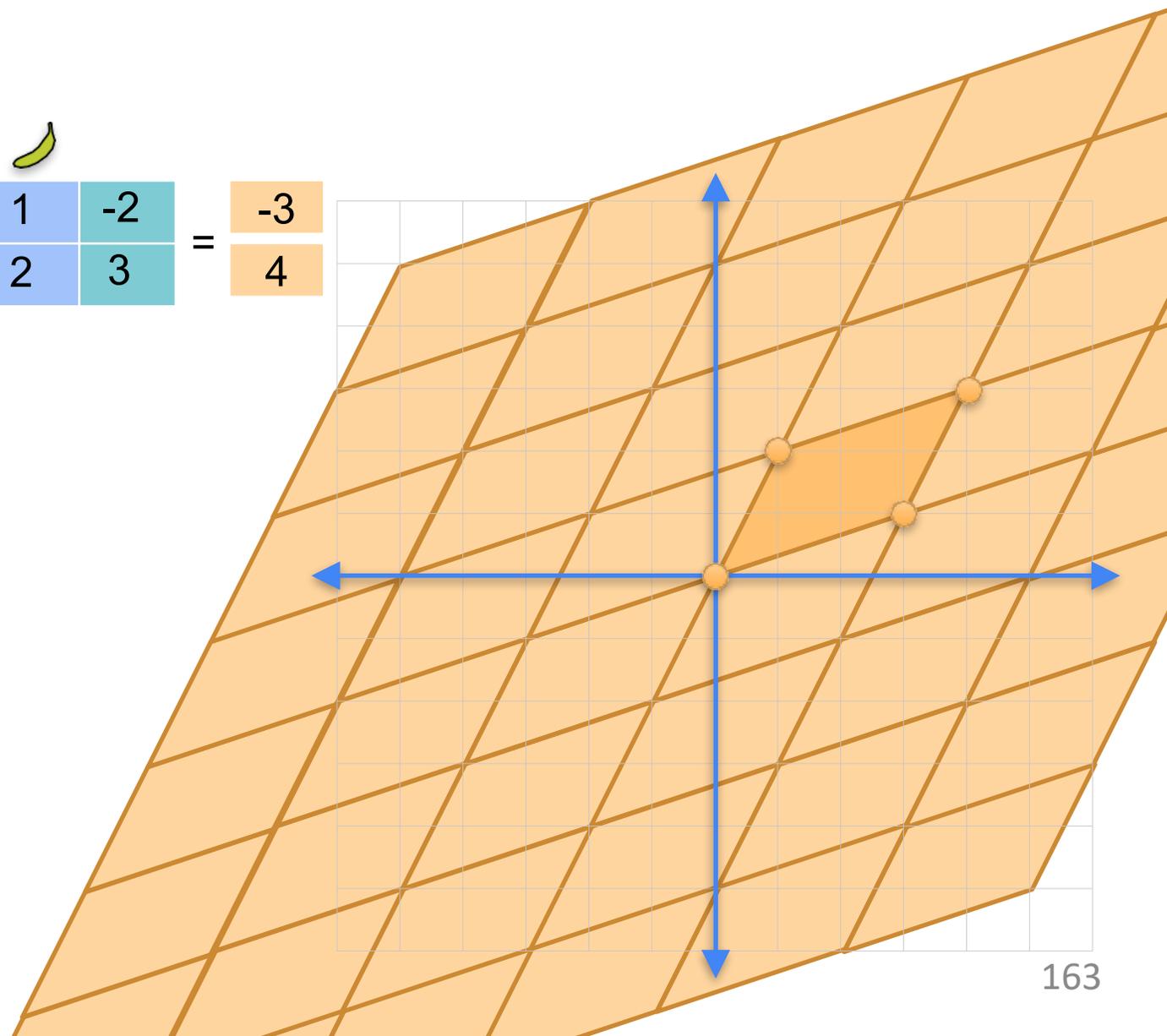
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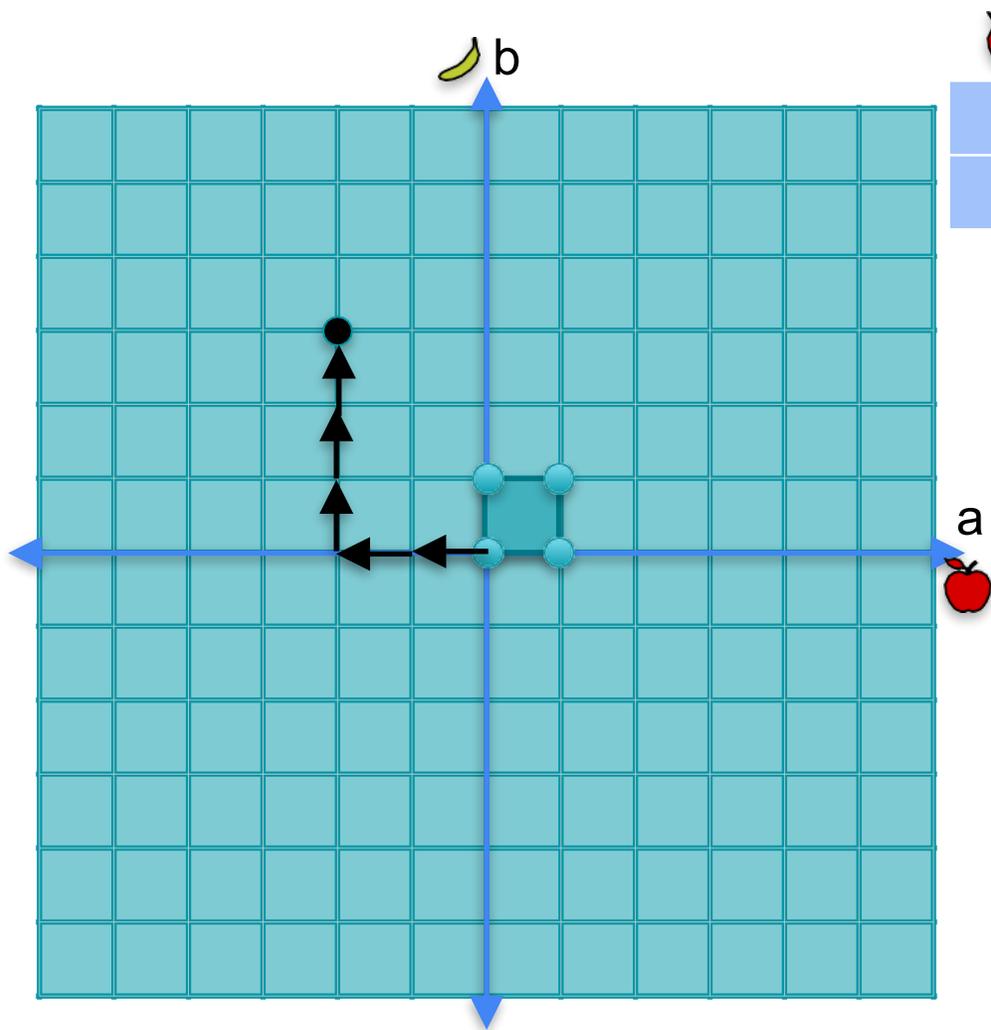
			
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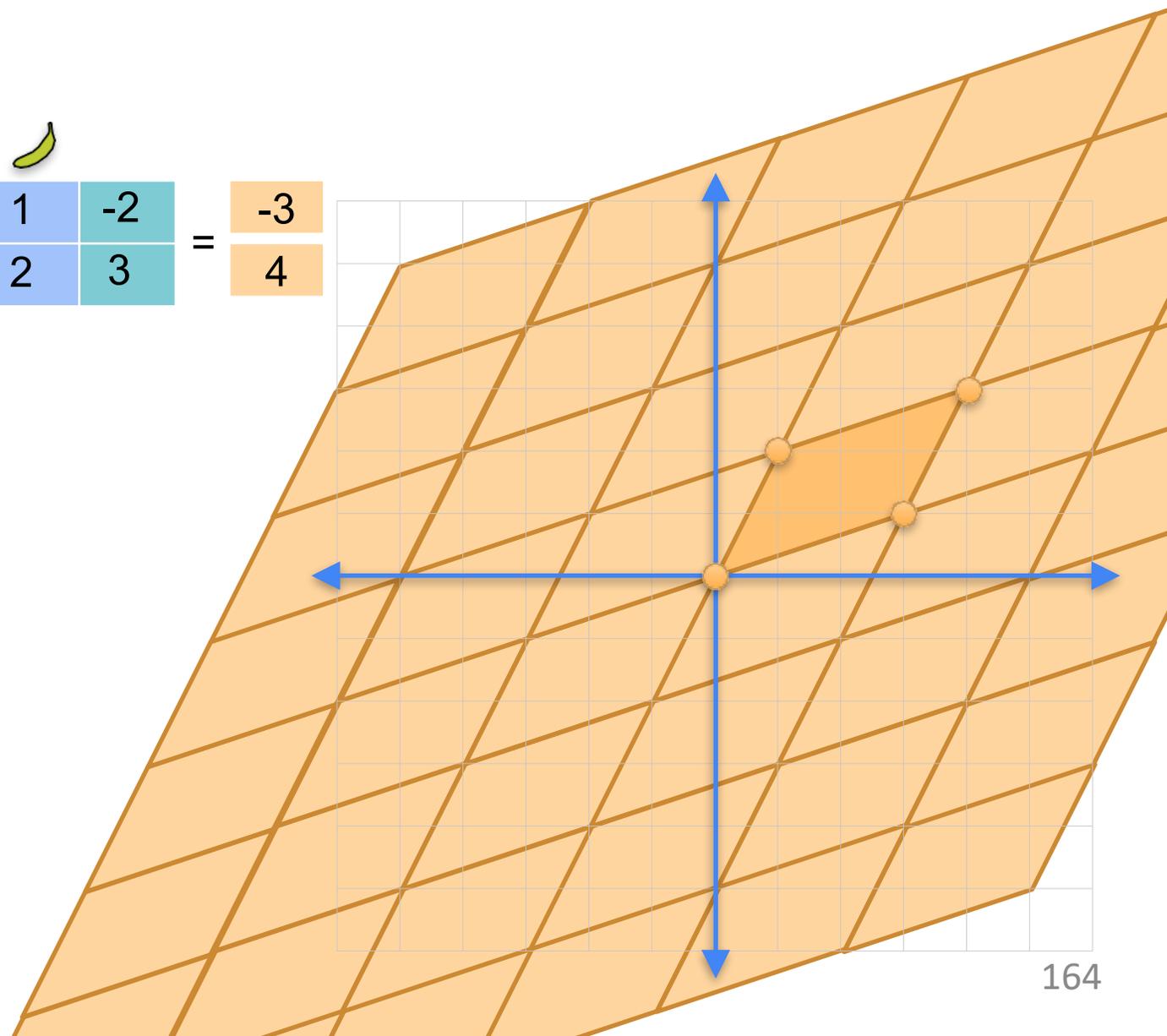
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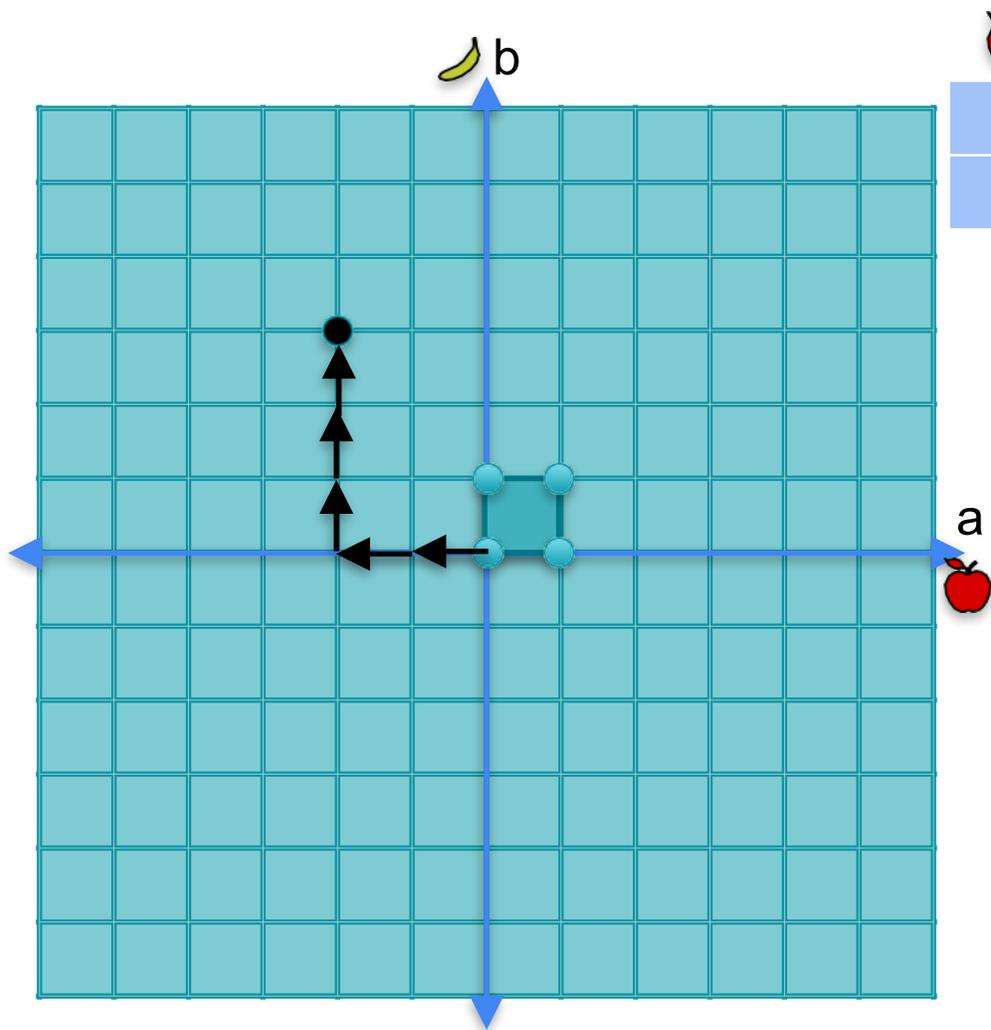
			
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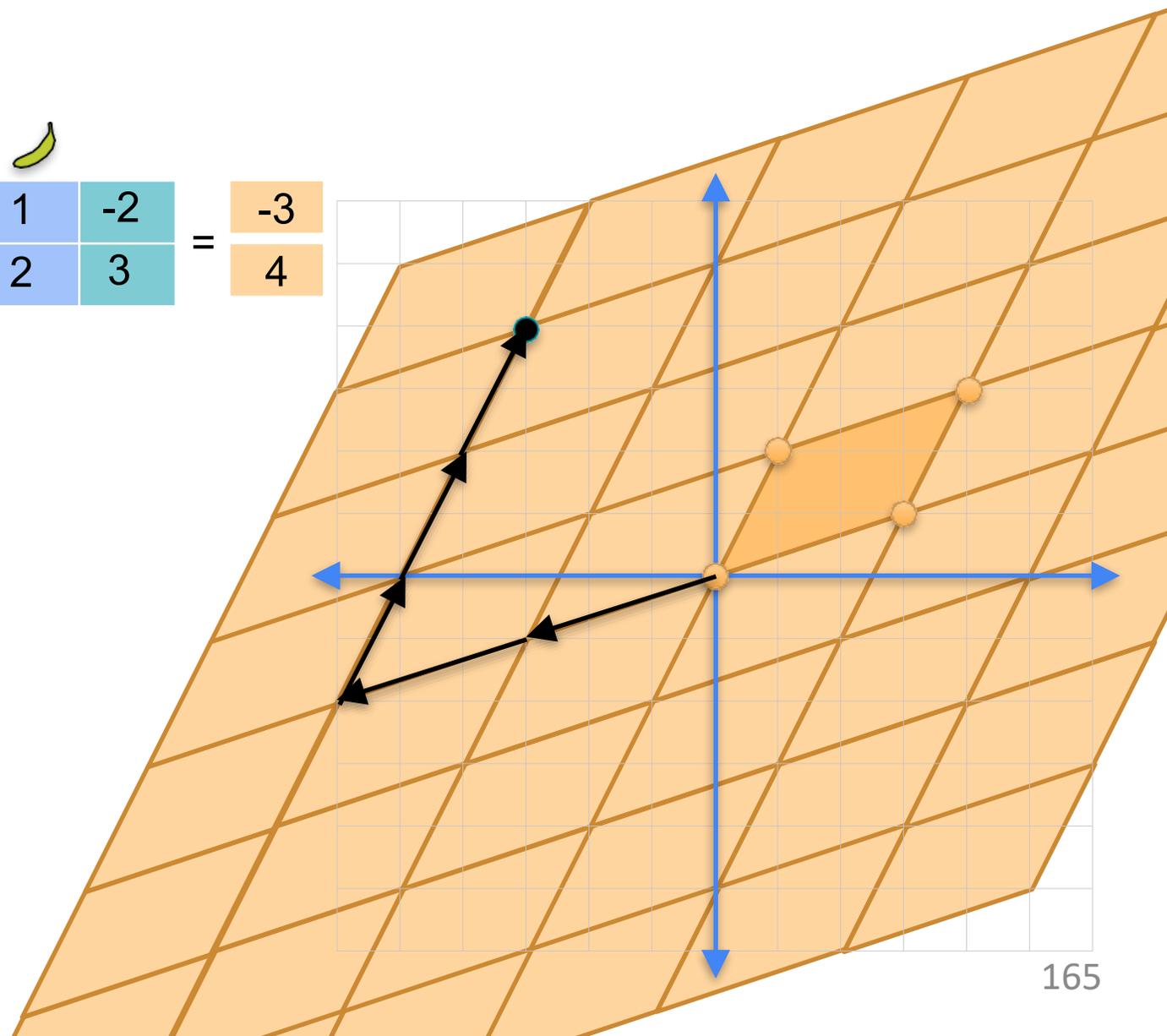
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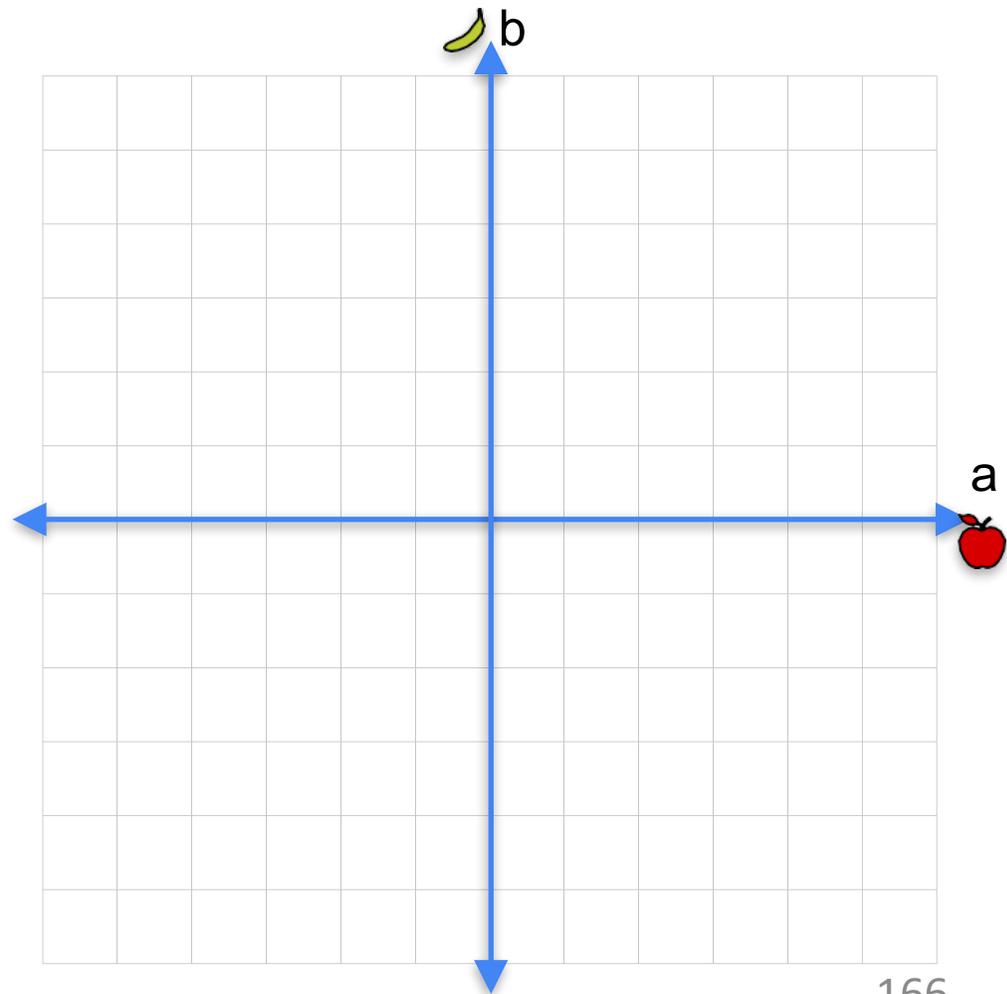
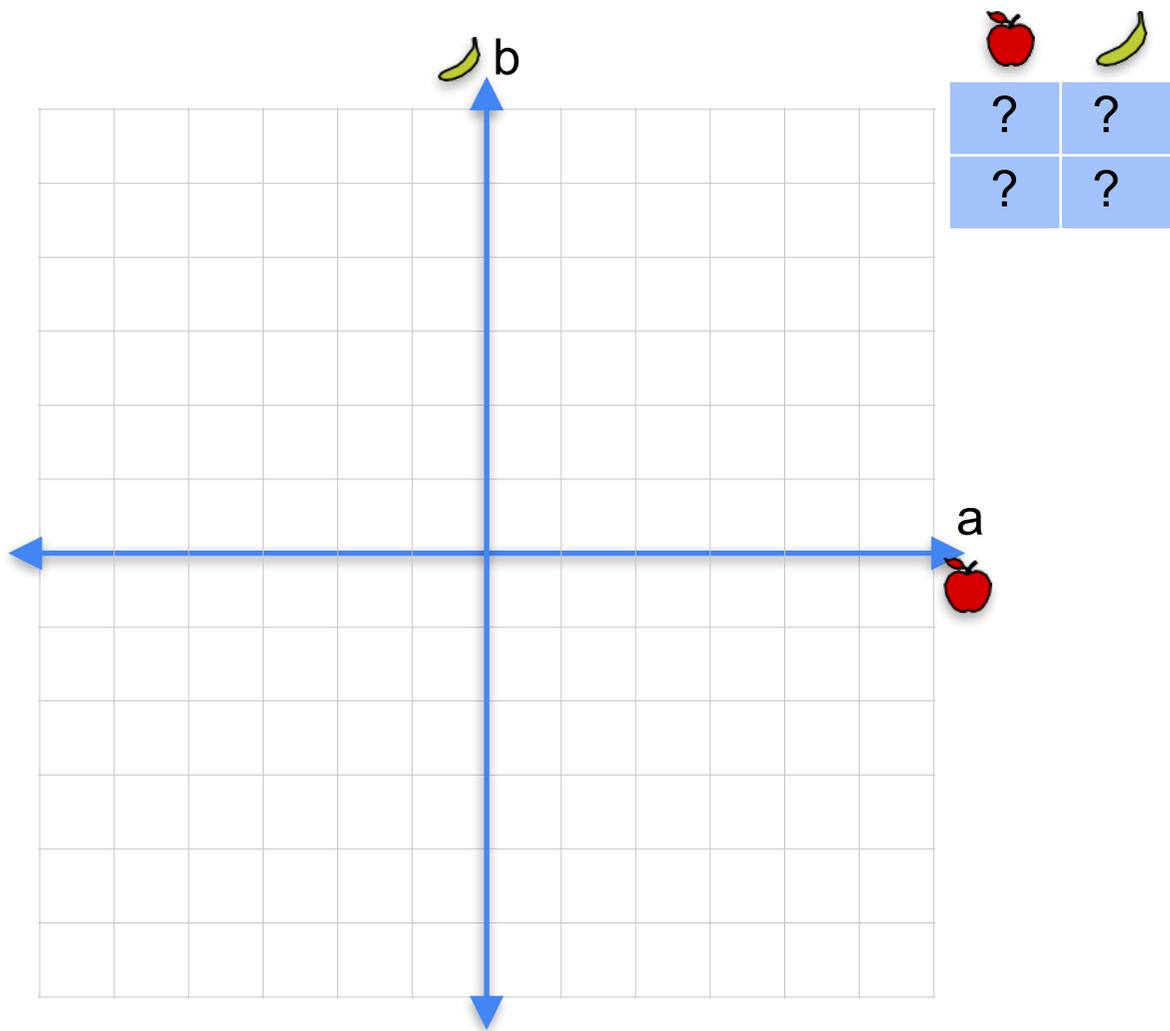
			
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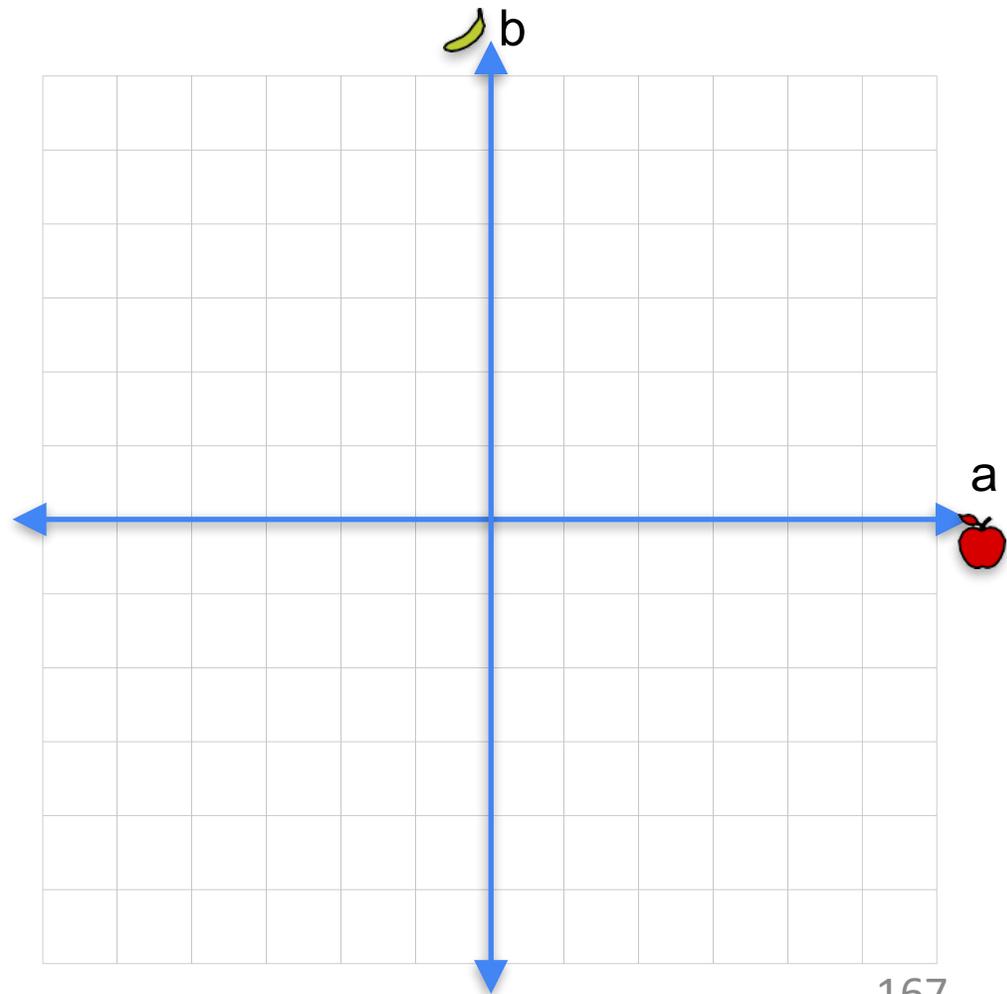
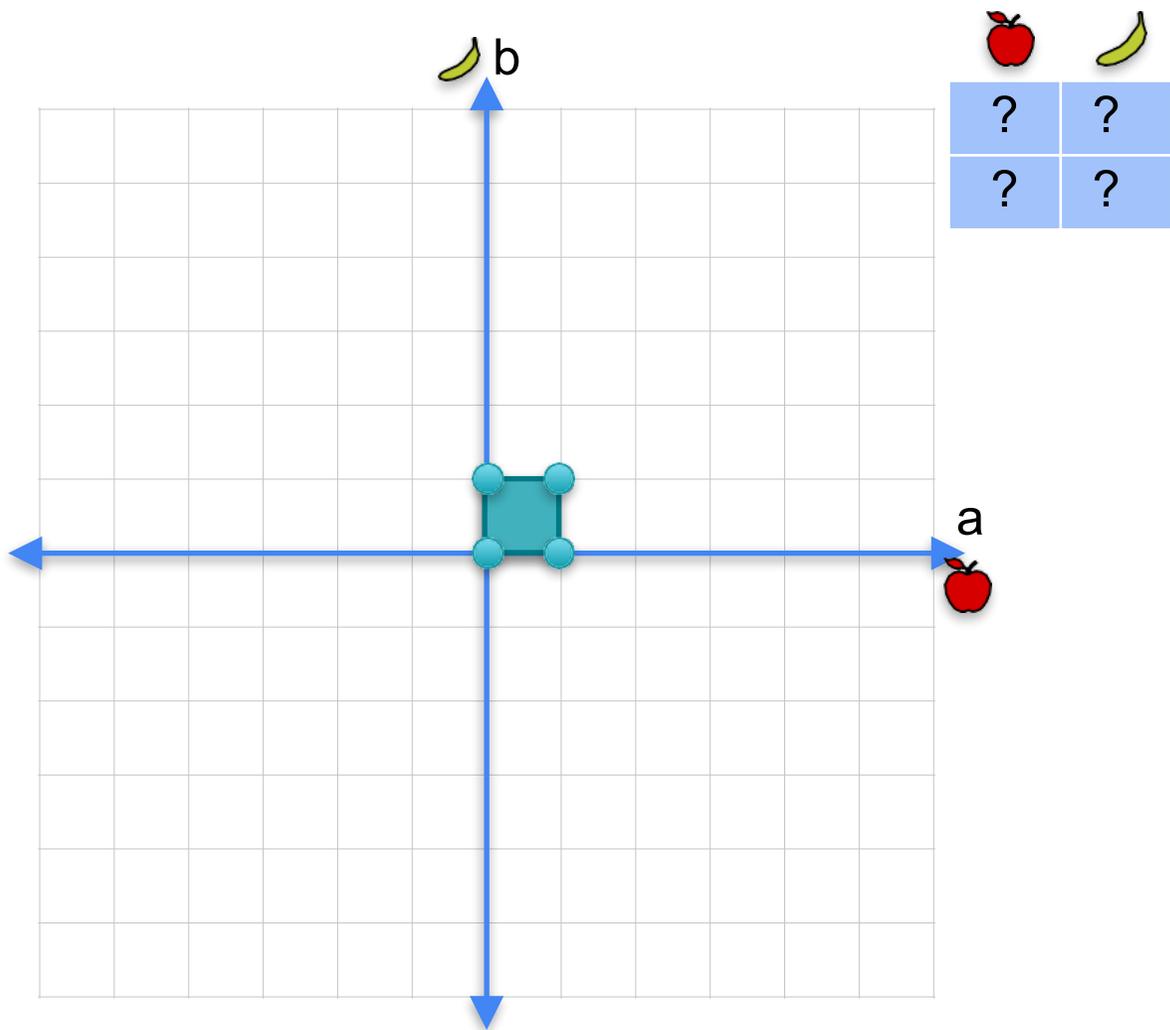
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线性变换是矩阵



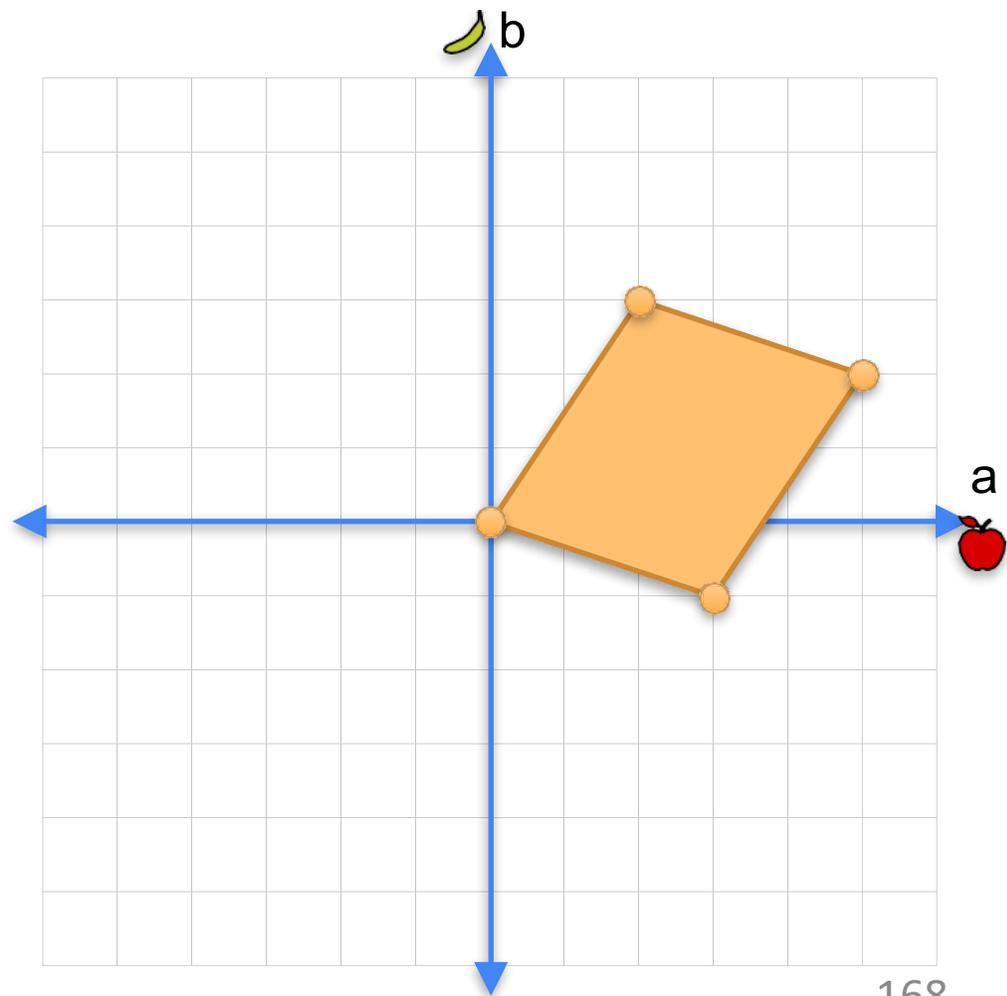
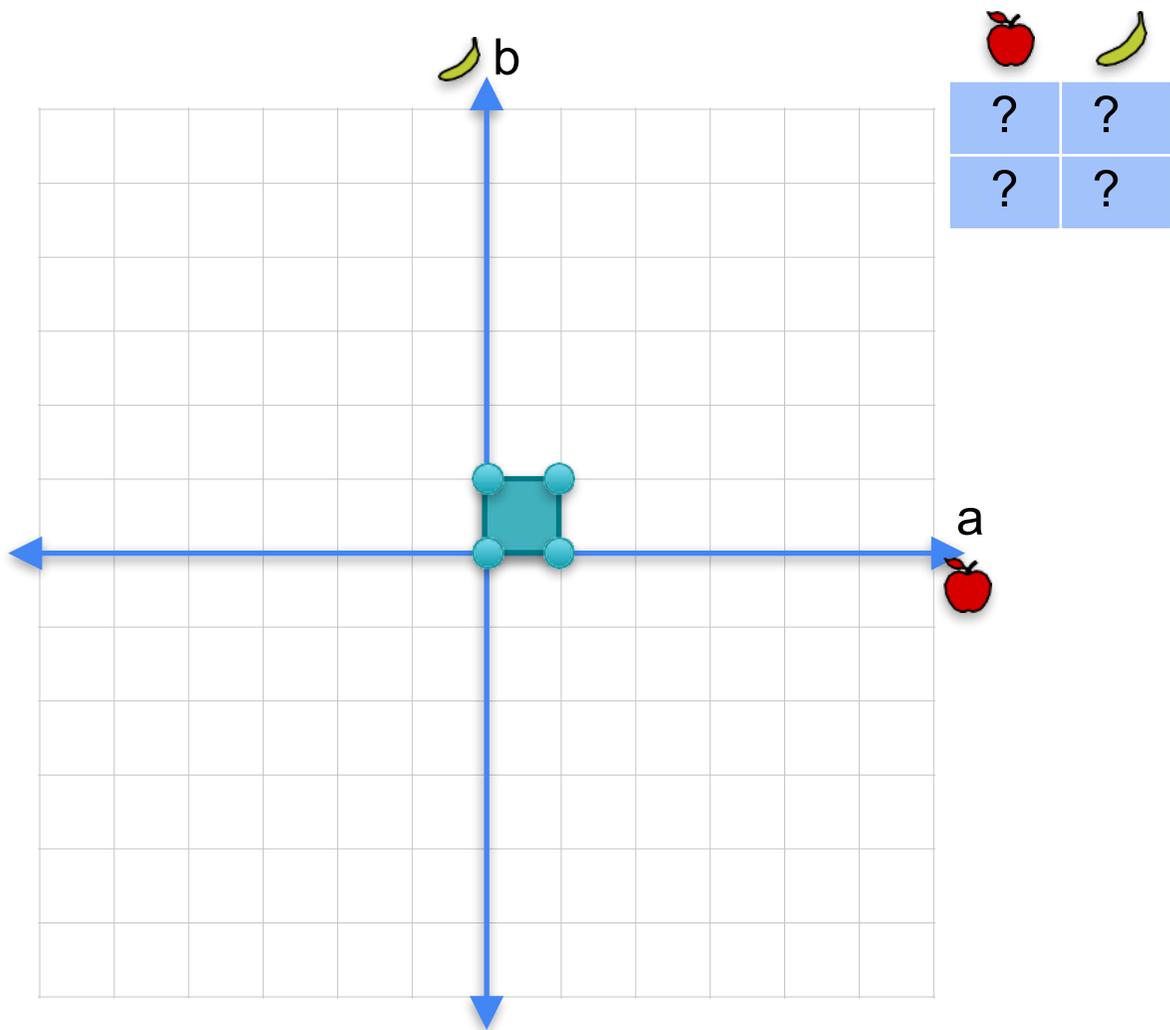
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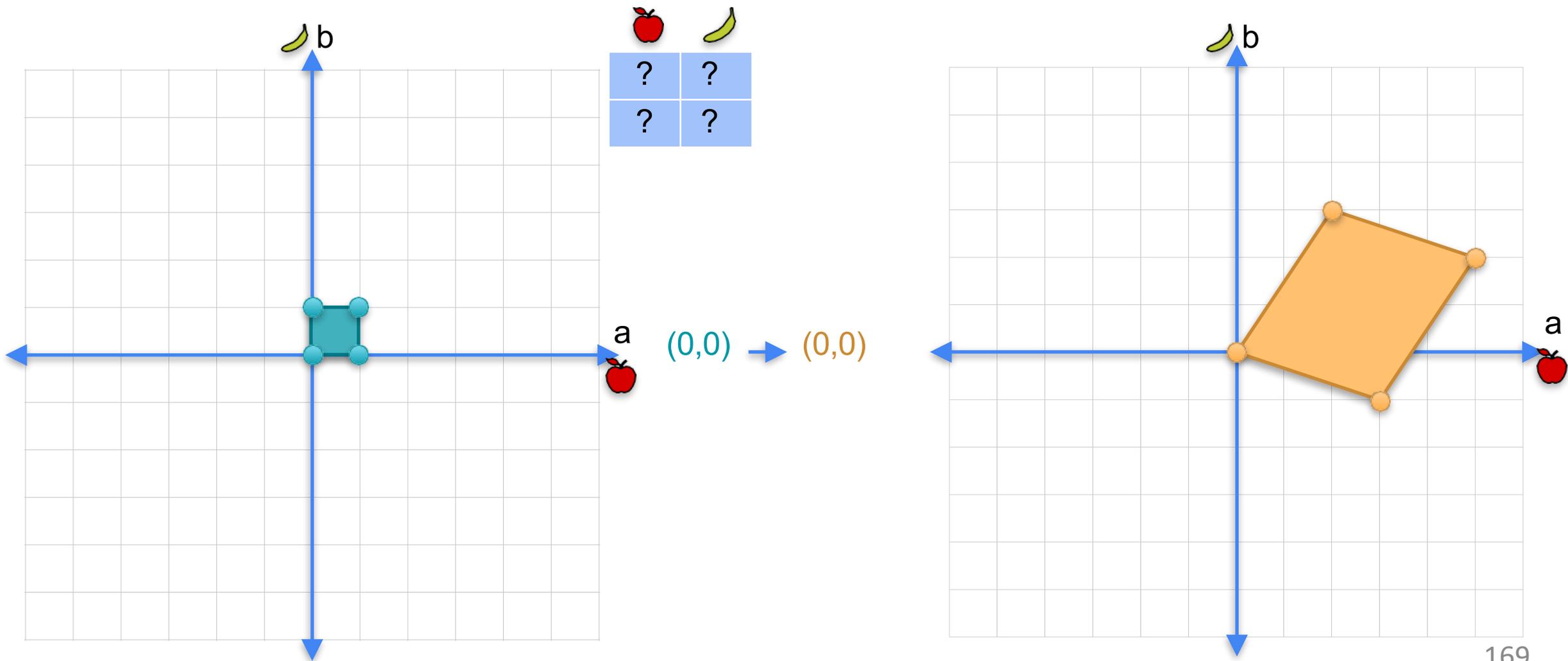
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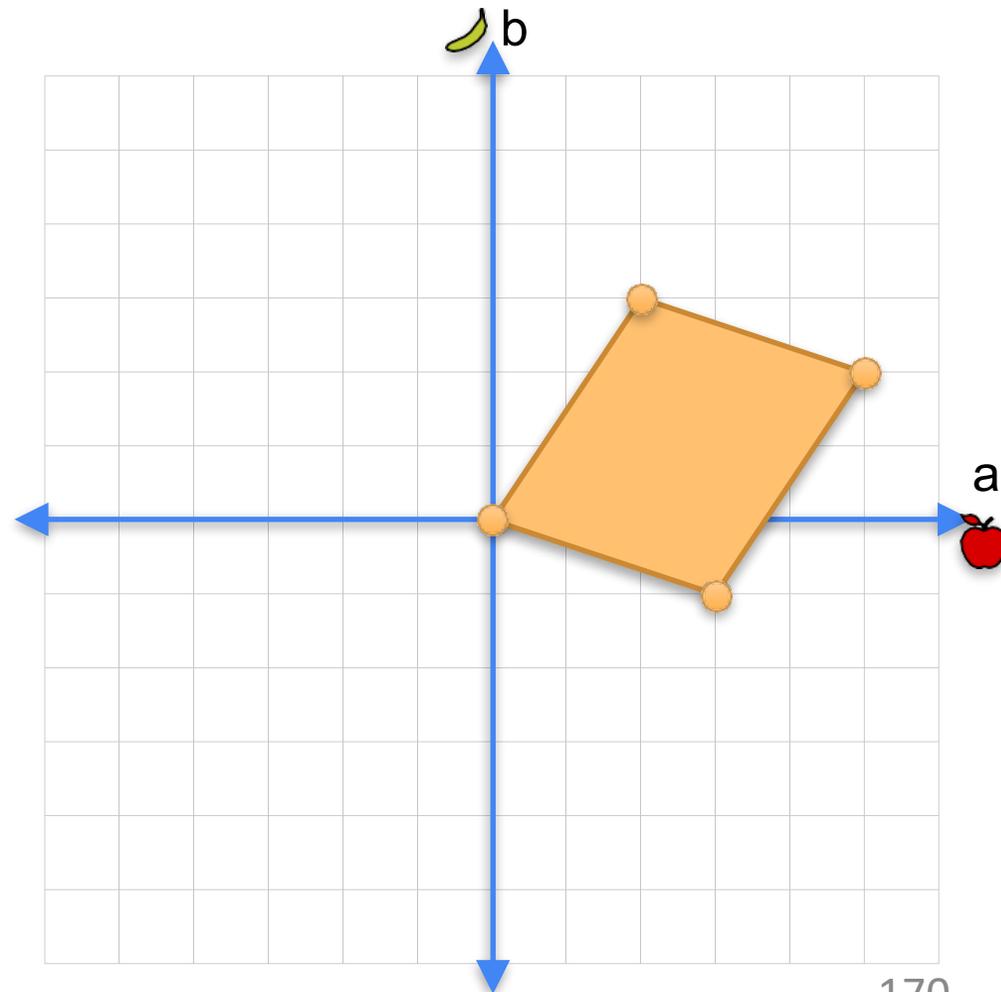
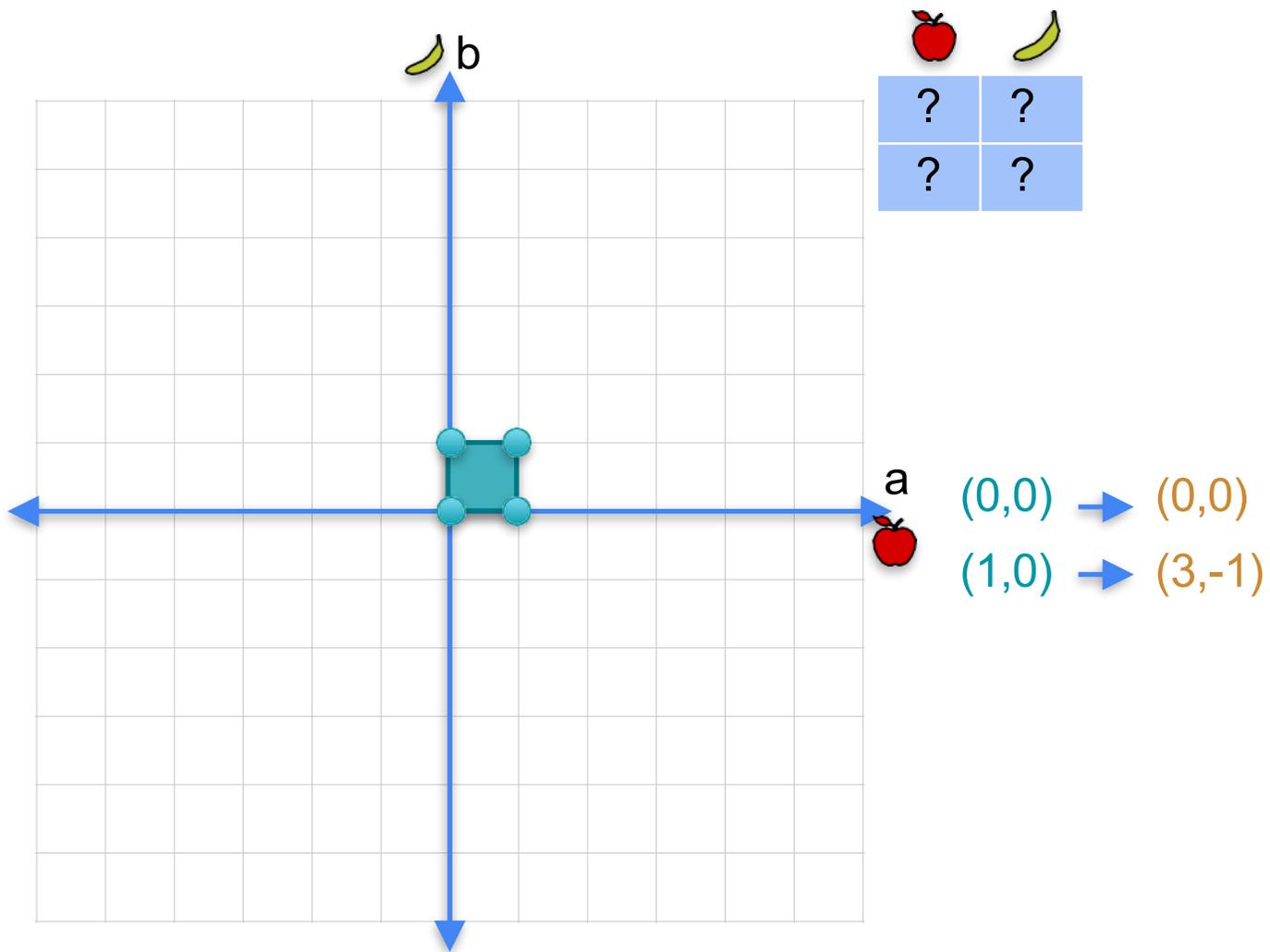
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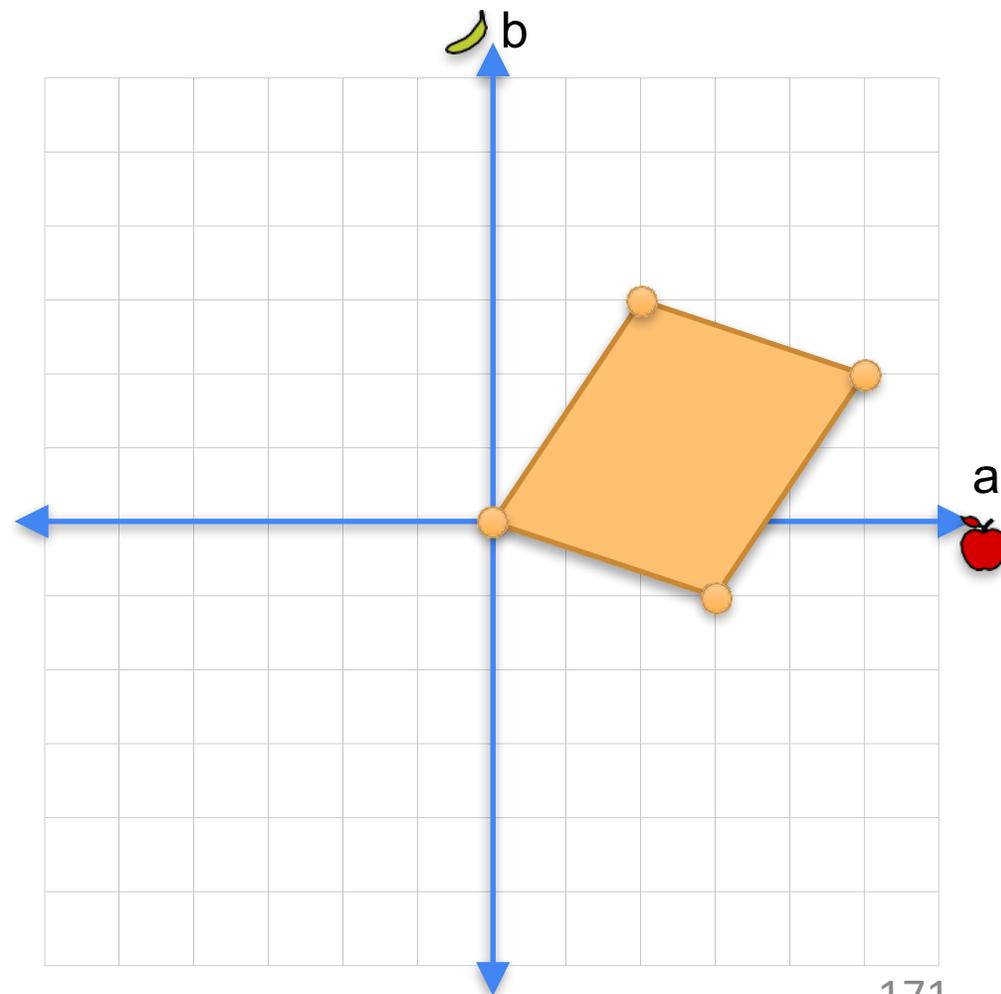
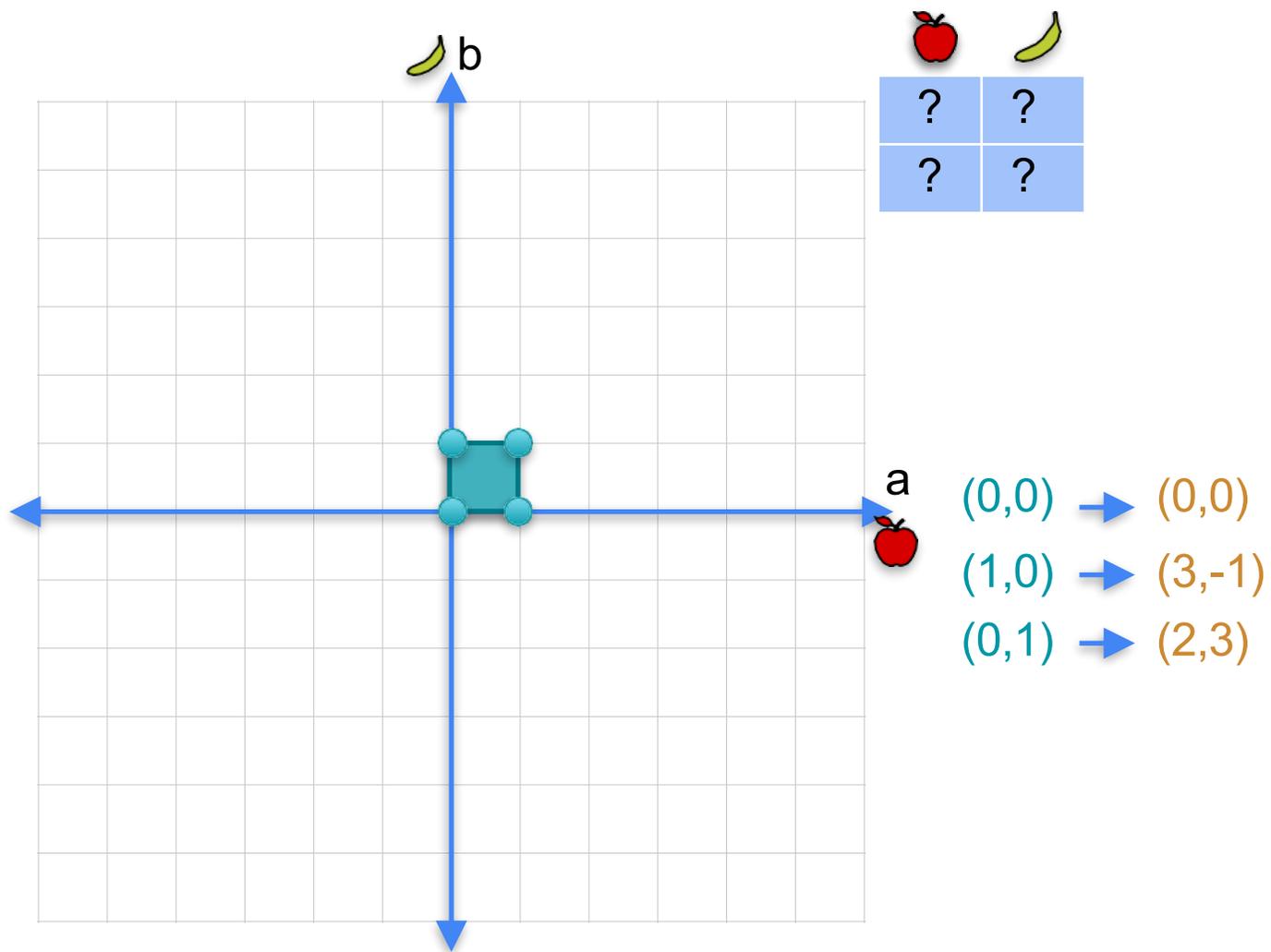
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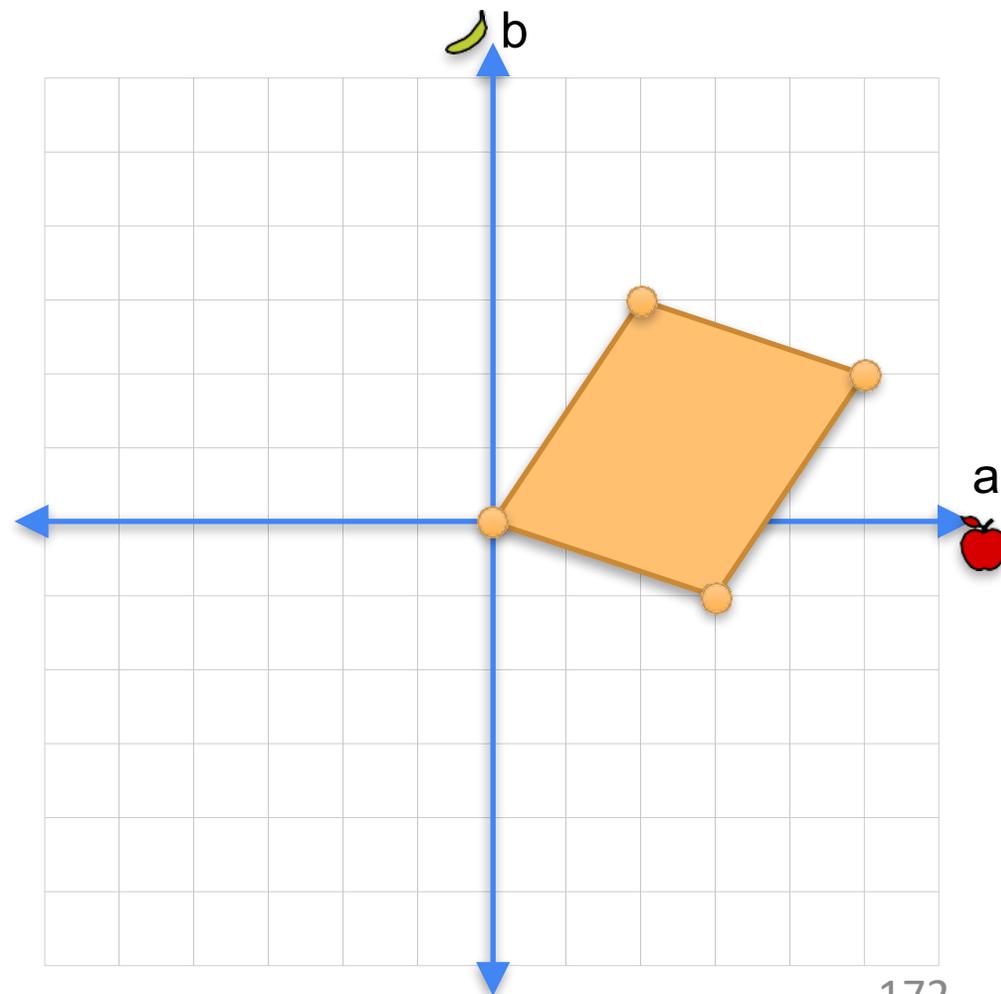
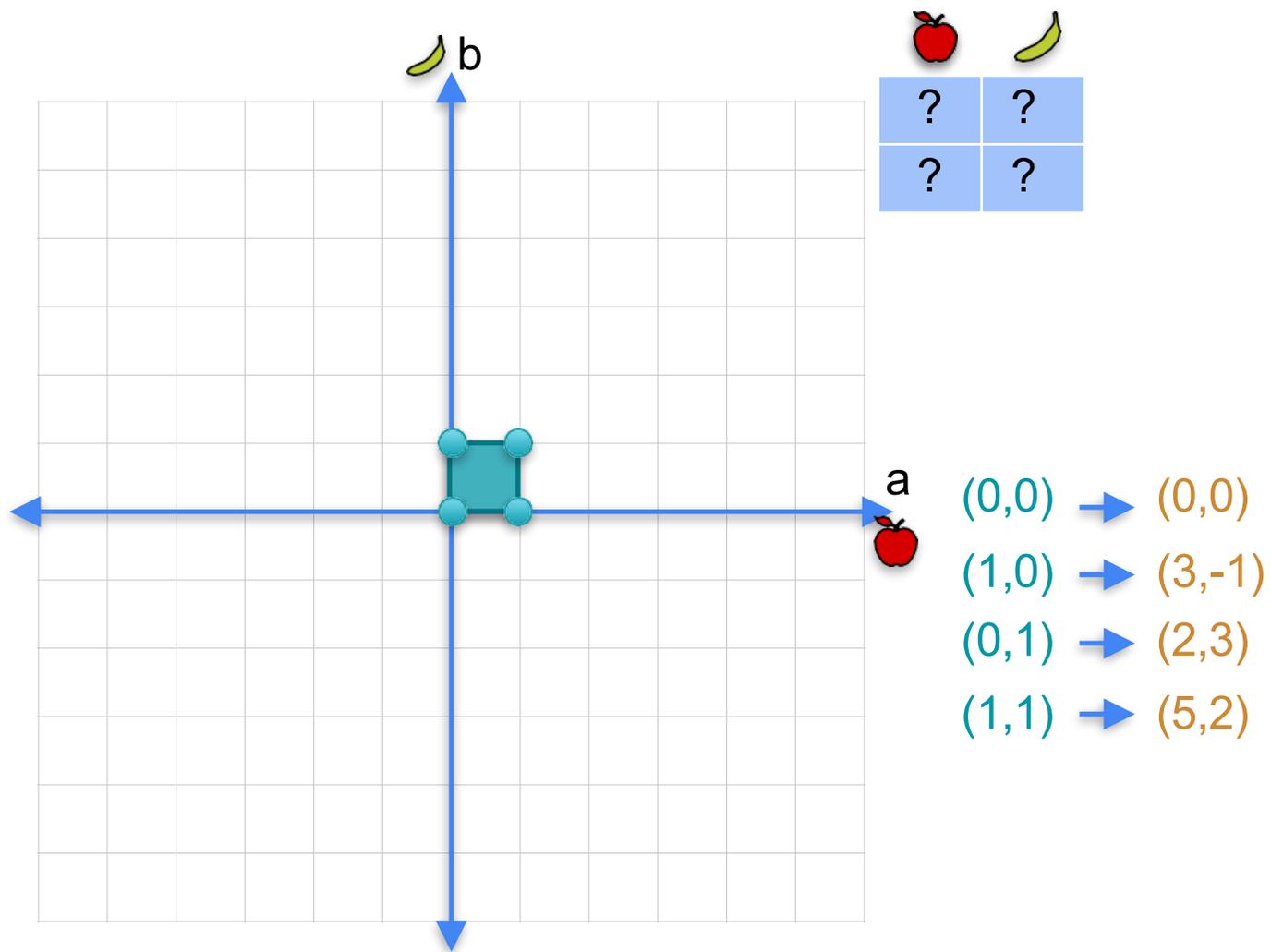
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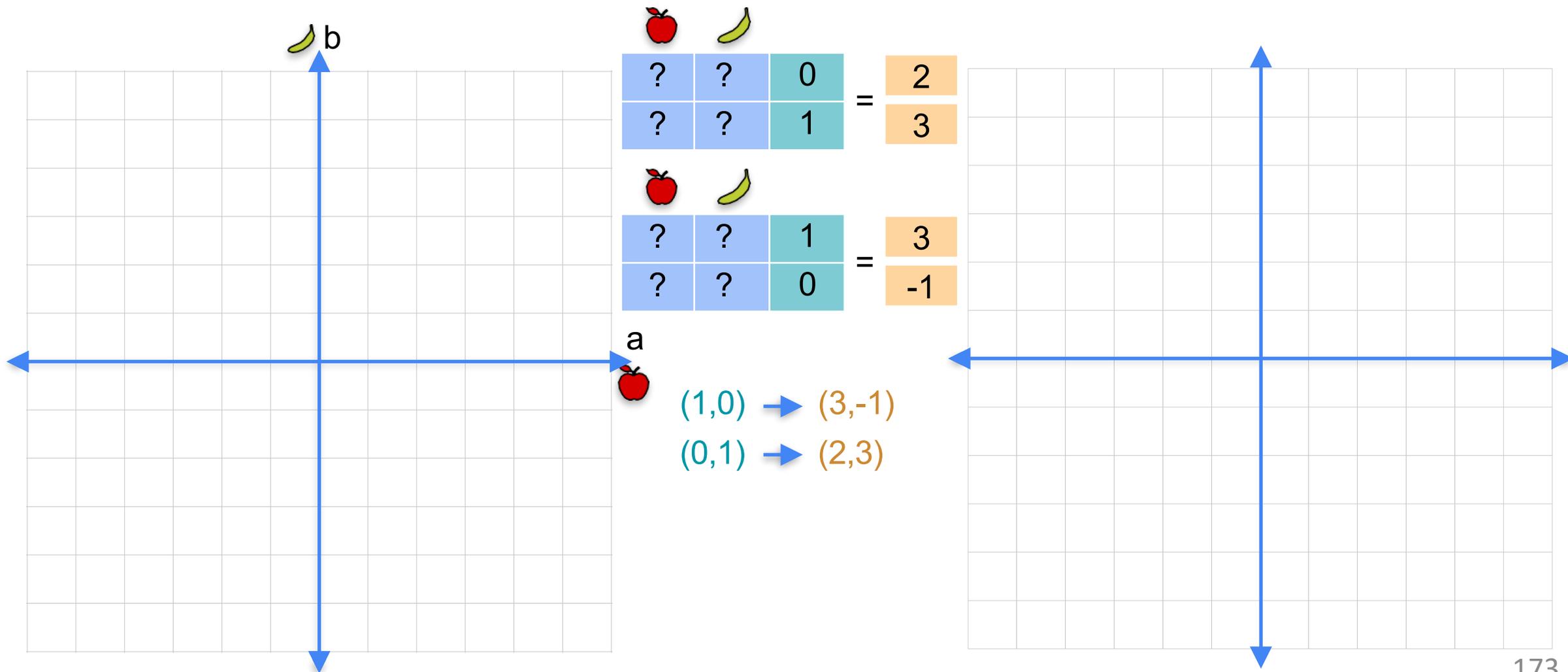
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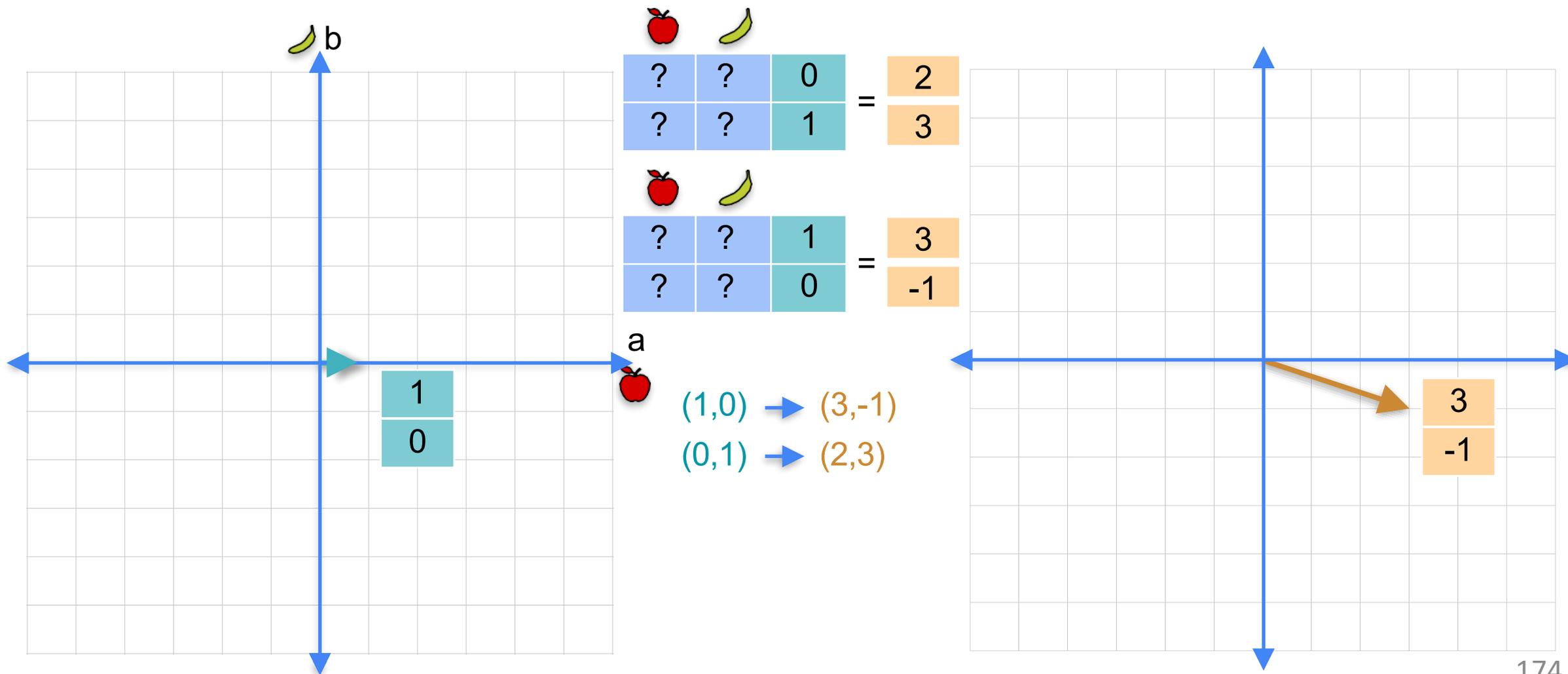
3. 线性变换和矩阵乘法

线性变换是矩阵



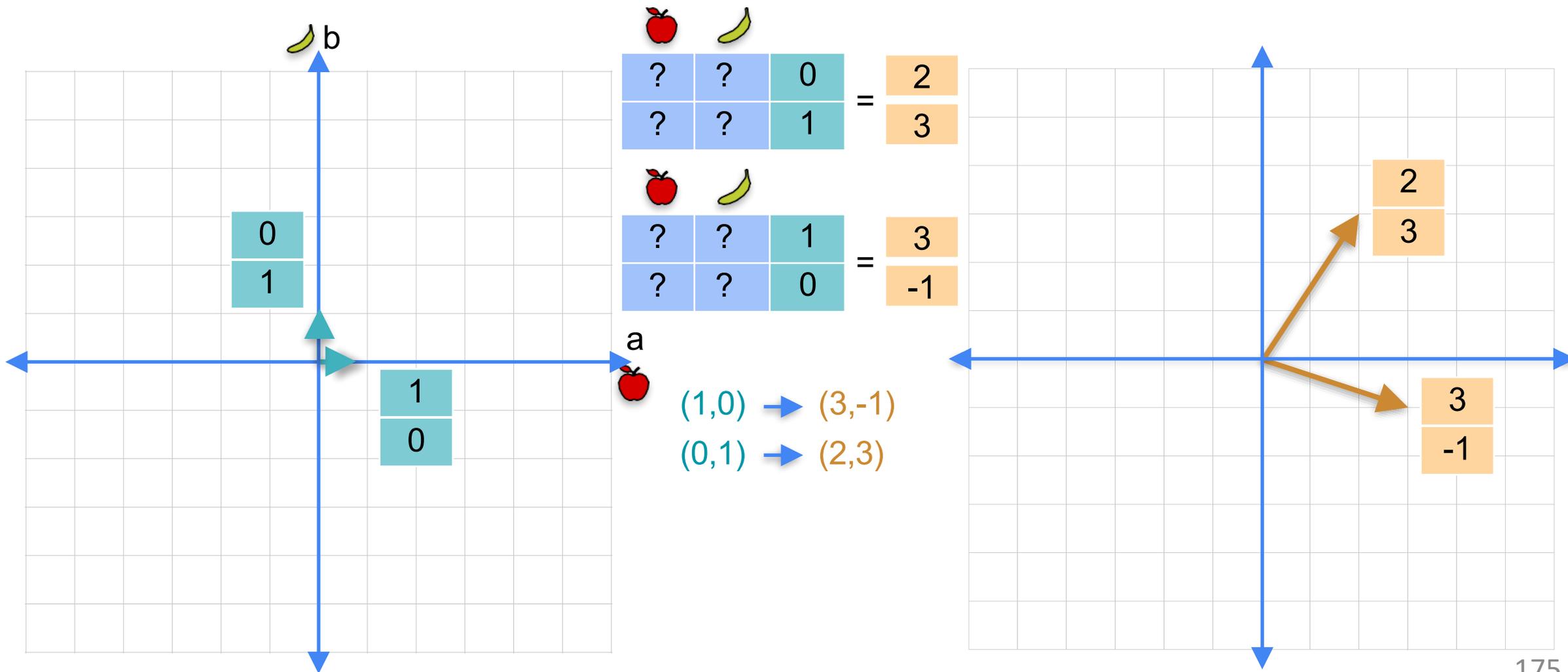
3. 线性变换和矩阵乘法

线性变换是矩阵



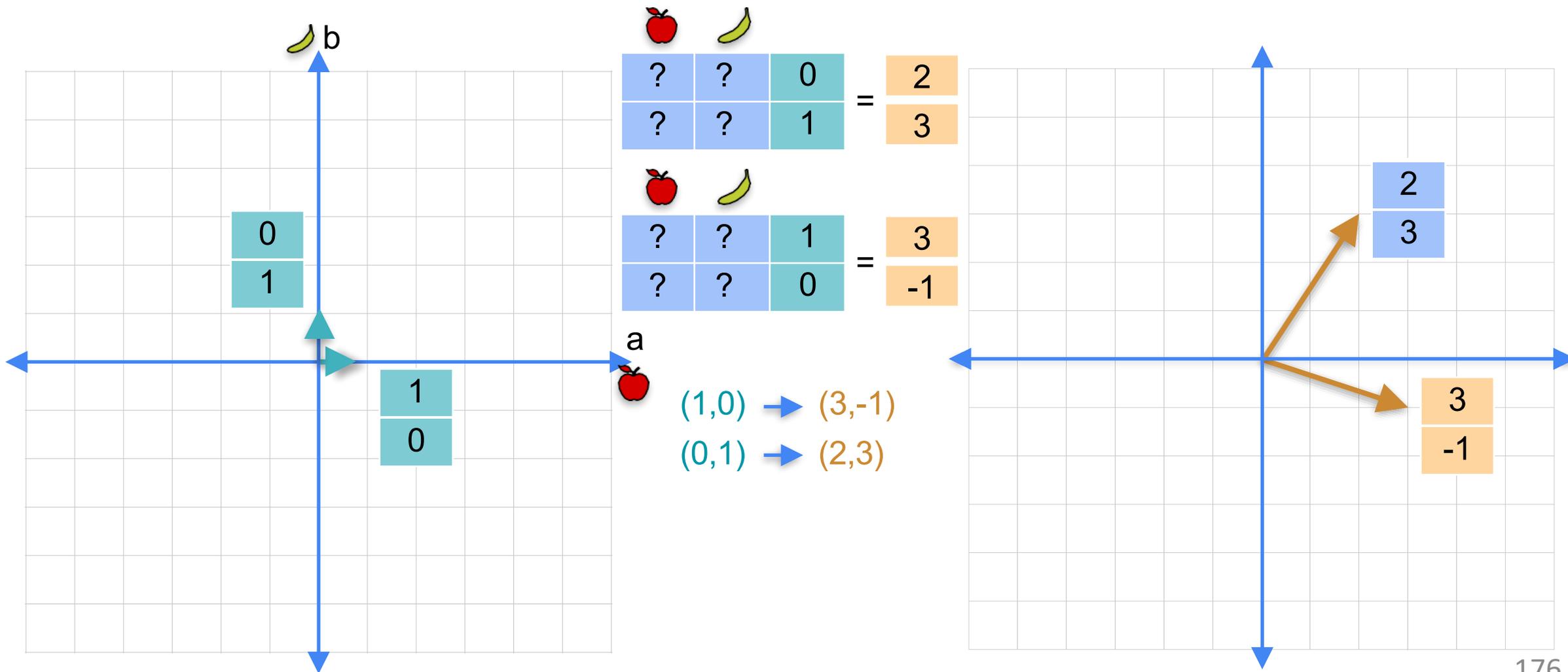
3. 线性变换和矩阵乘法

线性变换是矩阵



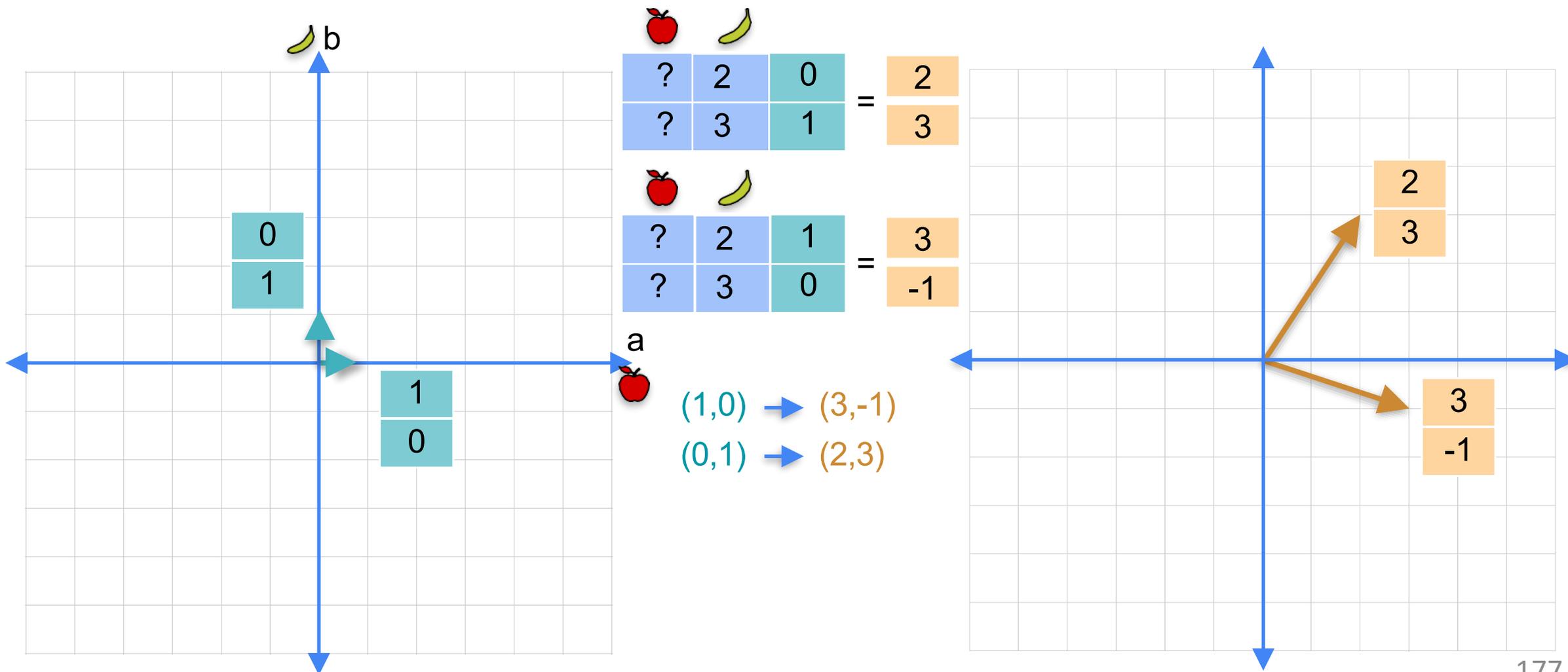
3. 线性变换和矩阵乘法

线性变换是矩阵



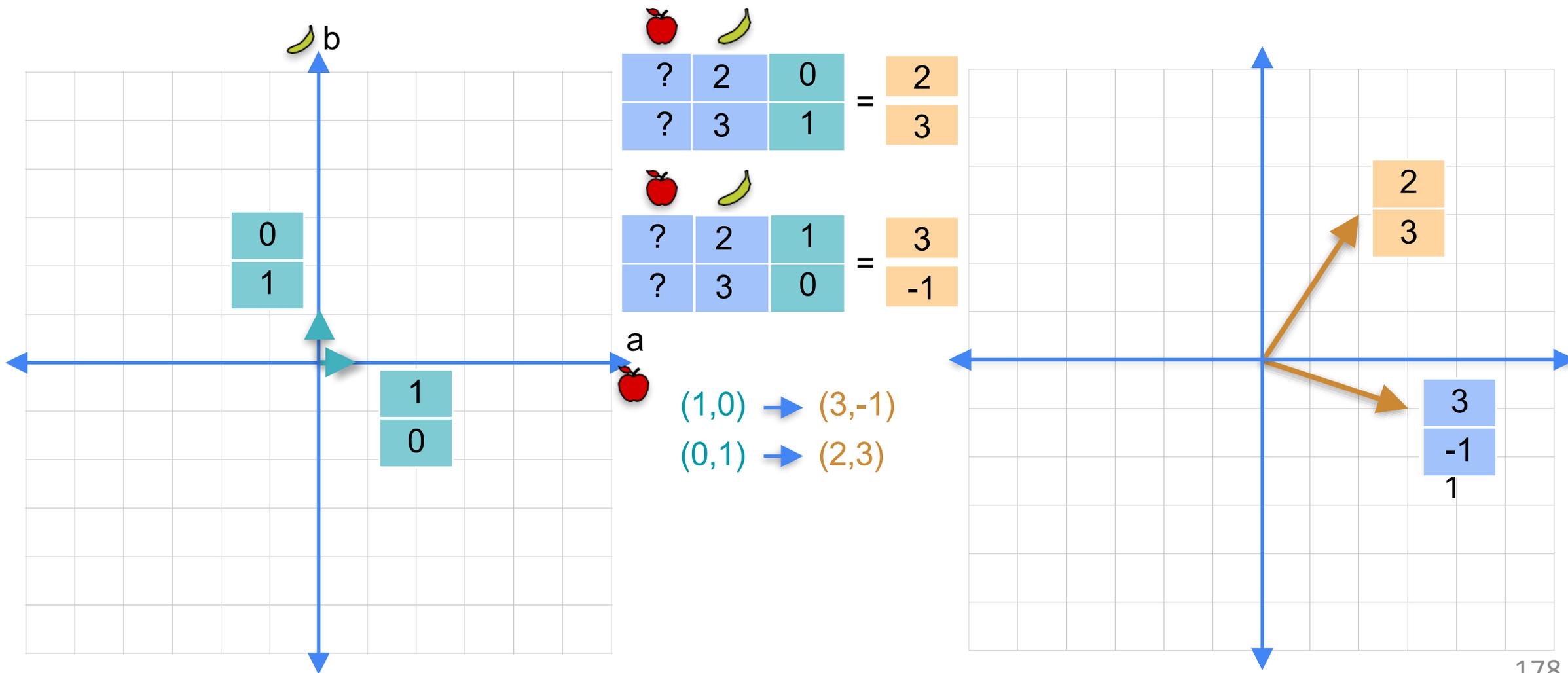
3. 线性变换和矩阵乘法

线性变换是矩阵



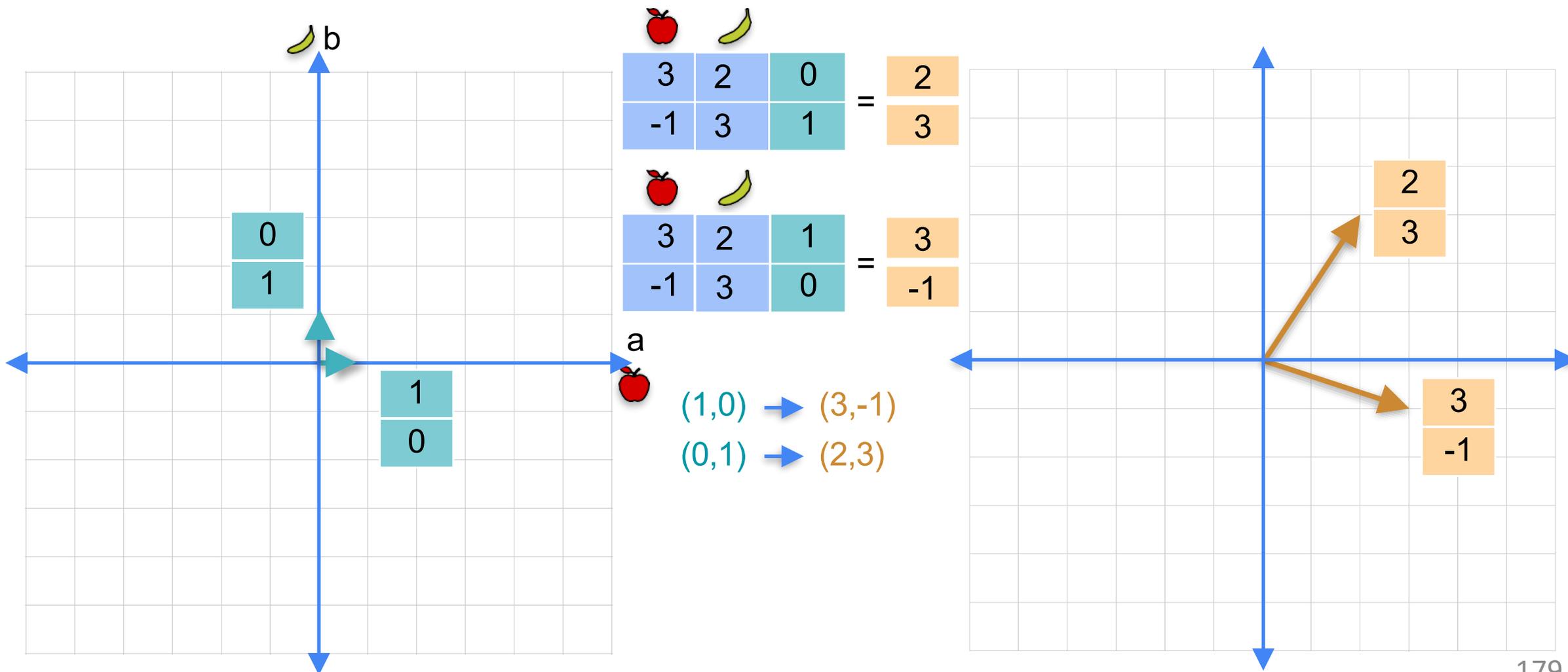
3. 线性变换和矩阵乘法

线性变换是矩阵

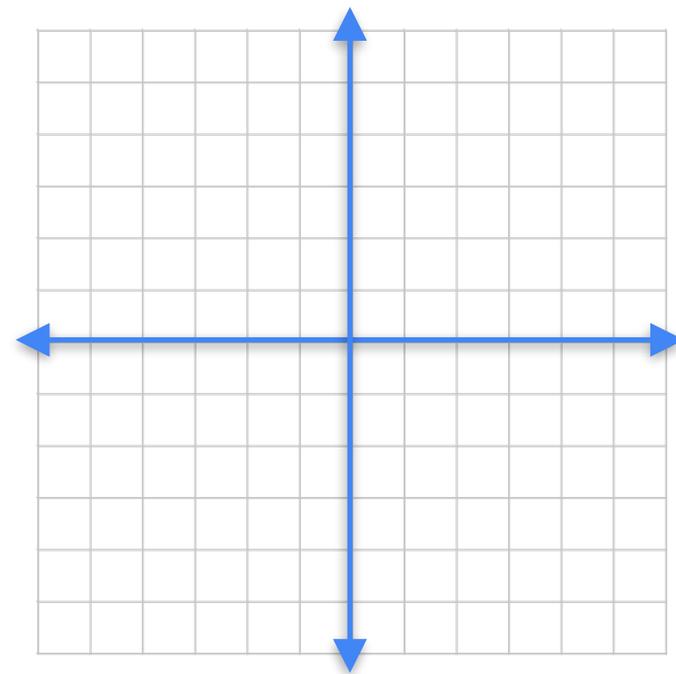
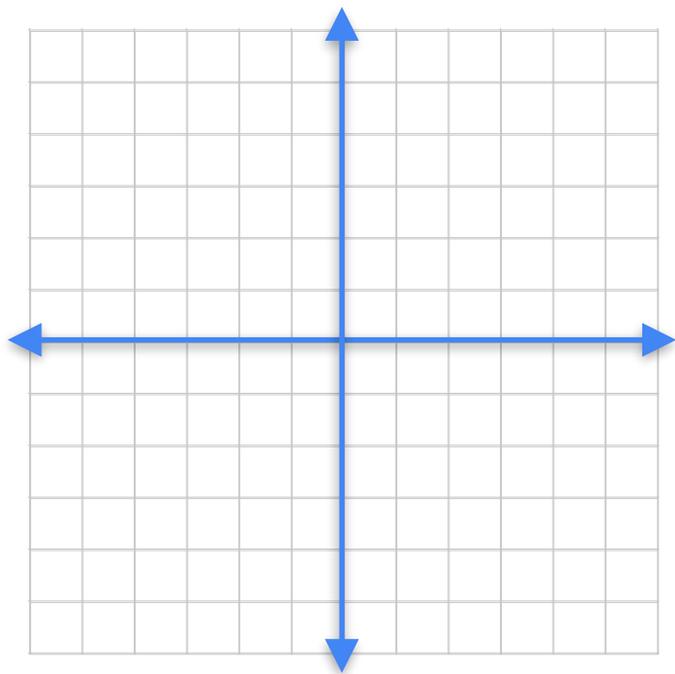


3. 线性变换和矩阵乘法

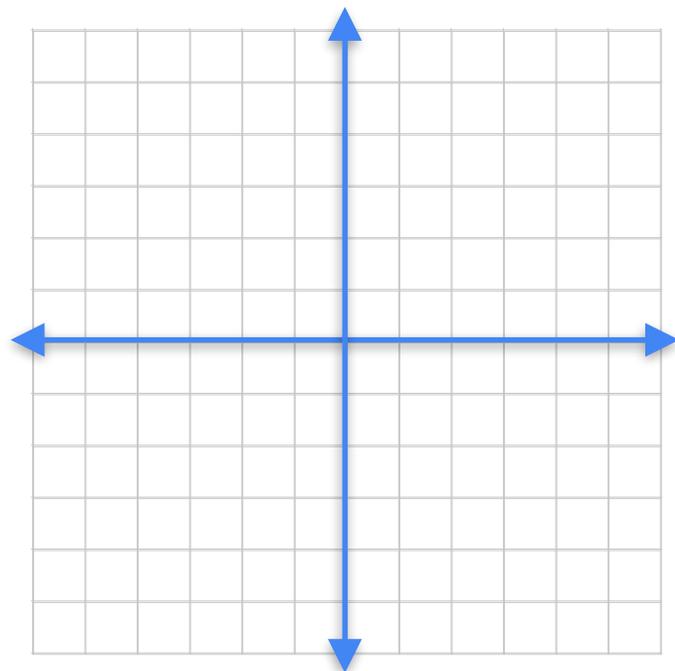
线性变换是矩阵



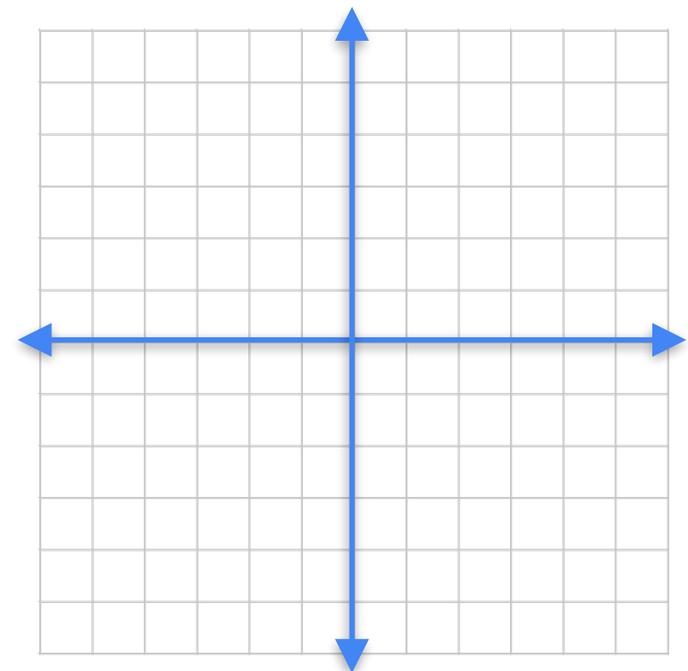
多次线性变换



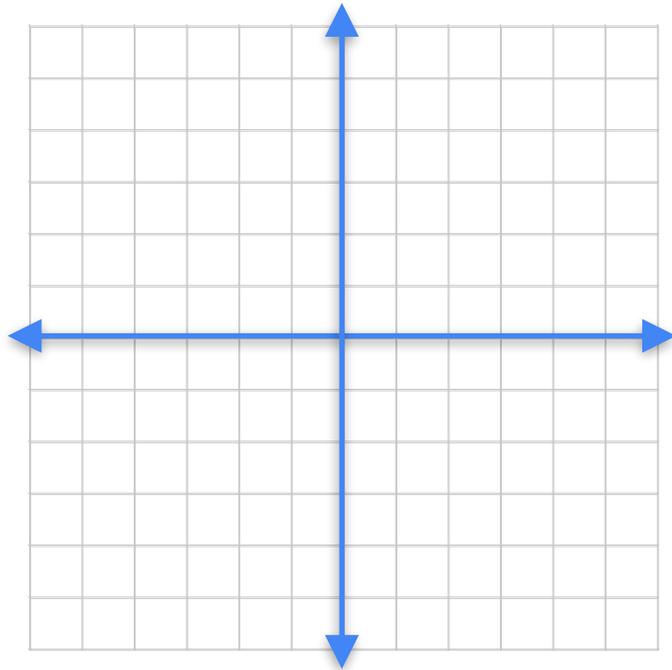
多次线性变换



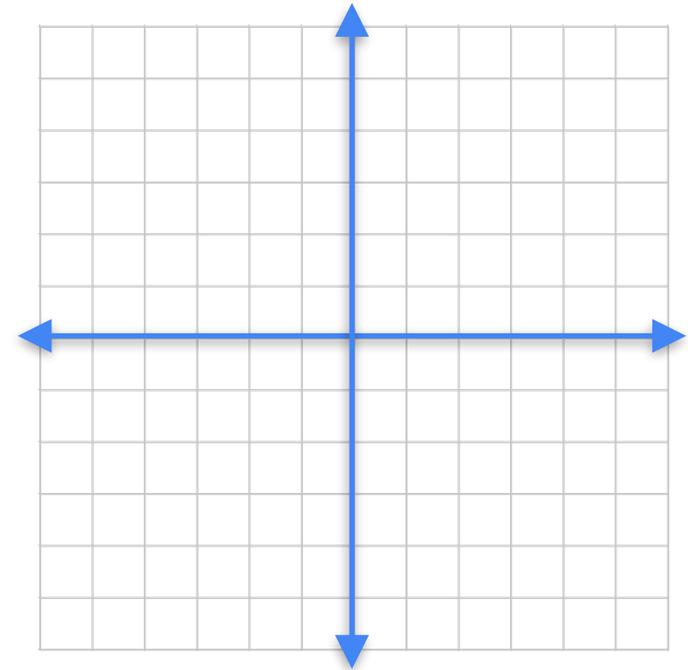
3	1
1	2



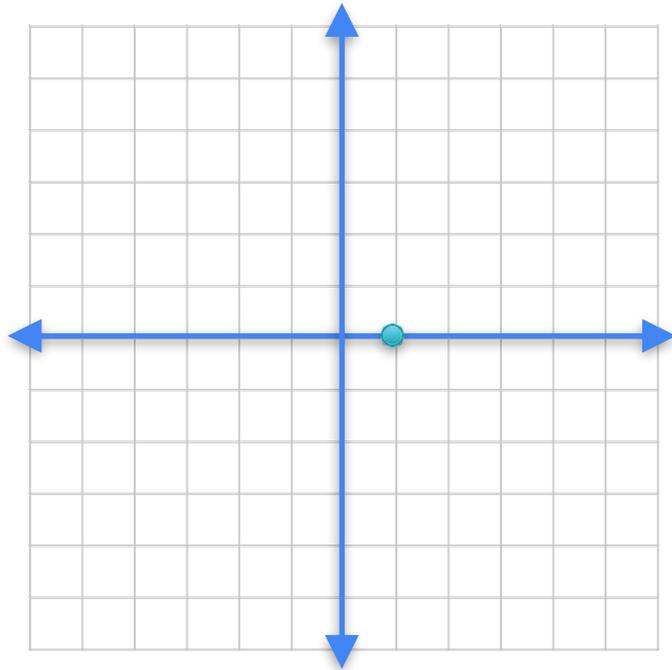
多次线性变换



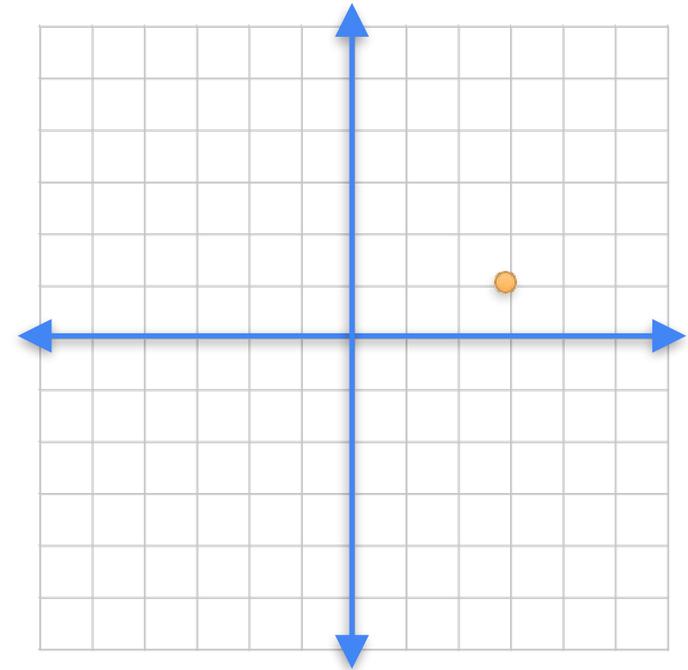
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



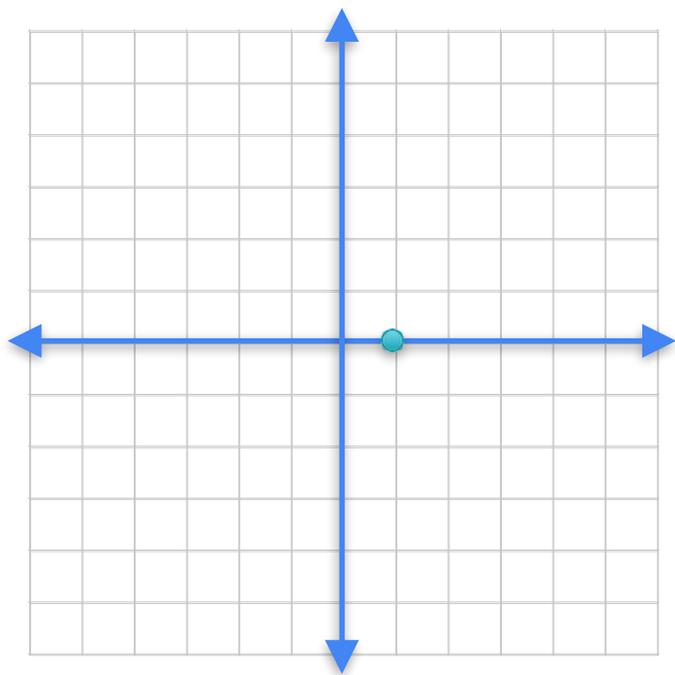
多次线性变换



$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

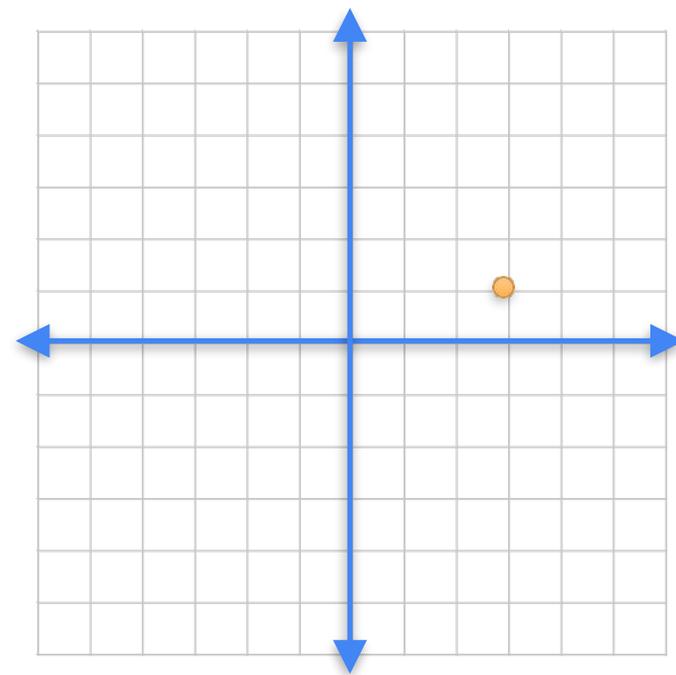


多次线性变换

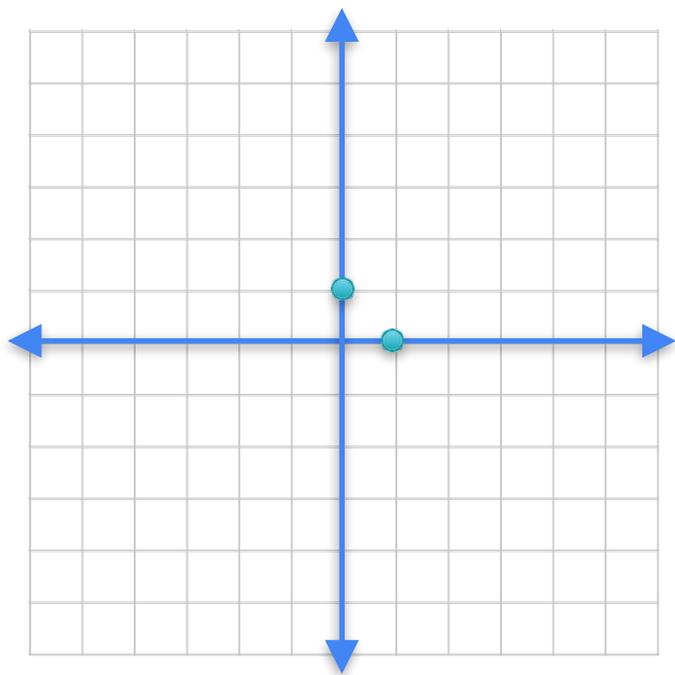


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

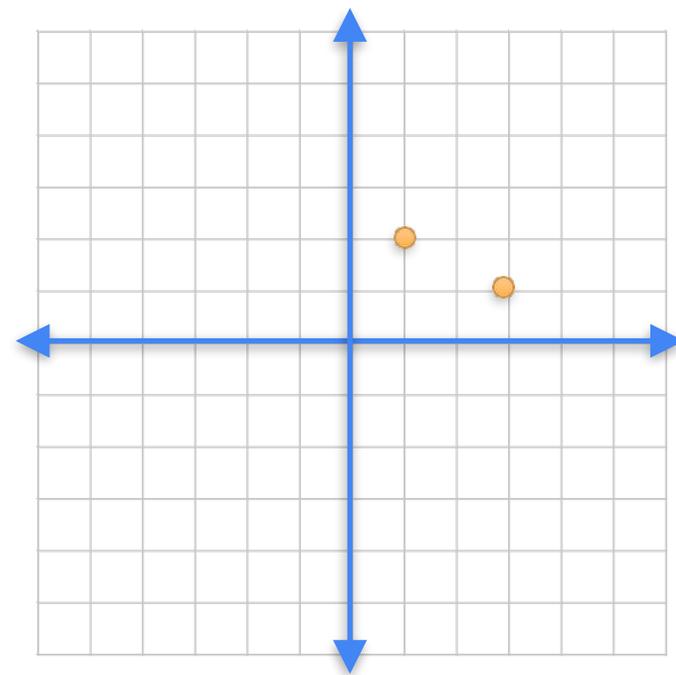


多次线性变换

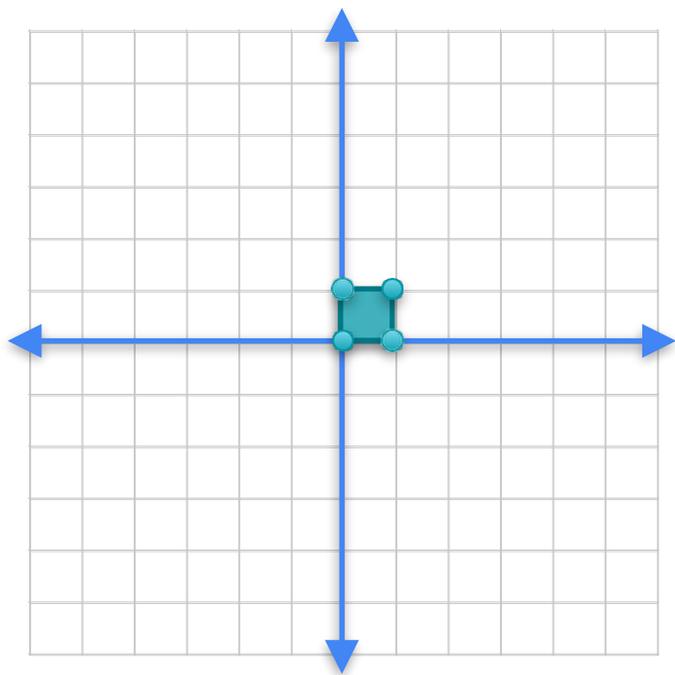


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

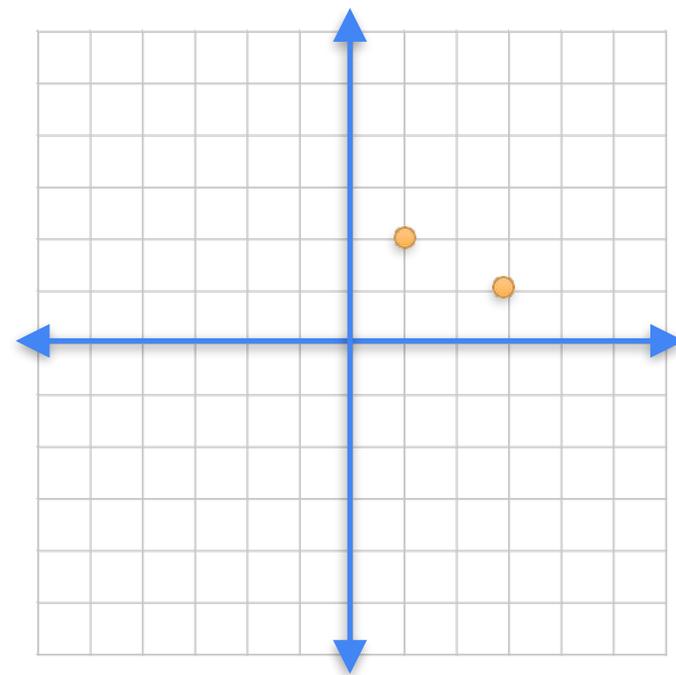


多次线性变换

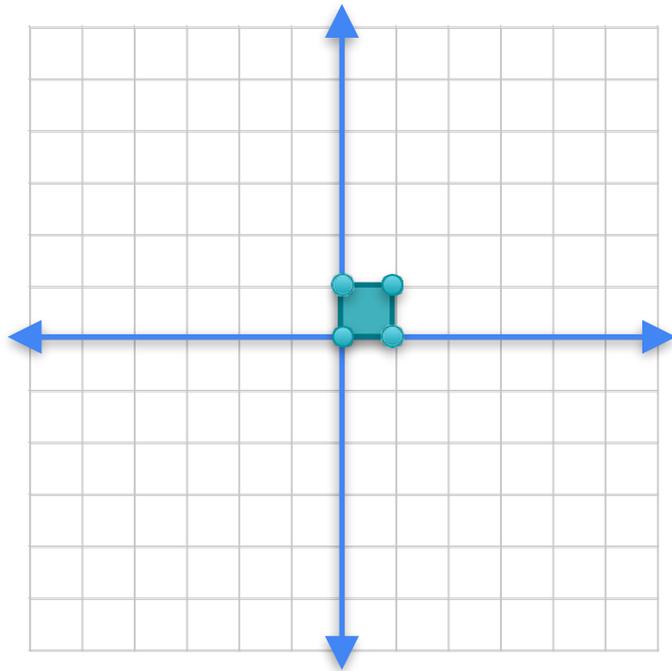


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

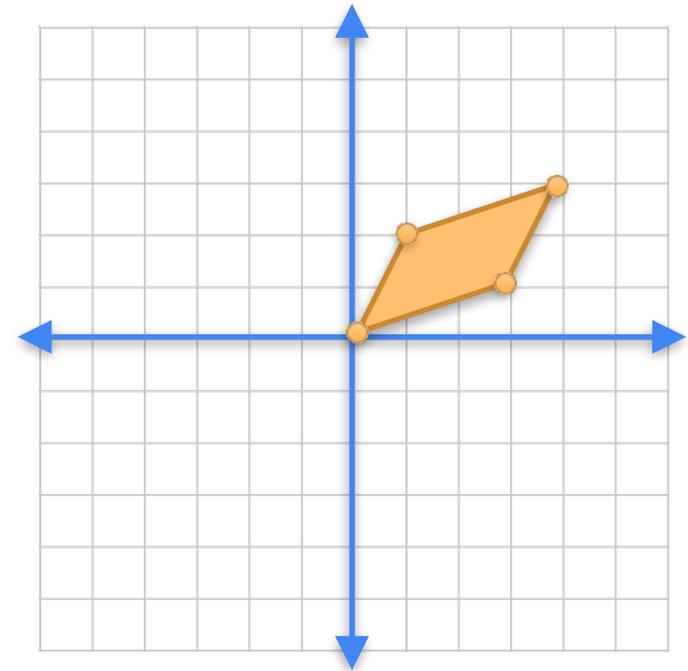


多次线性变换



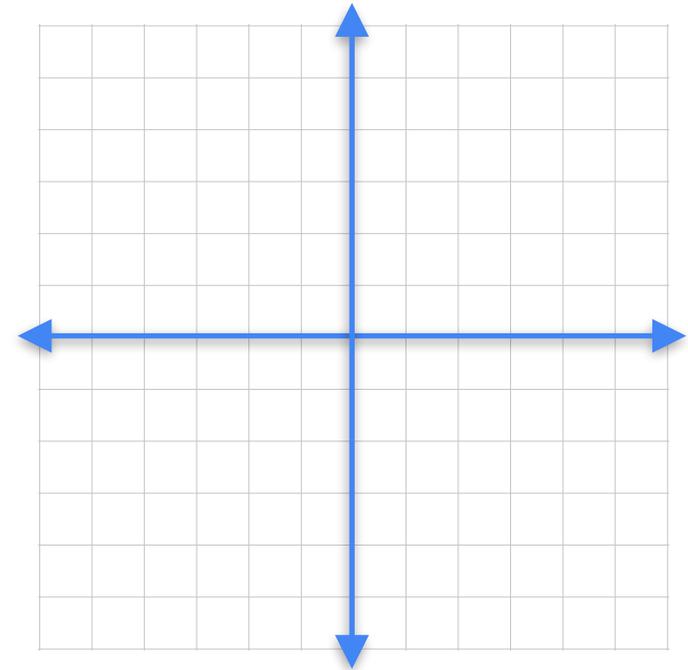
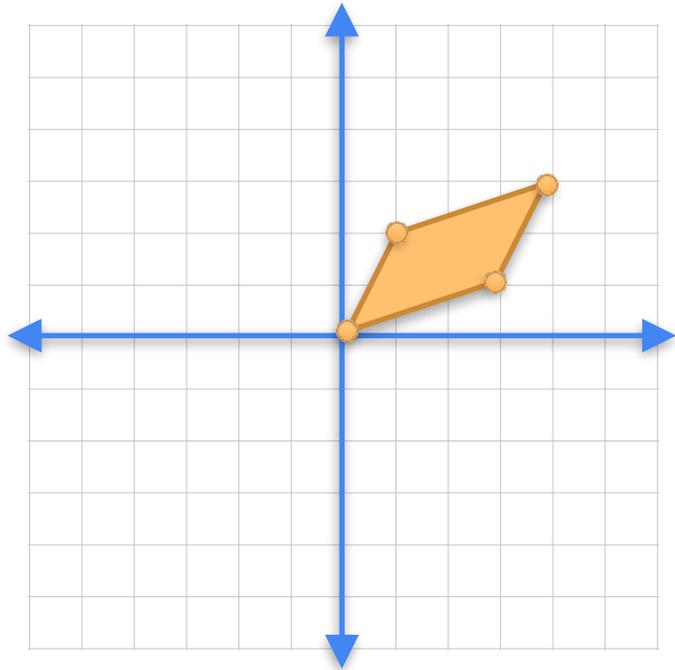
$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



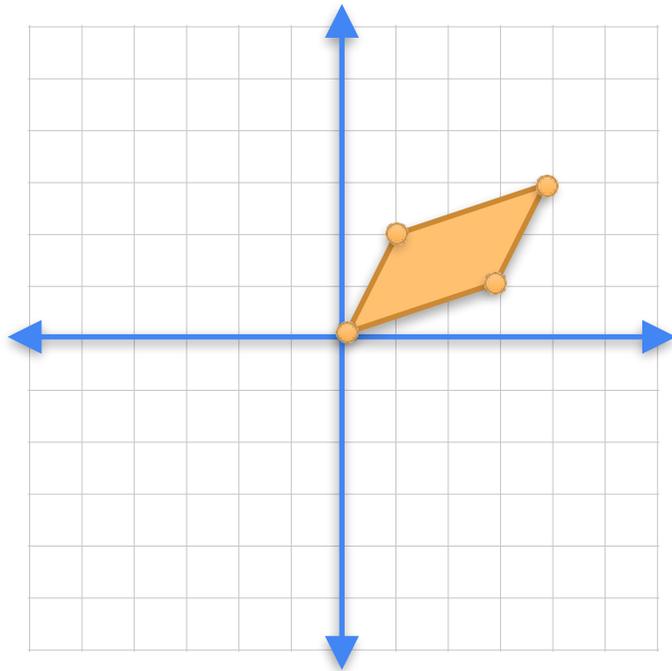
3. 线性变换和矩阵乘法

多次线性变换

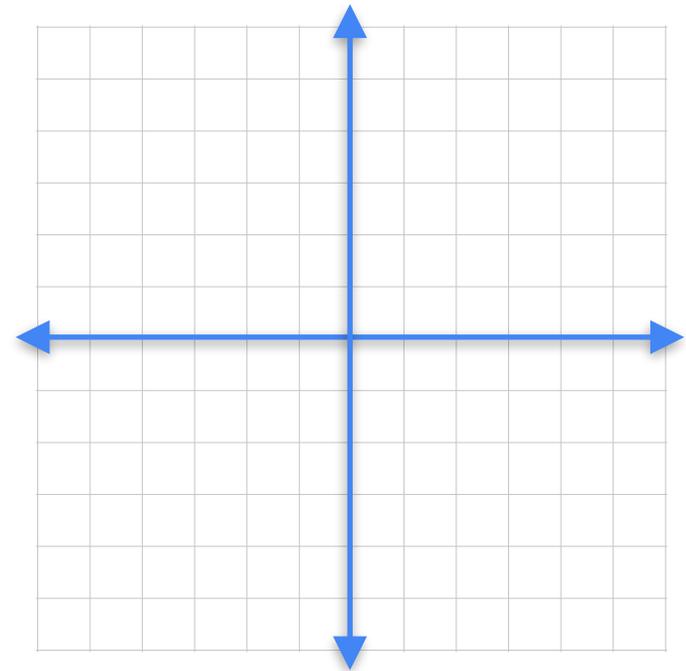


3. 线性变换和矩阵乘法

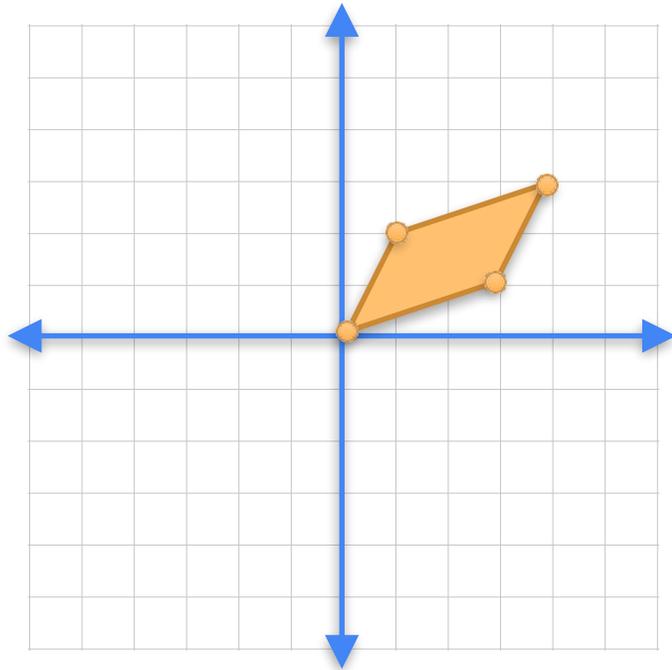
多次线性变换



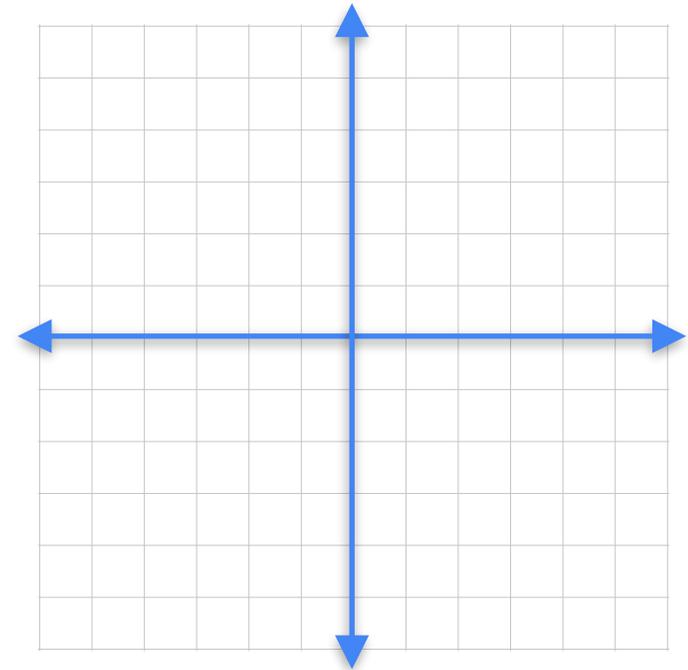
2	-1
0	2



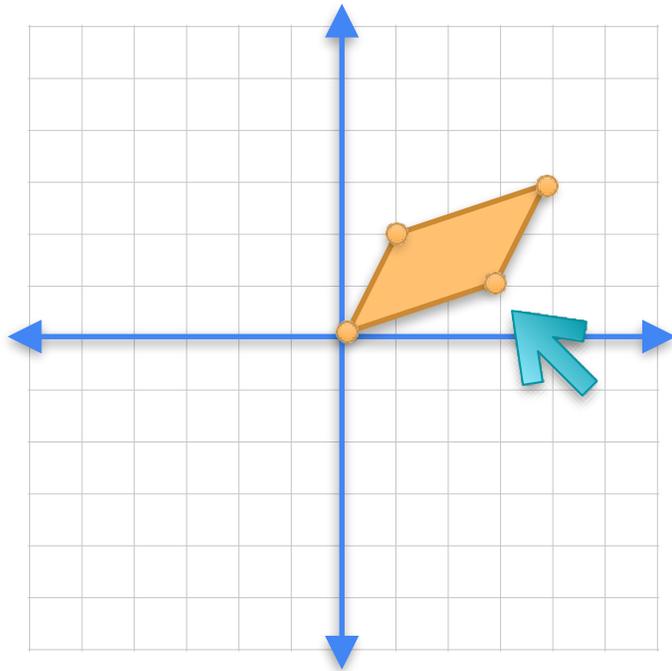
多次线性变换



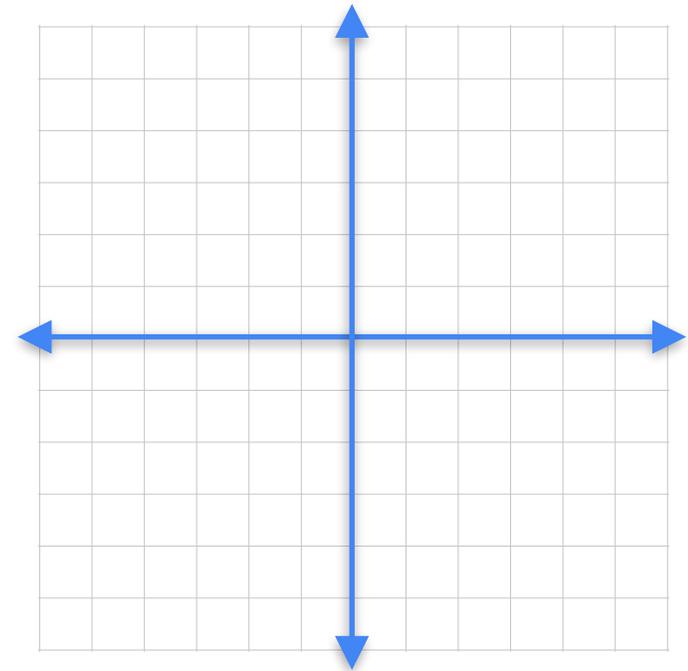
$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



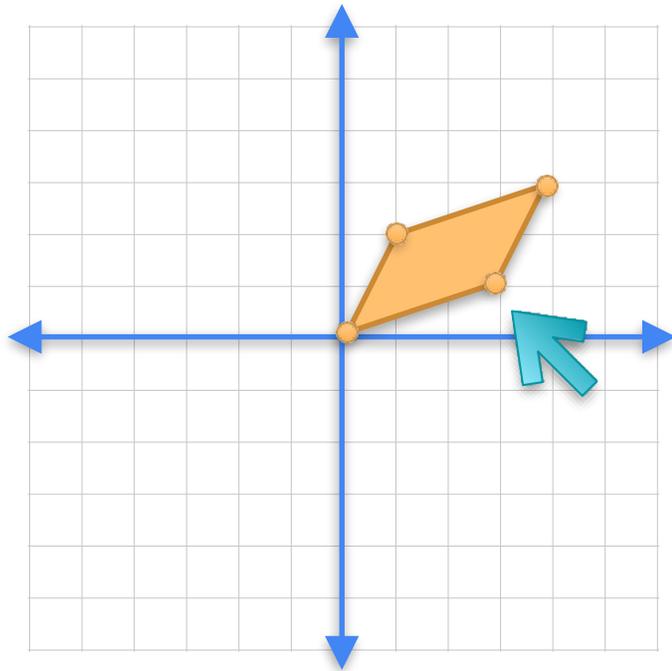
多次线性变换



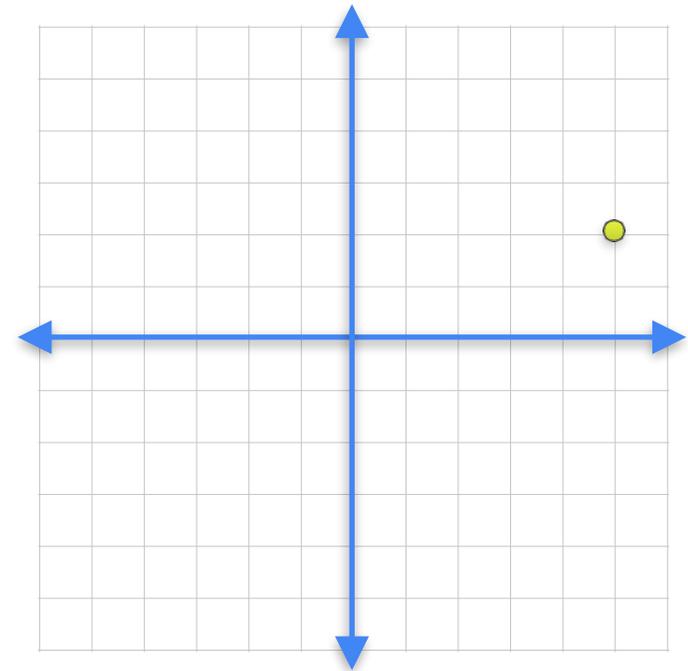
$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$



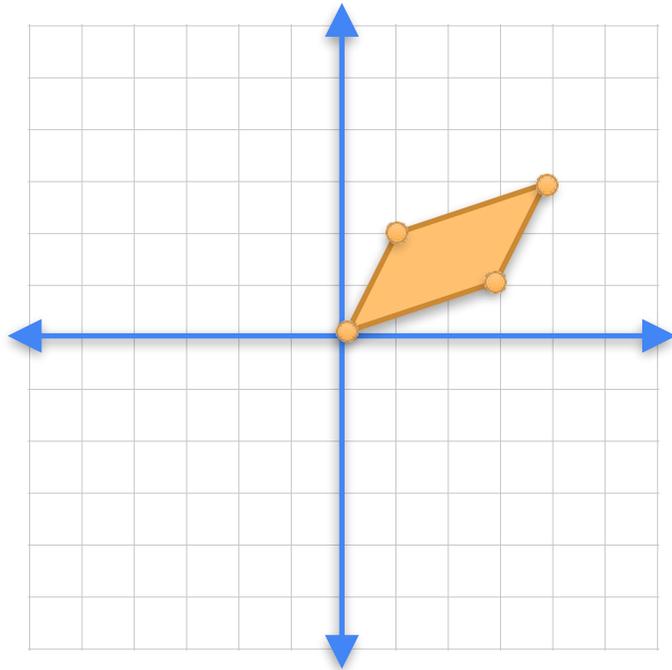
多次线性变换



$$\begin{bmatrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

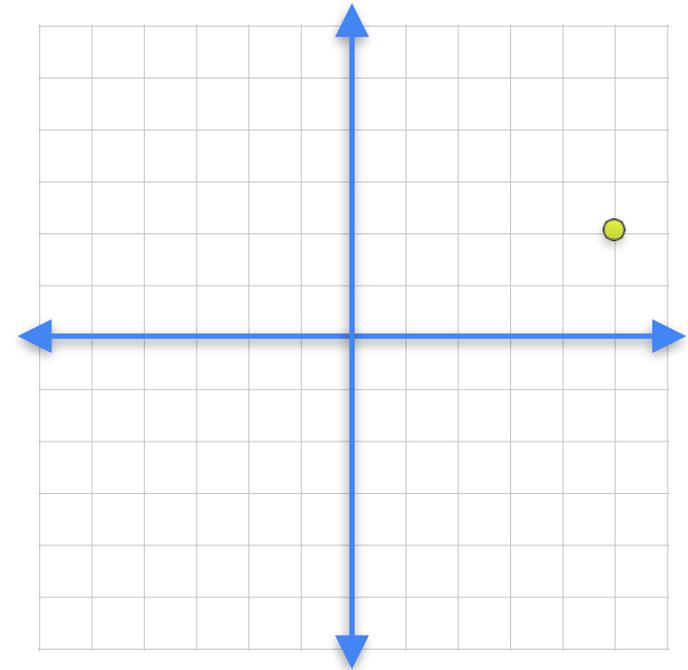


多次线性变换

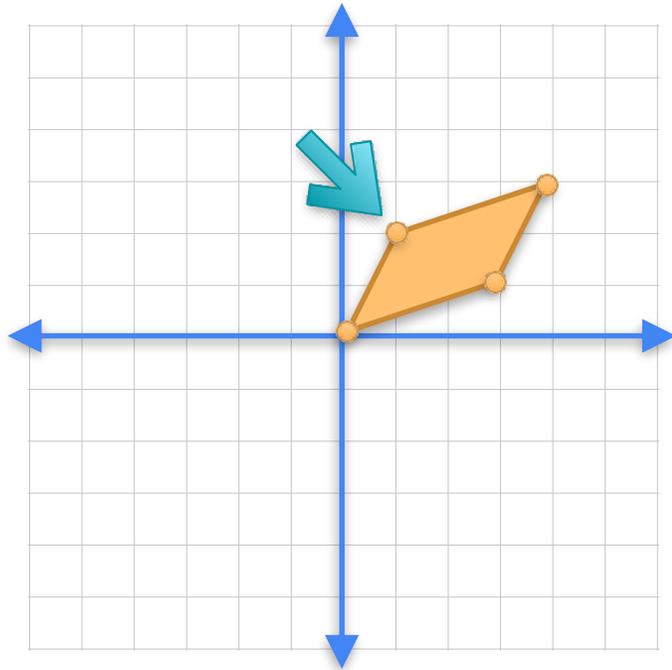


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

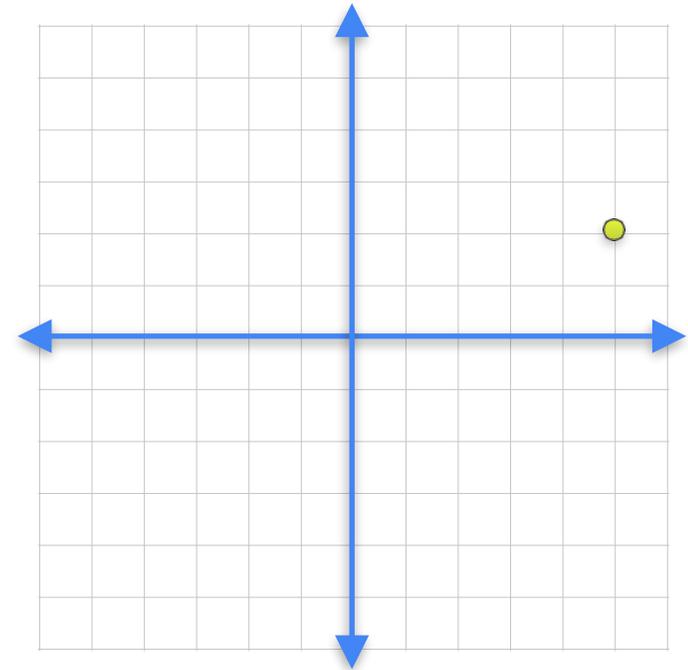


多次线性变换

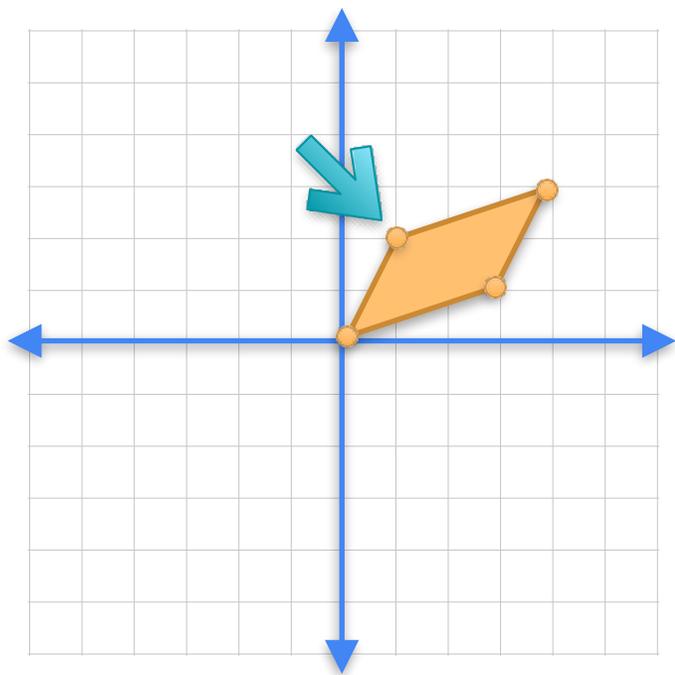


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

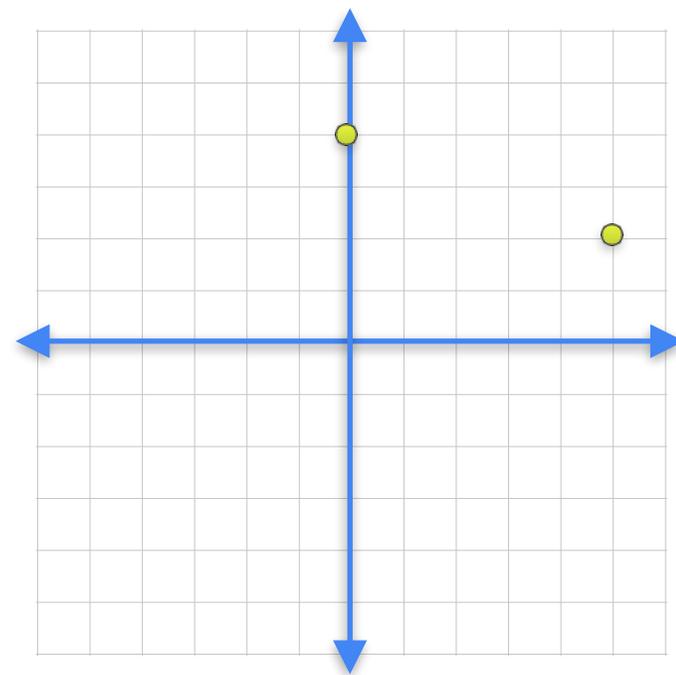


多次线性变换

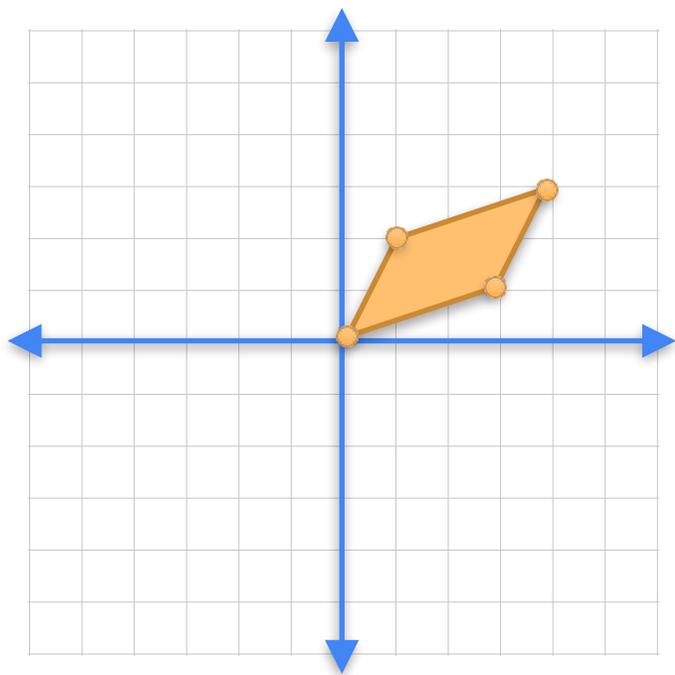


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

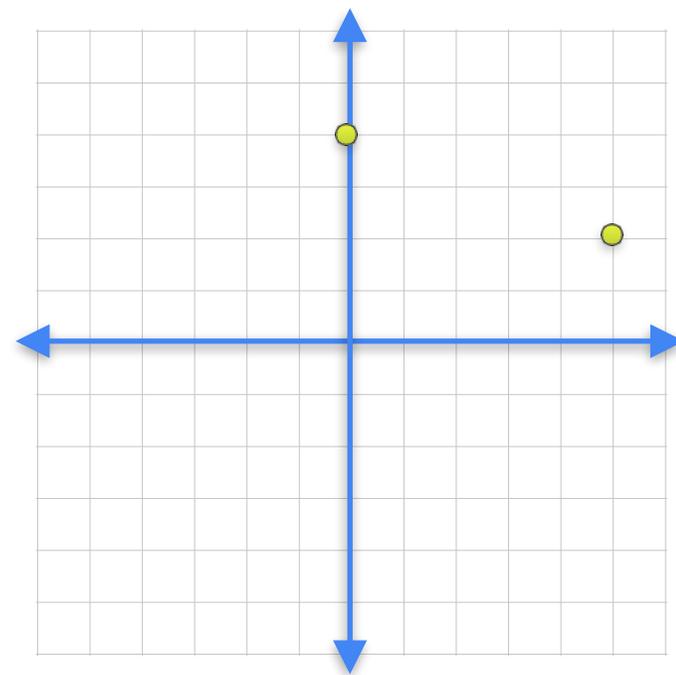


多次线性变换

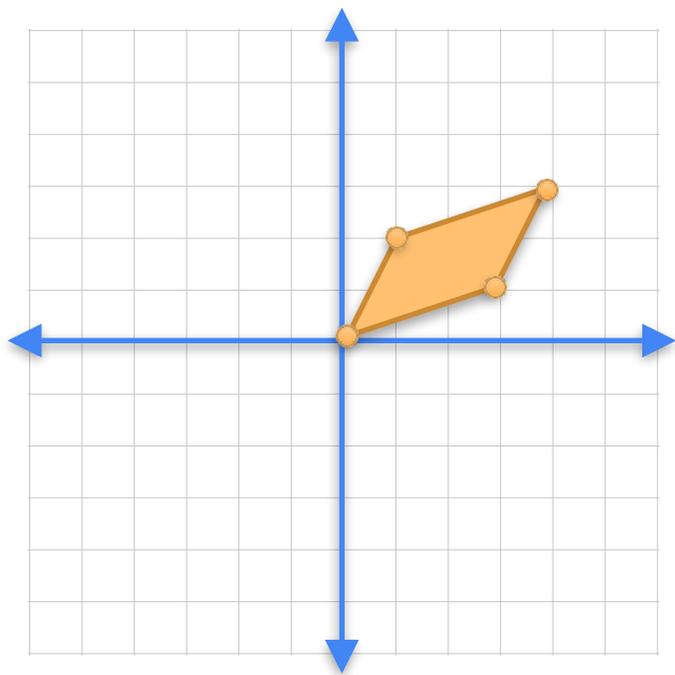


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

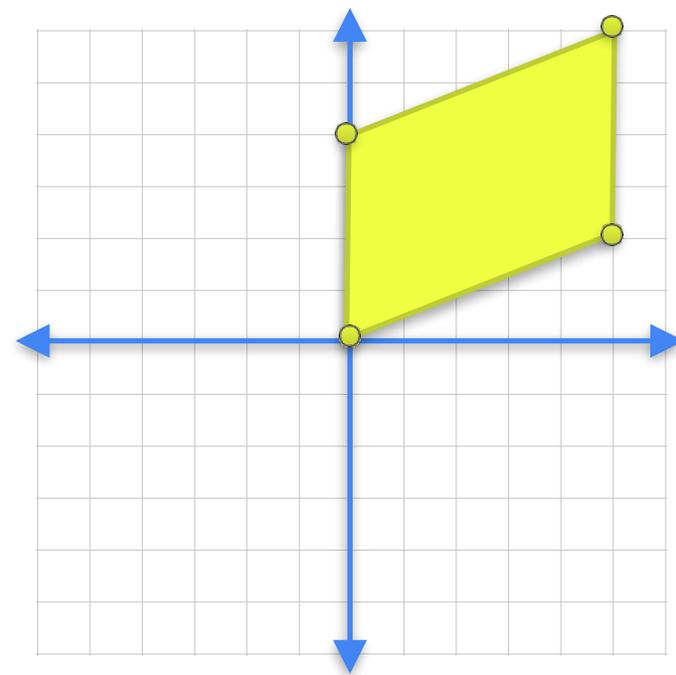


多次线性变换

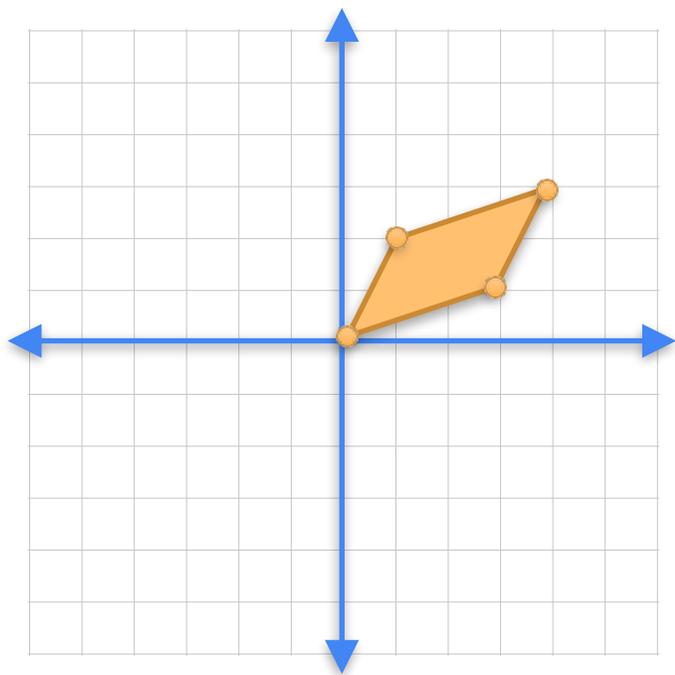


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

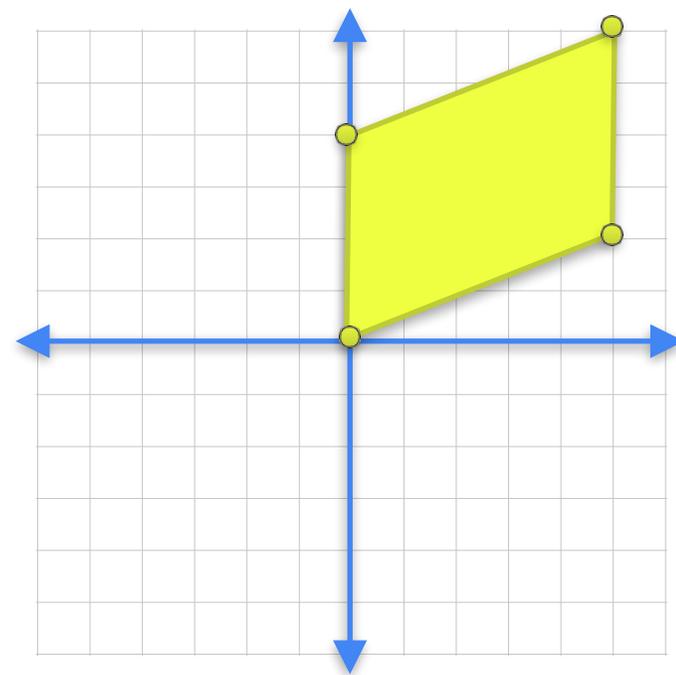


多次线性变换

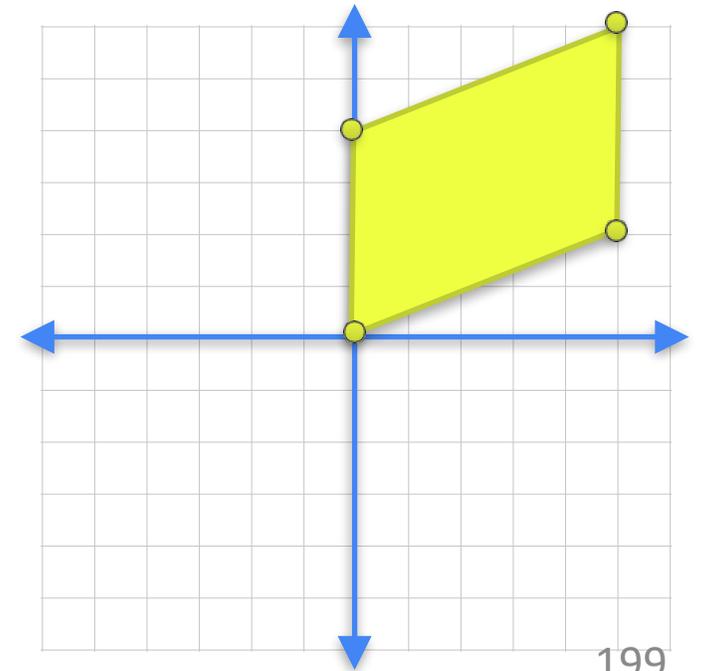
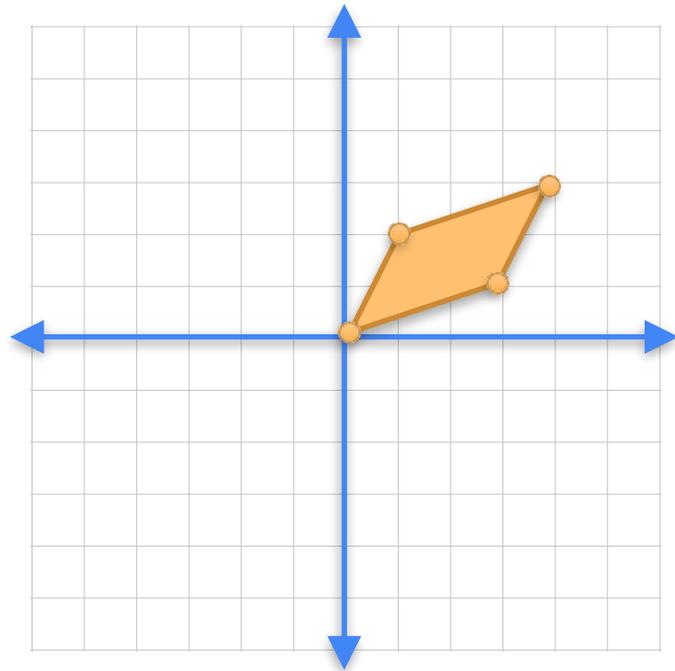
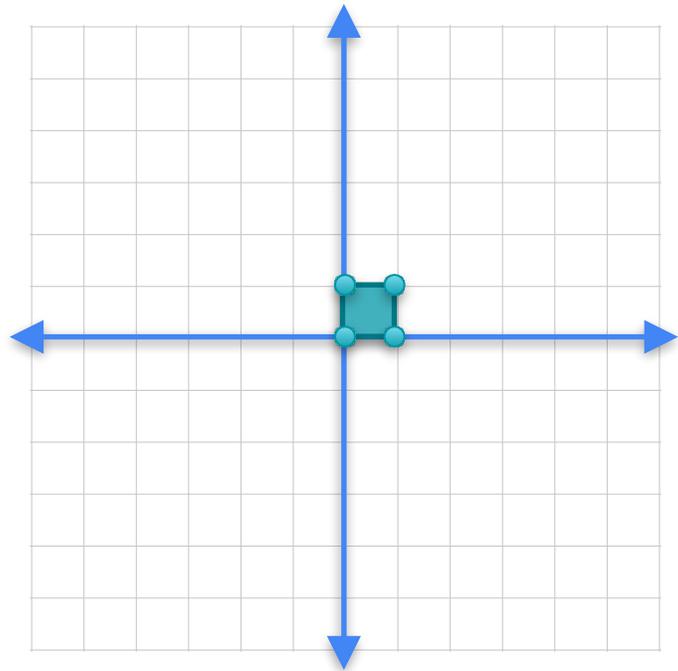


$$\begin{array}{|c|c|c|} \hline 2 & -1 & 3 \\ \hline 0 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|} \hline 5 \\ \hline 2 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 2 & -1 & 1 \\ \hline 0 & 2 & 2 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline 4 \\ \hline \end{array}$$

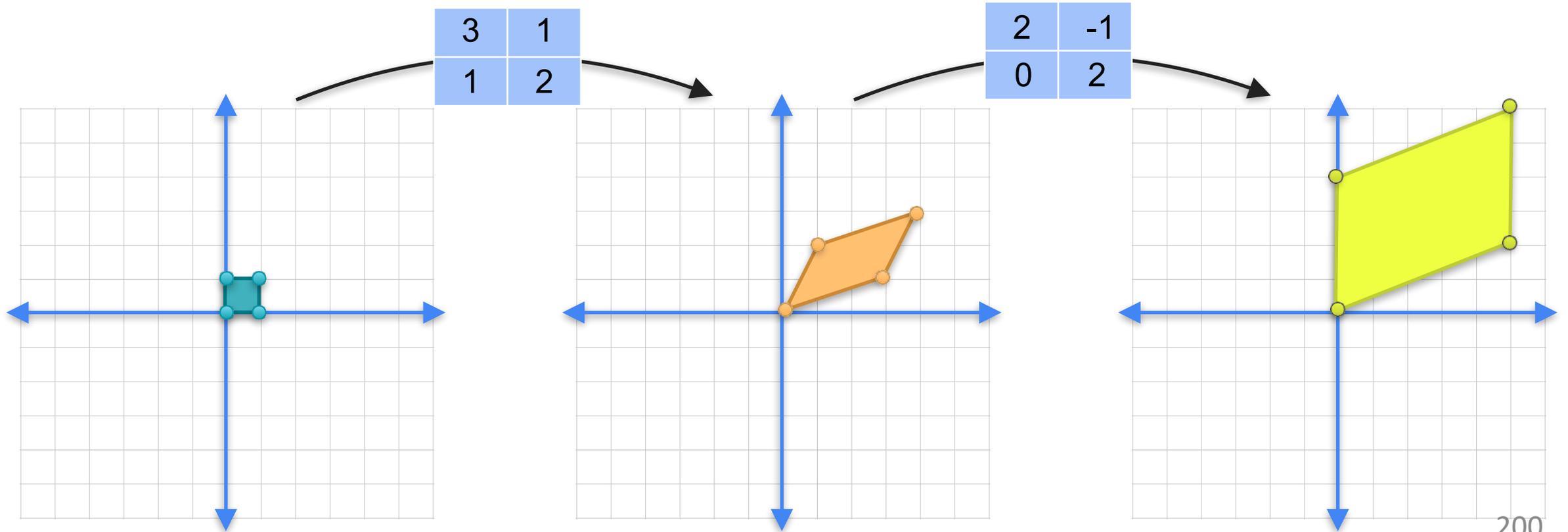


多次线性变换

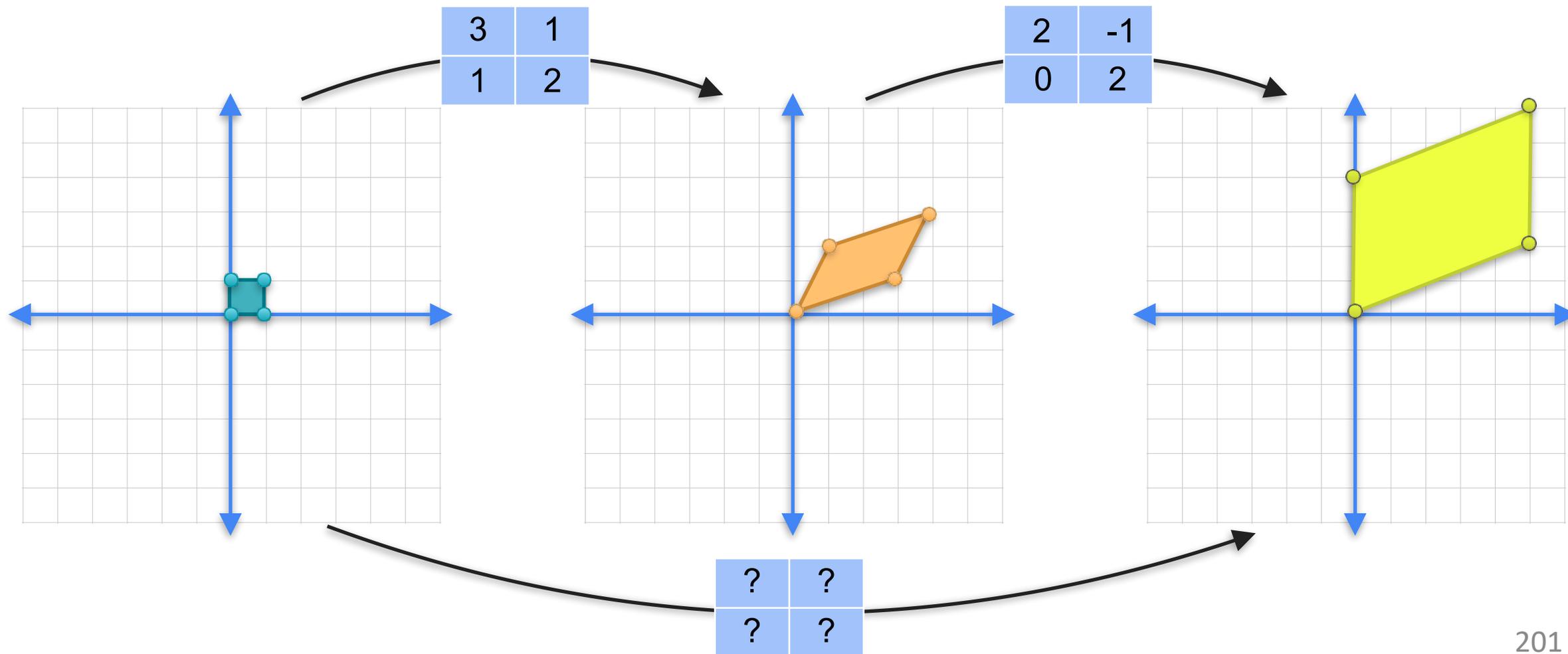


3. 线性变换和矩阵乘法

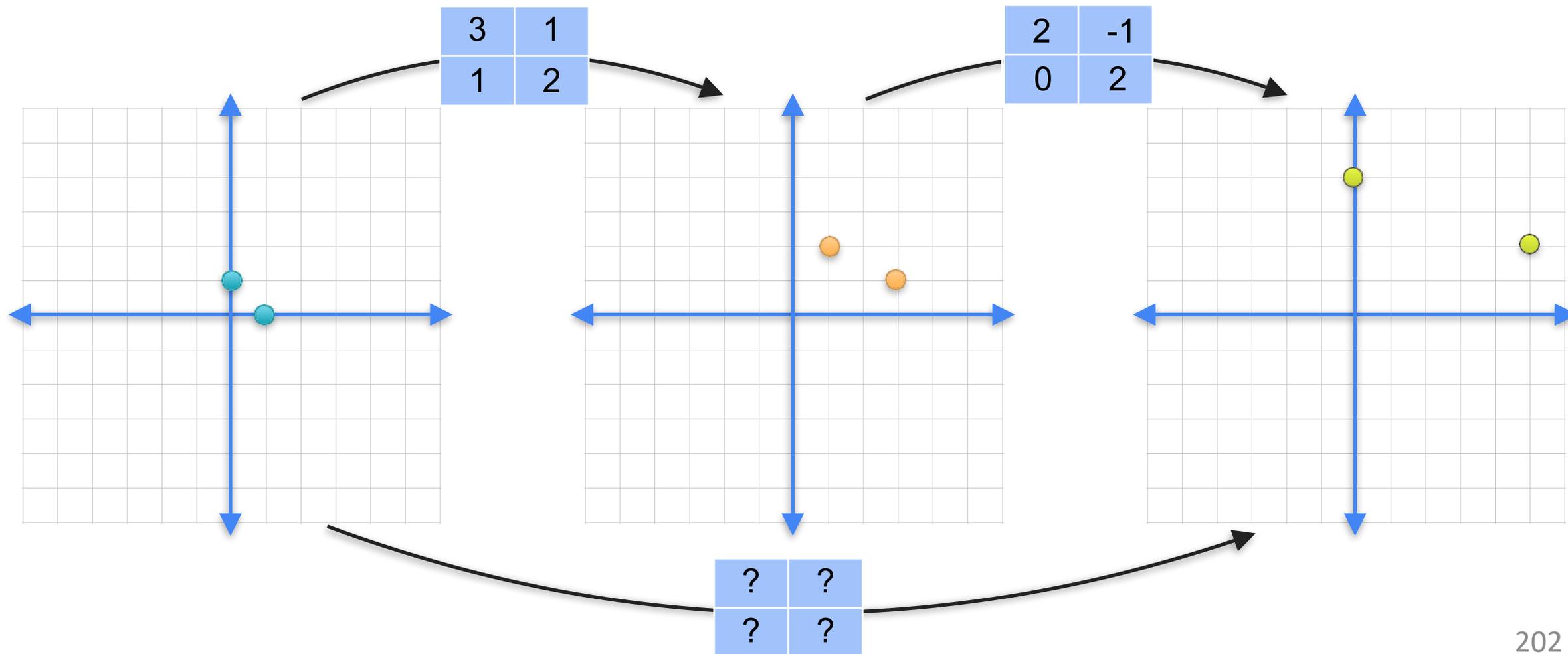
多次线性变换



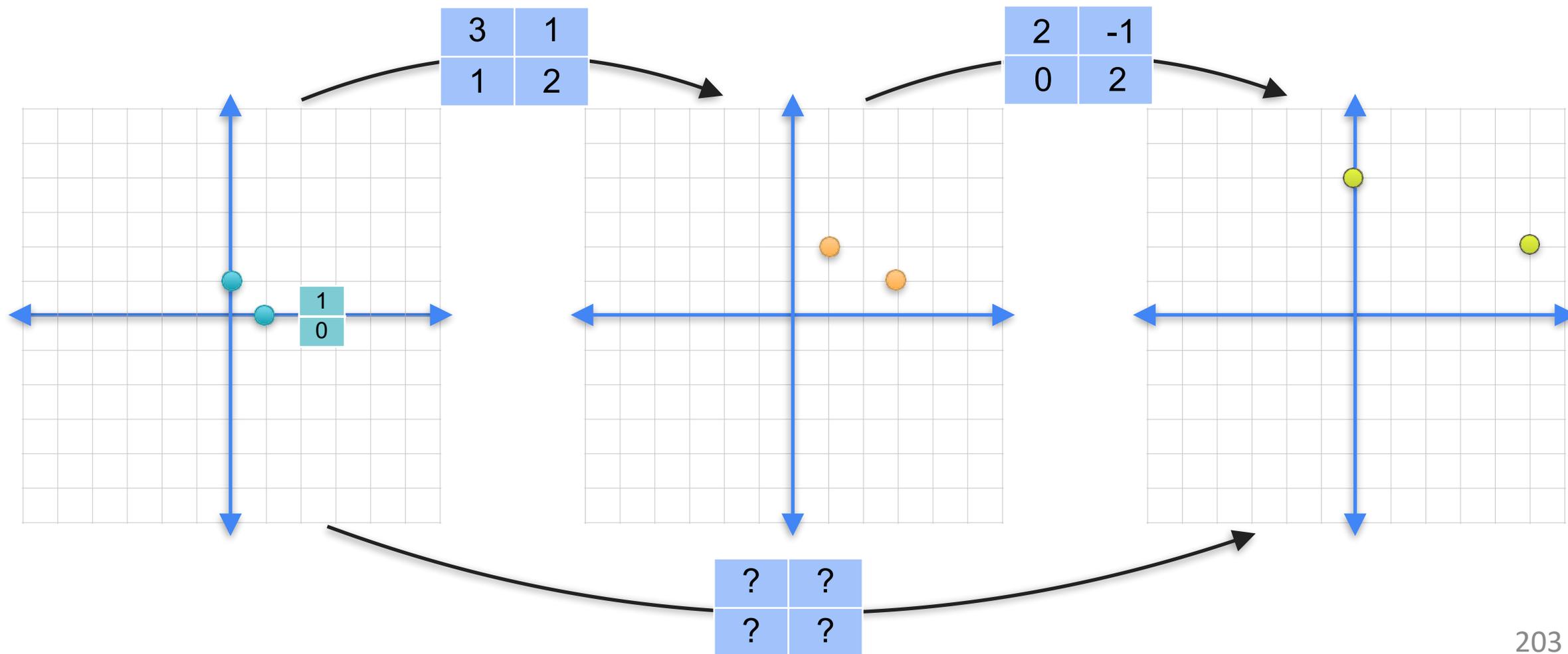
多次线性变换



多次线性变换

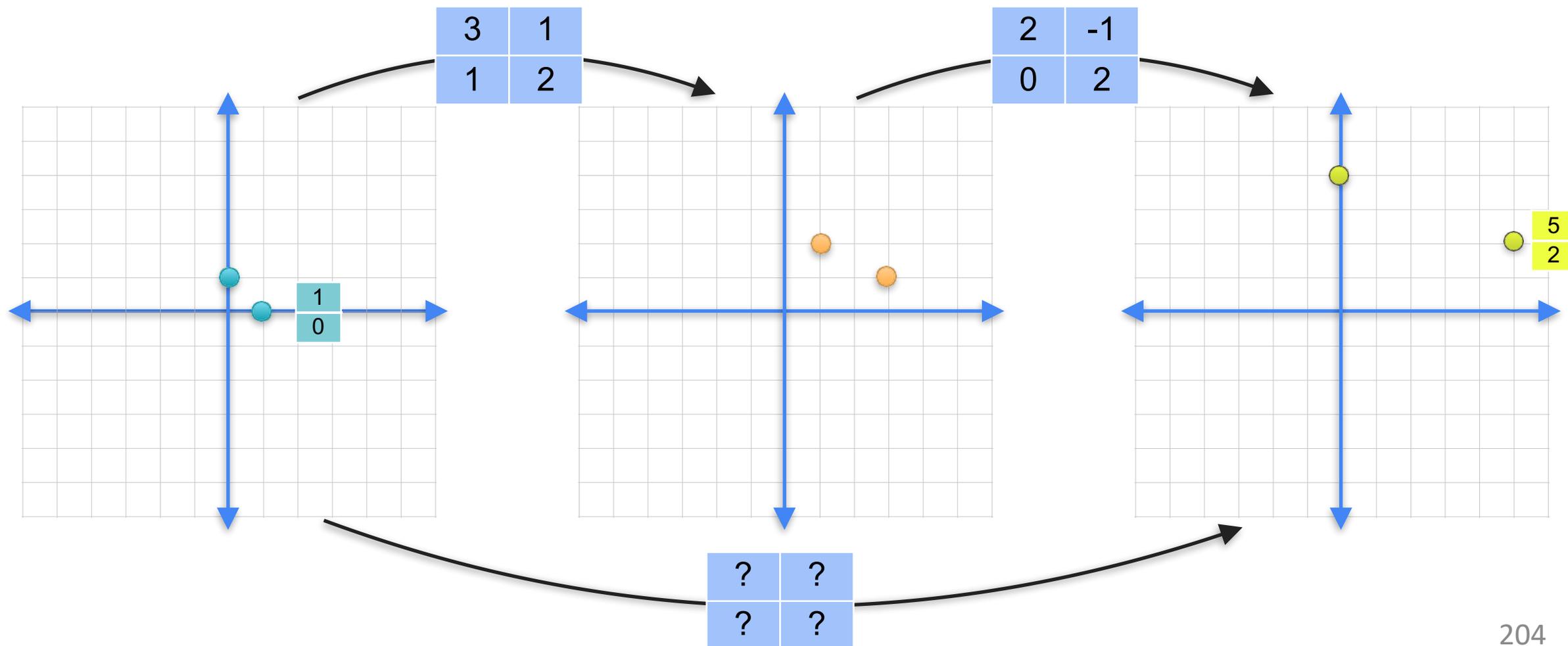


多次线性变换



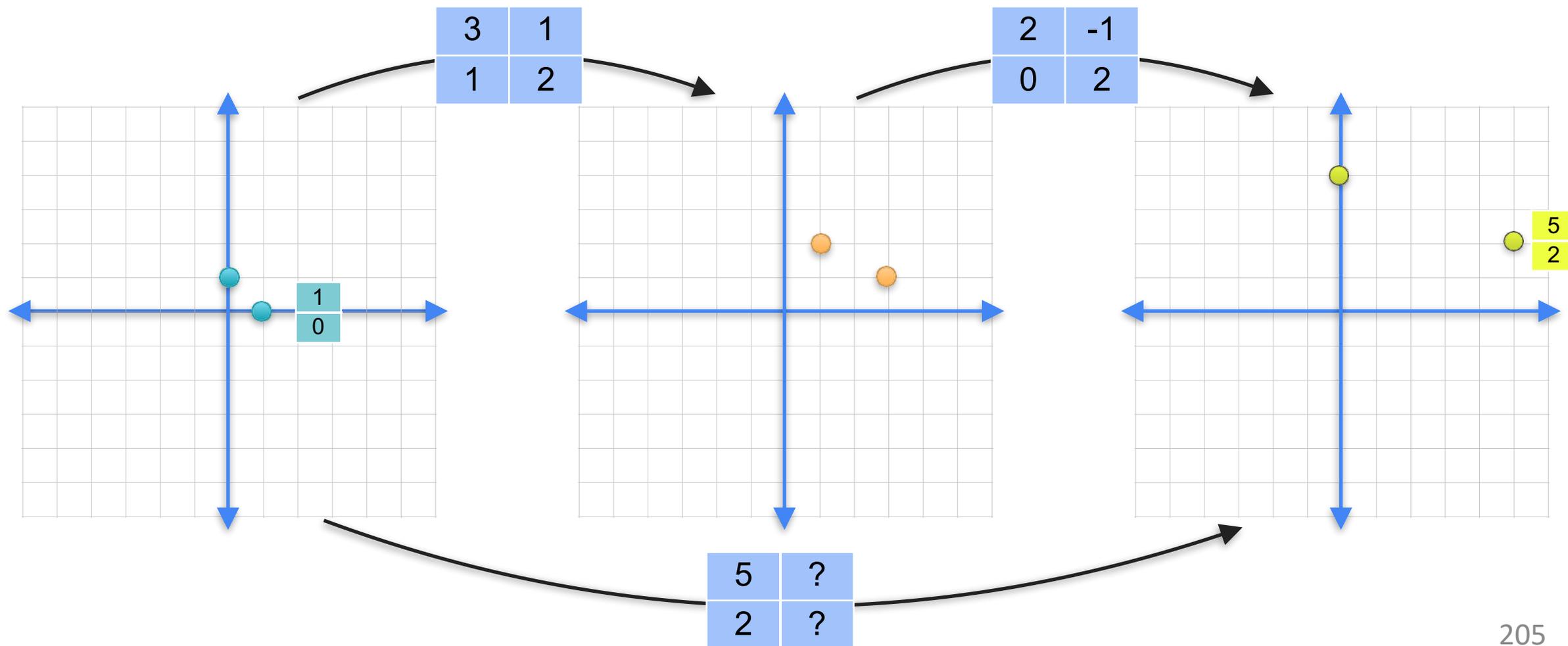
3. 线性变换和矩阵乘法

多次线性变换



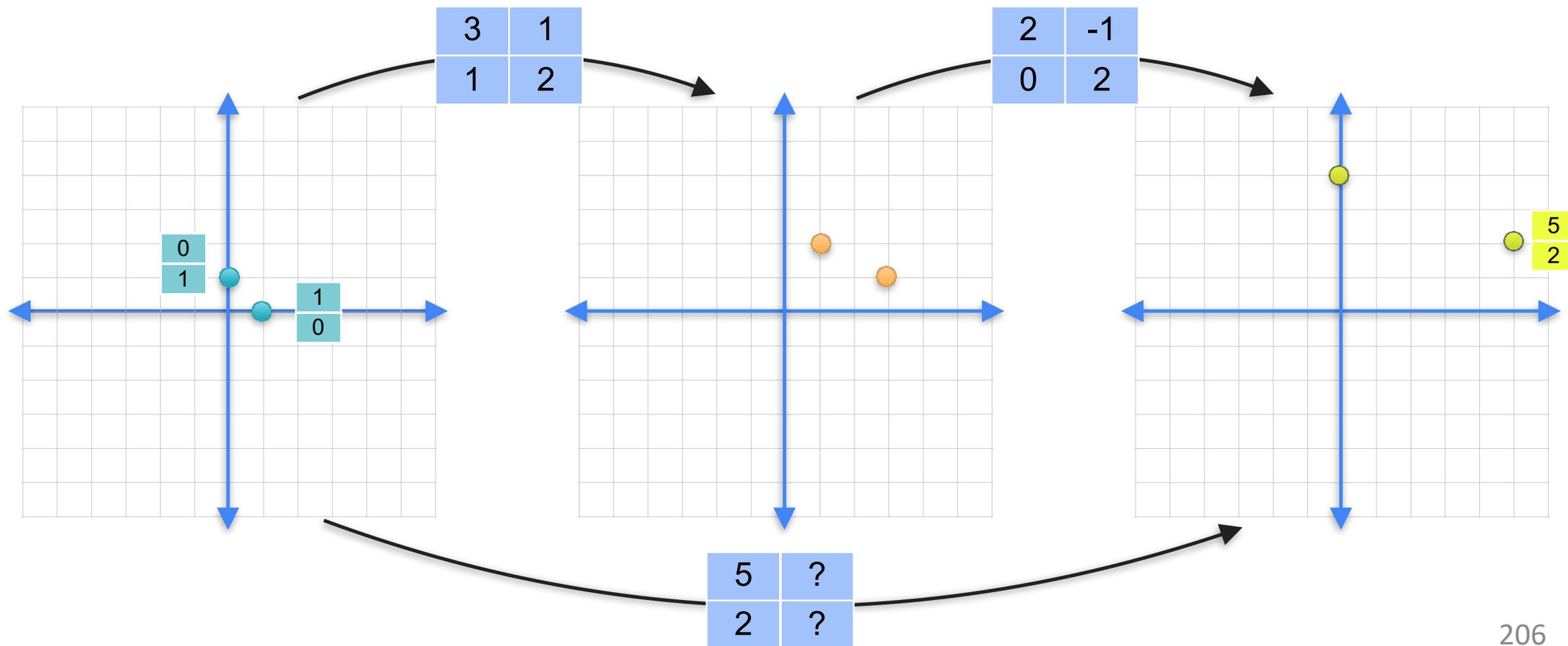
3. 线性变换和矩阵乘法

多次线性变换



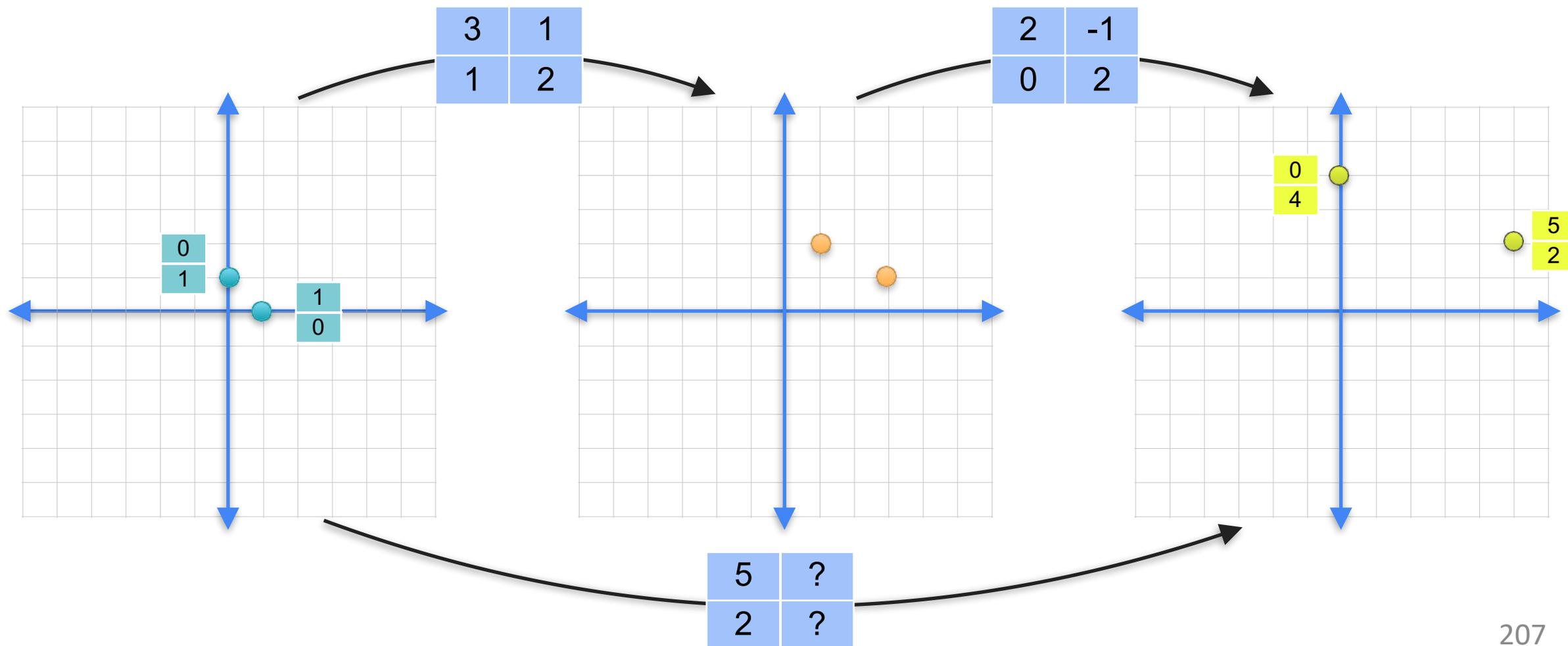
3. 线性变换和矩阵乘法

多次线性变换



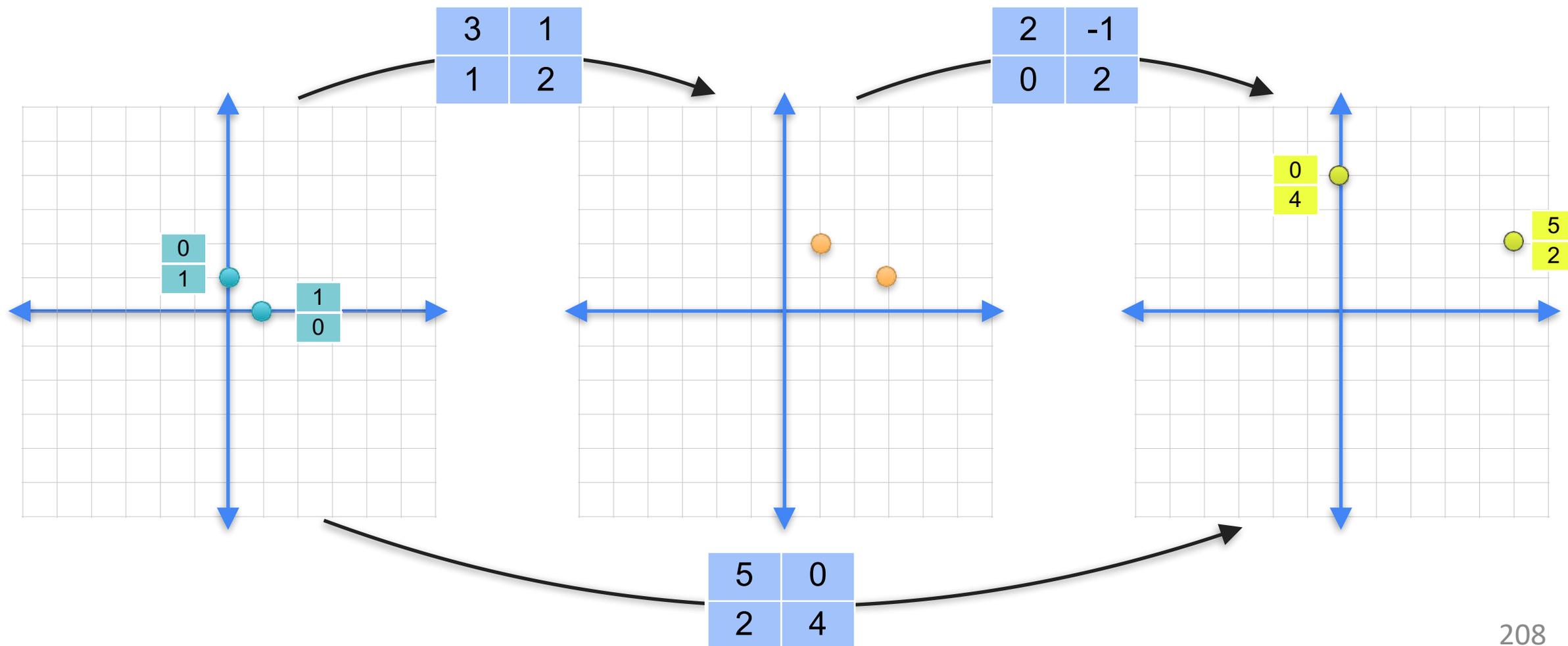
3. 线性变换和矩阵乘法

多次线性变换



3. 线性变换和矩阵乘法

多次线性变换



矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

矩阵乘法

First
↓

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

矩阵乘法

$$\begin{array}{cc} \text{Second} & \text{First} \\ \downarrow & \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

3. 线性变换和矩阵乘法

矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 & 1 \\ 0 & 2 & 3 & 1 \\ 2 & -1 & 1 & 2 \\ 0 & 2 & 1 & 2 \end{bmatrix}$$

3. 线性变换和矩阵乘法

矩阵乘法

The diagram illustrates the multiplication of two 2x2 matrices. The first matrix is represented by two teal-colored row vectors: $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$. The second matrix is represented by two orange-colored column vectors: $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$. An equals sign follows, leading to a 2x2 grid of four blue boxes. The top-left box contains the scalar result 5. The top-right box shows the first teal row vector multiplied by the first orange column vector, resulting in the scalar 5. The bottom-left box shows the second teal row vector multiplied by the first orange column vector, resulting in the scalar 2. The bottom-right box shows the second teal row vector multiplied by the second orange column vector, resulting in the scalar 2.

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \\ \begin{bmatrix} 0 & 2 \end{bmatrix} & \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \end{bmatrix}$$

矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

The diagram illustrates the calculation of the product of two matrices. The first matrix (teal) has elements 2, -1, 0, 2. The second matrix (orange) has elements 3, 1, 1, 2. The result is a blue 2x2 matrix with elements 5, 0, 0, 2. The bottom row of the result is shown as a teal row [0, 2] multiplied by an orange column [3, 1] to get 0, and a teal row [0, 2] multiplied by an orange column [1, 2] to get 2.

3. 线性变换和矩阵乘法

矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix} \end{bmatrix}$$

矩阵乘法

$$\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 2 & 4 \end{bmatrix}$$

3.线性变换和矩阵乘法

单位矩阵乘法

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

3. 线性变换和矩阵乘法

单位矩阵乘法

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

3. 线性变换和矩阵乘法

单位矩阵乘法

$$\begin{array}{|c|c|c|c|c|c|} \hline 1 & 0 & 0 & 0 & 0 & a \\ \hline 0 & 1 & 0 & 0 & 0 & b \\ \hline 0 & 0 & 1 & 0 & 0 & c \\ \hline 0 & 0 & 0 & 1 & 0 & d \\ \hline 0 & 0 & 0 & 0 & 1 & e \\ \hline \end{array} = \begin{array}{|c|} \hline a \\ \hline b \\ \hline c \\ \hline d \\ \hline e \\ \hline \end{array}$$

3. 线性变换和矩阵乘法

单位矩阵乘法

The diagram illustrates the multiplication of a 5x5 identity matrix by a column vector. The identity matrix is shown as a 5x5 grid of numbers, with the diagonal elements being 1 and the off-diagonal elements being 0. The column vector consists of five elements: a, b, c, d, and e. The result of the multiplication is a column vector with the same five elements: a, b, c, d, and e. A callout box shows the first row of the identity matrix, [1, 0, 0, 0, 0], multiplied by the column vector, resulting in the value 'a'.

1	0	0	0	0	a
0	1	0	0	0	b
0	0	1	0	0	c
0	0	0	1	0	d
0	0	0	0	1	e

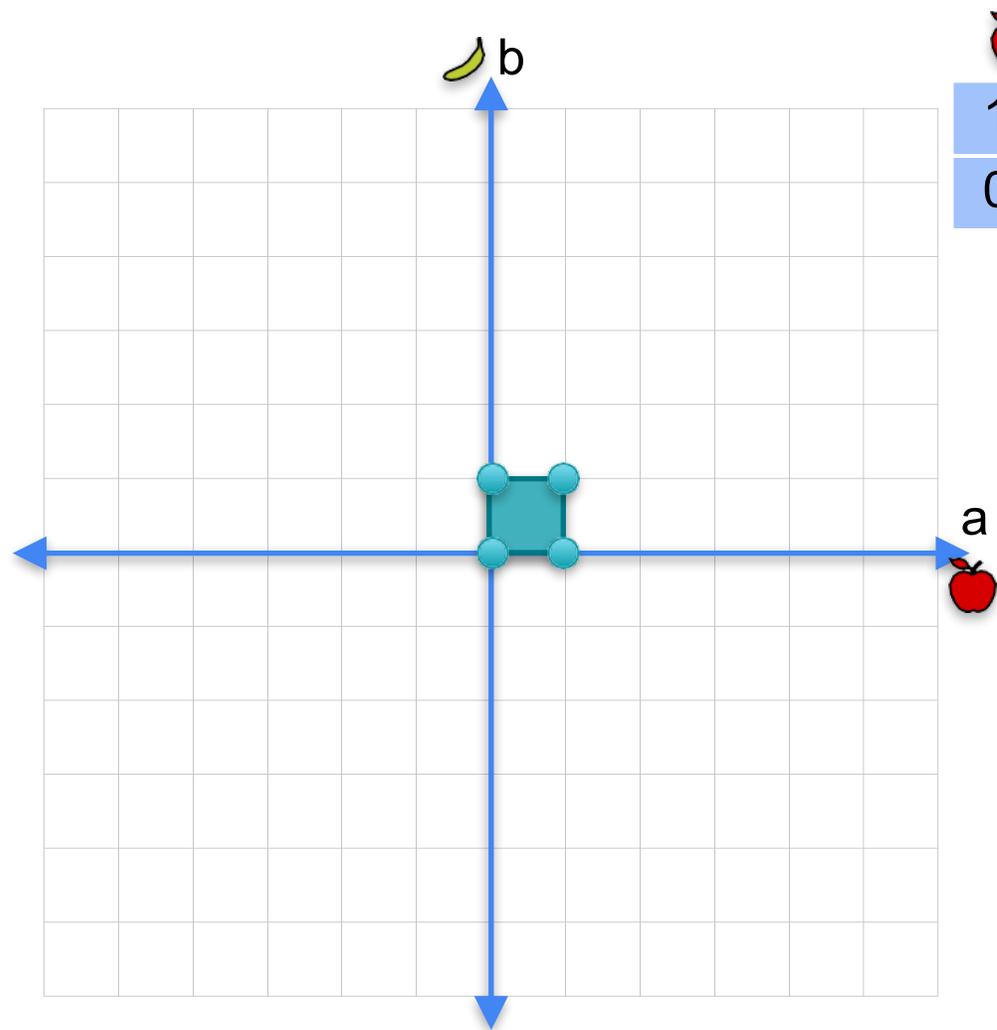
=

a
b
c
d
e

Callout: $[1 \ 0 \ 0 \ 0 \ 0] \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = a$

3. 线性变换和矩阵乘法

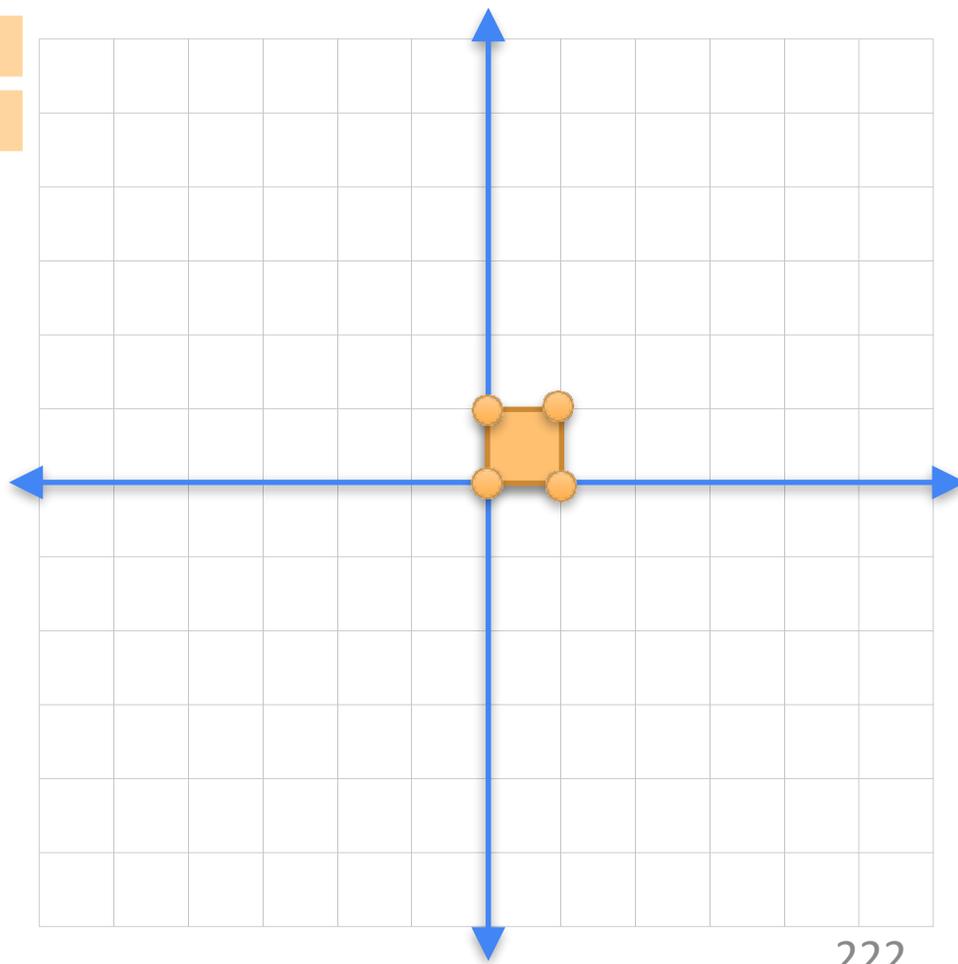
单位矩阵乘法



			
1	0	x	x
0	1	y	y

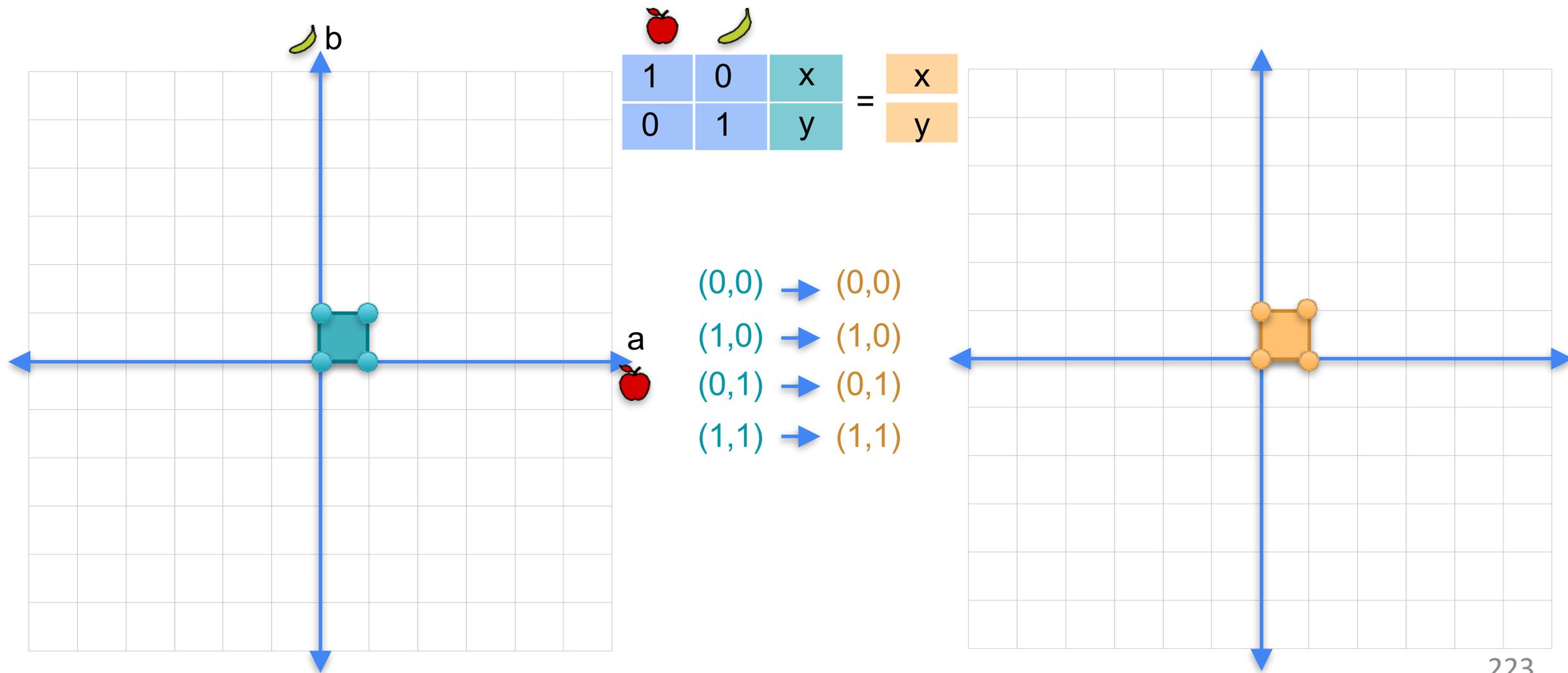
=

x
y



3. 线性变换和矩阵乘法

单位矩阵乘法



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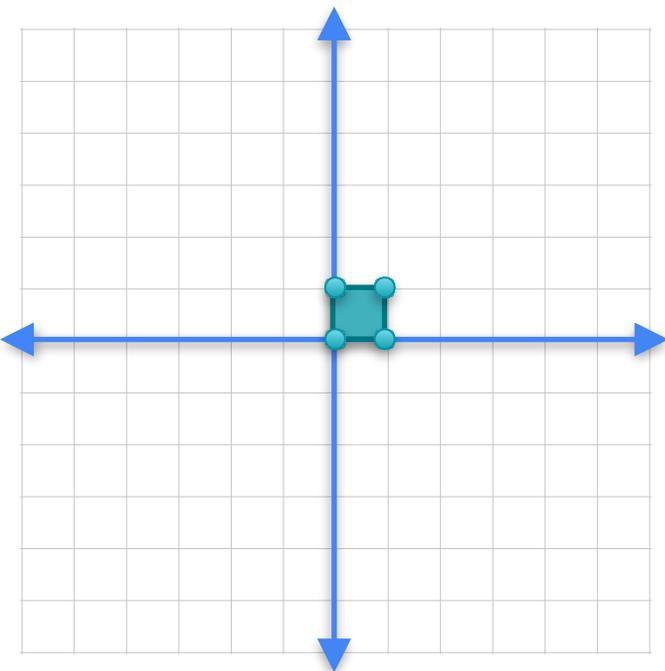
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4.逆矩阵

逆矩阵

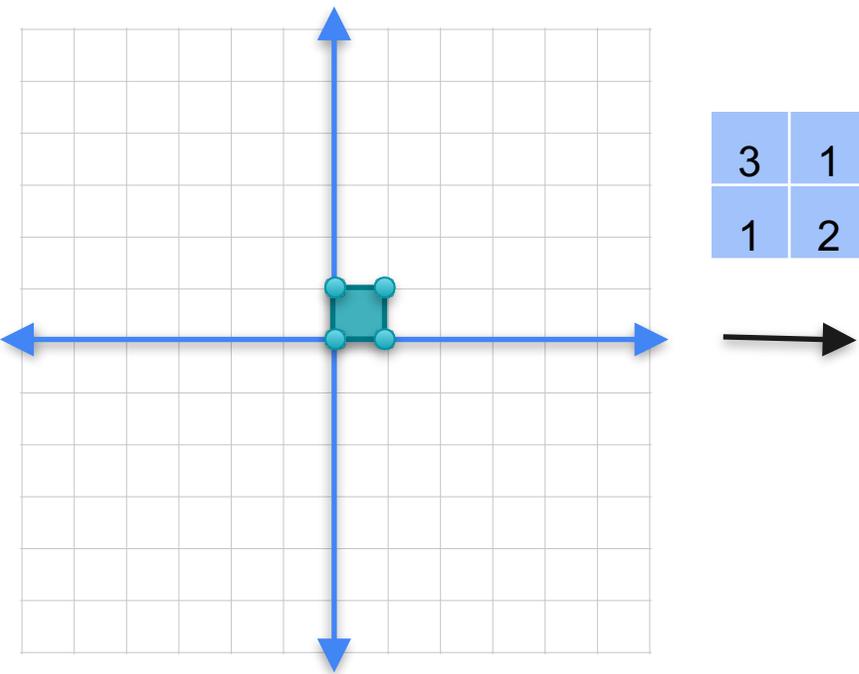
4. 逆矩阵

逆矩阵



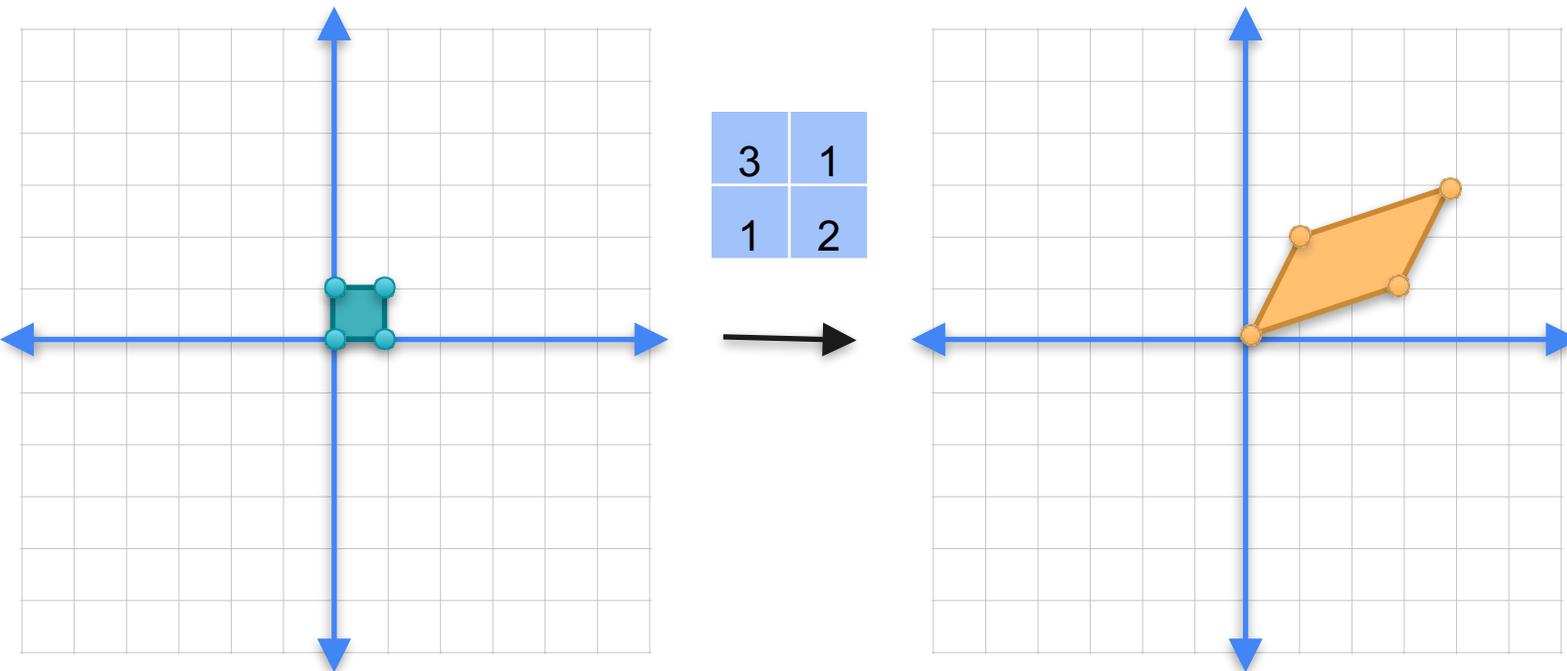
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逆矩阵



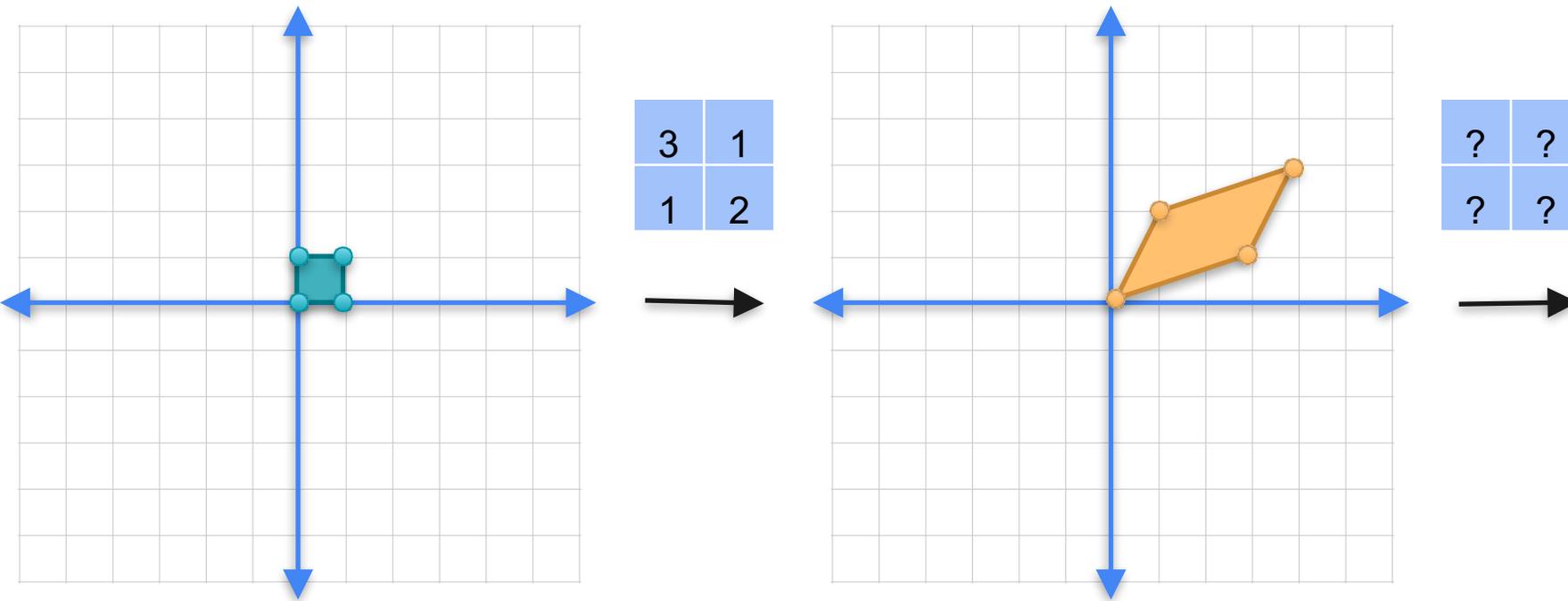
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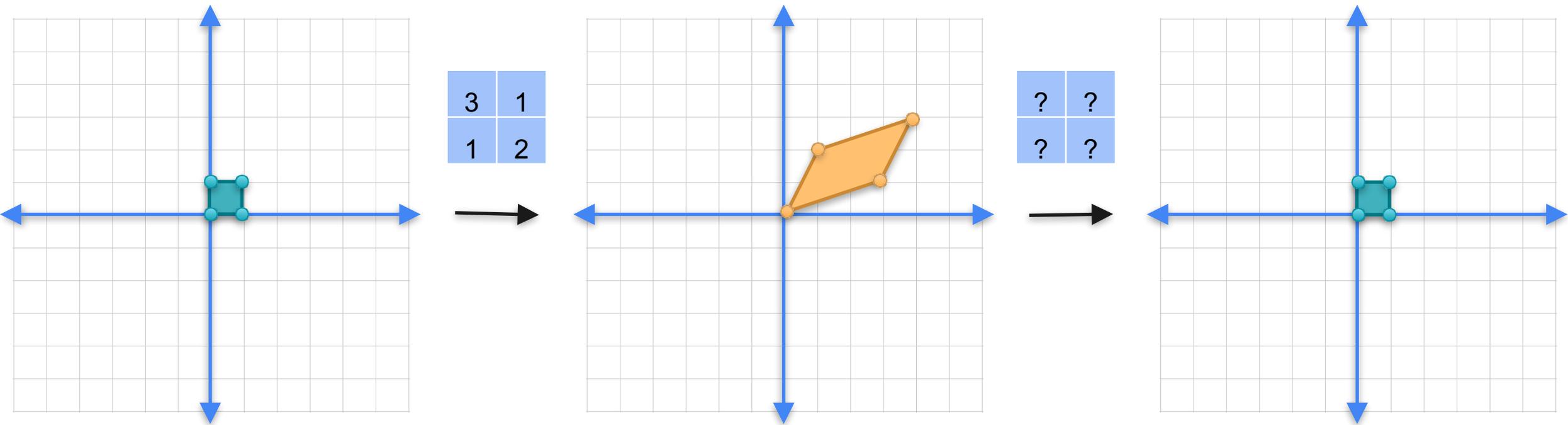
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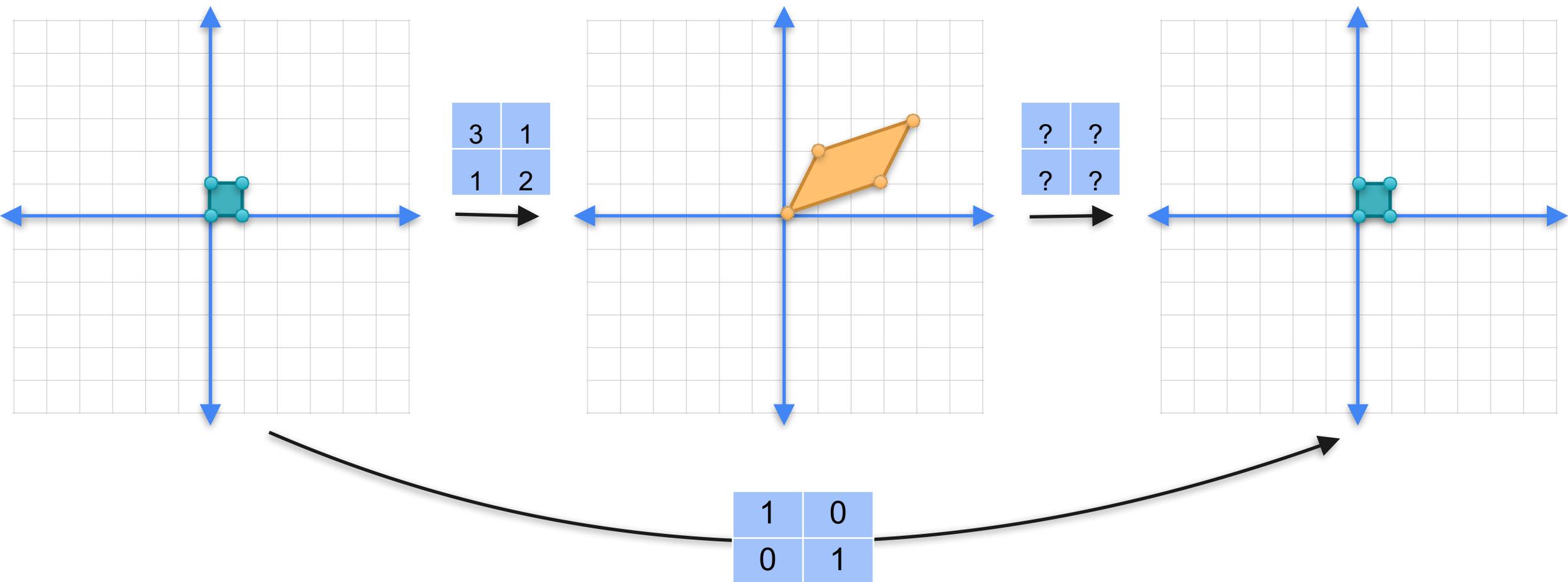
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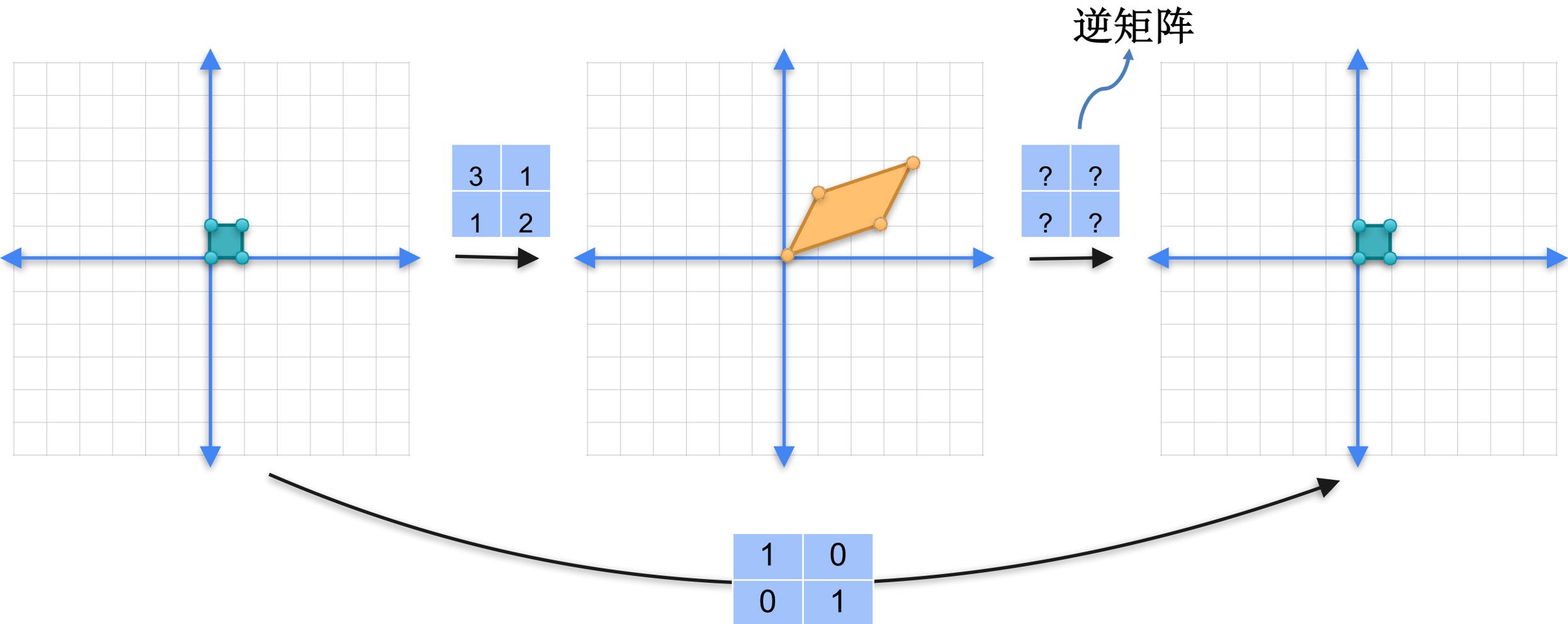
4. 逆矩阵

逆矩阵



4. 逆矩阵

逆矩阵



4. 逆矩阵

逆矩阵

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

逆矩阵

4.逆矩阵

逆矩阵

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

逆矩阵

如果存在另一个方阵 B ，使得 $AB = I_n$ 成立，则方阵 A 称为可逆的。这时候可以证明也 $BA = I_n$ 成立，可将矩阵 B 称为 A 的逆矩阵。一个矩阵 A 的逆矩阵如果存在的话，就是唯一的，通常记作 A^{-1}

4.逆矩阵

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详细计算方法可以参考Strang G. Introduction to linear algebra[M]. Wellesley-Cambridge Press, 2022.

- 01 向量及其属性
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5. 机器学习模型实例



用矩阵重写西瓜分类模型

5. 机器学习模型实例

用矩阵重写西瓜分类模型

数据

色泽 (x_1)	根蒂 (x_2)	敲声 (x_3)	好瓜 (y)
0 (青绿)	0 (蜷缩)	0 (浊响)	0 (是)
1 (乌黑)	0 (蜷缩)	0 (浊响)	0 (是)
0 (青绿)	1 (硬挺)	1 (清脆)	1 (否)
1 (乌黑)	2 (稍蜷)	2 (沉闷)	1 (否)

5. 机器学习模型实例

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样本

5. 机器学习模型实例

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样本

特征

5. 机器学习模型实例

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标签

5. 机器学习模型实例

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特征

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样本

模型

$$y = ax_1 + bx_2 + cx_3 + d$$

5. 机器学习模型实例

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模型

$$y = ax_1 + bx_2 + cx_3 + d$$

\hat{y} 是预测值

代入 x_1, x_2, x_3 $\left\{ \begin{array}{l} \text{若 } y \leq 0, \hat{y} = 0 \\ \text{若 } y > 0, \hat{y} = 1 \end{array} \right.$

用矩阵重写西瓜分类模型

数据矩阵

x_1	x_2	x_3	y
0	0	0	0
1	0	0	0
0	1	1	1
1	2	2	1

模型

$$y = ax_1 + bx_2 + cx_3 + d$$

\hat{y} 是预测值

代入 x_1, x_2, x_3 $\left\{ \begin{array}{l} \text{若 } y \leq 0, \hat{y} = 0 \\ \text{若 } y > 0, \hat{y} = 1 \end{array} \right.$

用矩阵重写西瓜分类模型

扩展数据矩阵

x_1	x_2	x_3	bias	y
0	0	0	1	0
1	0	0	1	0
0	1	1	1	1
1	2	2	1	1

模型

$$y = ax_1 + bx_2 + cx_3 + d$$

\hat{y} 是预测值

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5.机器学习模型实例

用矩阵重写西瓜分类模型

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5. 机器学习模型实例

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1	0	0	1	0
0	1	1	1	1
1	2	2	1	1

模型

$$y = ax_1 + bx_2 + cx_3 + d$$

$$y = \begin{matrix} x_1 & x_2 & x_3 & 1 \end{matrix} \begin{matrix} a \\ b \\ c \\ d \end{matrix}$$

5. 机器学习模型实例

用矩阵重写西瓜分类模型

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1	2	2	1	1

模型

$$y = ax_1 + bx_2 + cx_3 + d$$

$$y = \underbrace{\begin{bmatrix} x_1 & x_2 & x_3 & 1 \end{bmatrix}}_{\mathbf{x}} \cdot \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{\mathbf{w}^T}$$

5. 机器学习模型实例

用矩阵重写西瓜分类模型

扩展数据矩阵

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1	0	0	1	0
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1	2	2	1	1

模型

$$y = ax_1 + bx_2 + cx_3 + d$$

$$y = \underbrace{\begin{bmatrix} x_1 & x_2 & x_3 & 1 \end{bmatrix}}_{\mathbf{x}} \underbrace{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}_{\mathbf{w}^T}$$

$$y = \mathbf{xw}^T$$

延伸阅读（非必需）

- 吴恩达《机器学习数学基础（线性代数/微积分）》：

<https://www.coursera.org/specializations/mathematics-for-machine-learning-and-data-science>

- 线性代数的本质（Essense of Linear Algebra）系列

<https://www.bilibili.com/video/BV1Ys411k7yQ>



中国科学技术大学

University of Science and Technology of China

谢谢!

Slides are inspired by DeepLearning.AI